Master equation for collective spontaneous emission with quantized atomic motion

Phys. Rev. A **93**, 022124 (2016)

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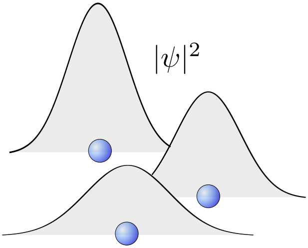
DPG Spring Meeting - 2 March 2016

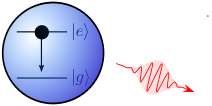
Motivation

Quantization of the atomic motion	$\overset{?}{\Longrightarrow}$	Internal atomic dynamics

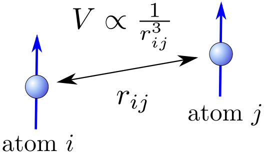


cold atoms

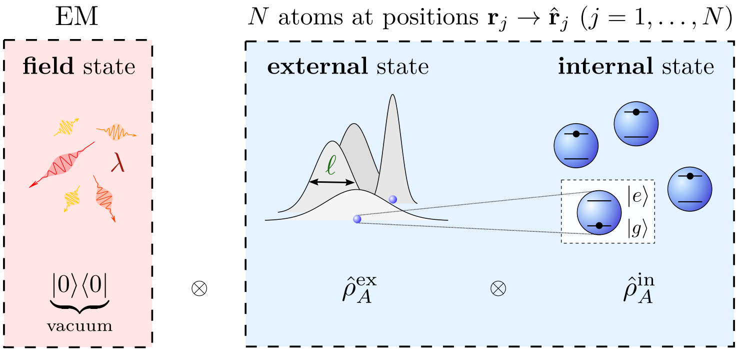
- photon emission



- virtual photon exchange



System



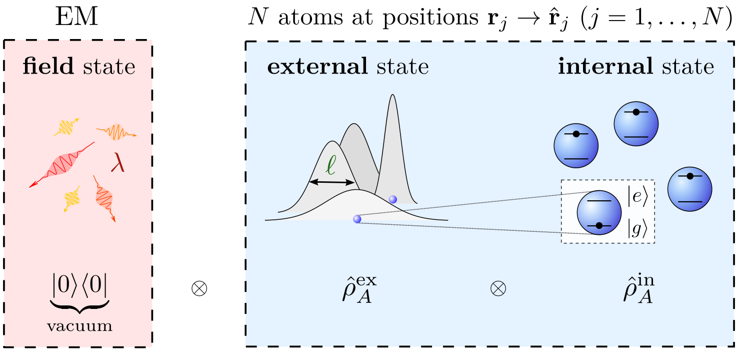
Lamb-Dicke parameter

$$\eta = 2\pi \left(\frac{\ell}{\lambda} \right)$$

ℓ : typical size of the atomic wavepacket

λ : wavelength of the emitted radiation

System



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The Lamb-Dicke parameter

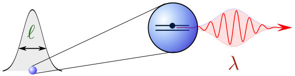
Lamb-Dicke parameter

$$\eta = 2\pi \left(\frac{\ell}{\lambda} \right)$$

$\eta \ll 1$

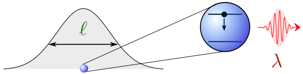
small recoil

(**Lamb-Dicke regime**)



$\eta \gg 1$

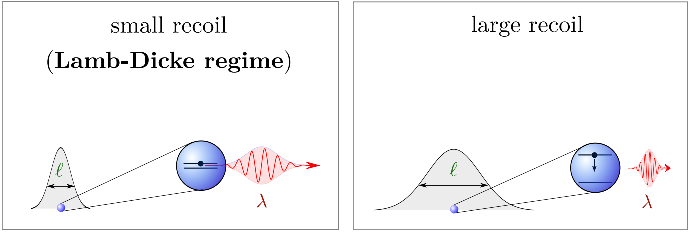
large recoil



Description of internal dynamics

Previous works

- Most of theoretical methods
  - are restricted to the Lamb-Dicke regime
    See e. g. PRL **104**, 043003 (2010); PRA **84**, 043825 (2011)
  - account either for recoil or indistinguishability, but not for both
    See e. g. PRA **51**, 3959 (1995); PRA **53**, 390 (1996); PRA **59**, 3797 (1999)



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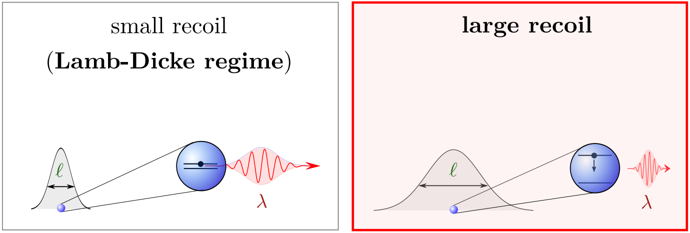
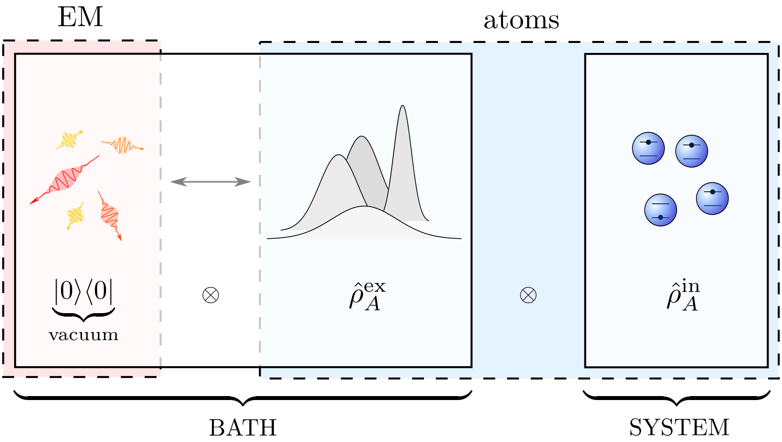
Our contribution

- **Master equation for internal degrees of freedom beyond the Lamb-Dicke regime**



Derivation of a master equation

System-bath decomposition



Derivation of a master equation

Timescales and approximations

$$\underbrace{\tau_B}_{\text{S-B correlation}} \lesssim \underbrace{\tau_S}_{\text{internal dynamics}} \ll \underbrace{\tau_R}_{\text{relaxation}} \ll \underbrace{\tau_M}_{\text{motional dynamics}}$$

- $\tau_B \ll \tau_R$: Born-Markov condition (for field degrees of freedom)
  $\Rightarrow$ no memory effects (no photon absorption)

- $\tau_S \ll \tau_R$: RWA condition
  $\Rightarrow$ only energy conserving transitions

- $\tau_R \ll \tau_M$: Born condition (for motional degrees of freedom)
  $\Rightarrow$ freezing of the intrinsic motional dynamics ($\hat{\mathbf{r}}_j(t) \longrightarrow \hat{\mathbf{r}}_j$)

Master equation

$$\frac{d\hat{\rho}_A^{\text{in}}(t)}{dt} = -\frac{i}{\hbar}\left[\hat{H}_{\text{dd}}, \hat{\rho}_A^{\text{in}}(t)\right] + \mathcal{D}\left[\hat{\rho}_A^{\text{in}}(t)\right]$$

conservative part

$$\hat{H}_{\text{dd}} = \sum_{i\neq j}^{N} \hbar\Delta_{ij}\hat{\sigma}_+^{(i)}\hat{\sigma}_-^{(j)}$$

with dipole-dipole shifts

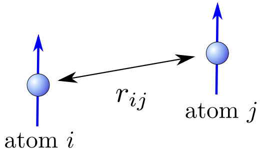
$$\Delta_{ij} = \int_{\mathbb{R}^3} \Delta^{\text{cl}}(\mathbf{r})\,\mathcal{F}_{\mathbf{r}}^{-1}\left[\mathcal{C}_{ij}^{\text{ex}}(\mathbf{k})\right] d\mathbf{r}$$

dissipative part

$$\mathcal{D}[\,\cdot\,] = \sum_{i,j}^{N} \gamma_{ij}\left(\hat{\sigma}_-^{(j)} \cdot \hat{\sigma}_+^{(i)} - \frac{1}{2}\left\{\hat{\sigma}_+^{(i)}\hat{\sigma}_-^{(j)}, \cdot\right\}\right)$$

with decay rates

$$\gamma_{ij} = \int_{\mathbb{R}^3} \gamma^{\text{cl}}(\mathbf{r})\,\mathcal{F}_{\mathbf{r}}^{-1}\left[\mathcal{C}_{ij}^{\text{ex}}(\mathbf{k})\right] d\mathbf{r}$$



Master equation

dipole-dipole shifts

$$\Delta_{ij} = \int_{\mathbb{R}^3} \Delta^{\mathrm{cl}}(\mathbf{r})\, \mathcal{F}_{\mathbf{r}}^{-1}\left[\mathcal{C}_{ij}^{\mathrm{ex}}(\mathbf{k})\right] d\mathbf{r}$$

classical
expression

decay rates

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classical
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Master equation

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decay rates

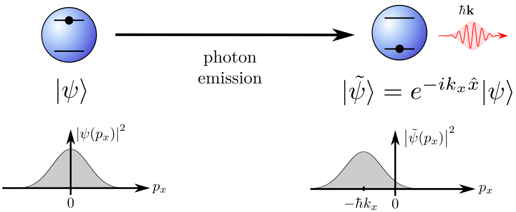
$$\gamma_{ij} = \int_{\mathbb{R}^3} \gamma^{\mathrm{cl}}(\mathbf{r})\, \mathcal{F}_{\mathbf{r}}^{-1}\left[\mathcal{C}_{ij}^{\mathrm{ex}}(\mathbf{k})\right] d\mathbf{r}$$

$$\mathcal{F}_{\mathbf{r}}^{-1}\left[\mathcal{C}_{ij}^{\mathrm{ex}}(\mathbf{k})\right] = \int_{\mathbb{R}^3} \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}}\, \mathcal{C}_{ij}^{\mathrm{ex}}(\mathbf{k})$$

motional correlation function

$$\mathcal{C}_{ij}^{\mathrm{ex}}(\mathbf{k}) = \left\langle \underline{e^{i\mathbf{k}\cdot\hat{\mathbf{r}}_{ij}}} \right\rangle_{\hat{\rho}_A^{\mathrm{ex}}}$$

recoil operator



Internal dynamics

Lamb-Dicke regime : $\eta \to 0$ (classical fixed atomic positions)

$$\Delta_{ij} \longrightarrow \Delta^{\mathrm{cl}}(\mathbf{r}_{ij}) = \frac{3\gamma_0}{4}\left[-p_{ij}\,\frac{\cos\xi_{ij}}{\xi_{ij}} + q_{ij}\left(\frac{\sin\xi_{ij}}{\xi_{ij}^2} + \frac{\cos\xi_{ij}}{\xi_{ij}^3}\right)\right]$$

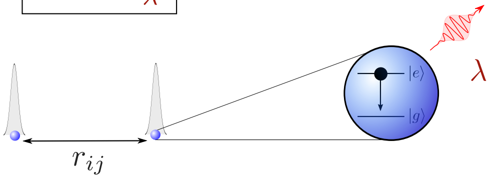
$$\gamma_{ij} \longrightarrow \gamma^{\mathrm{cl}}(\mathbf{r}_{ij}) = \frac{3\gamma_0}{2}\left[p_{ij}\,\frac{\sin\xi_{ij}}{\xi_{ij}} + q_{ij}\left(\frac{\cos\xi_{ij}}{\xi_{ij}^2} - \frac{\sin\xi_{ij}}{\xi_{ij}^3}\right)\right]$$

$$\boxed{\xi_{ij} = 2\pi\frac{r_{ij}}{\lambda}}$$

- p_{ij}, q_{ij} angular factors which depend on the dipolar transition (π or $\sigma^{\pm}$)
- γ_0 : single-atom spontaneous emission rate

Internal dynamics in Lamb-Dicke regime ($\eta \to 0$)

$$\xi_{ij} = 2\pi \frac{r_{ij}}{\lambda}$$

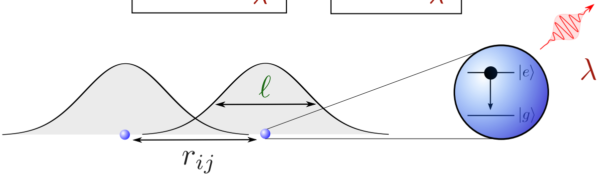


Regime	Relevant Phenomena
$1 \ll \xi_{ij}$	$\gamma_{ij} \simeq \gamma_0 \delta_{ij} \Rightarrow$ independant spontaneous emissions
$\xi_{ij} \ll 1$	$\gamma_{ij} \simeq \gamma_0 \Rightarrow$ cooperative effects

Internal dynamics beyond the Lamb-Dicke regime

$$\xi_{ij} = 2\pi \frac{r_{ij}}{\lambda}$$

$$\eta = 2\pi \frac{\ell}{\lambda}$$



Regime	Relevant Phenomena
$1 \lesssim \eta \ll \xi_{ij}$	recoil
$\eta \ll 1 \ll \xi_{ij}$	independent spontaneous emissions
$\eta \lesssim \xi_{ij} \ll 1$	cooperative effects
$1 \ll \xi_{ij} \lesssim \eta$	indistinguishability, recoil
$\xi_{ij} \ll 1 \lesssim \eta$	cooperative effects, recoil, indistinguishability
$\xi_{ij} \lesssim \eta \ll 1$	cooperative effects, indistinguishability

Internal dynamics beyond the Lamb-Dicke regime

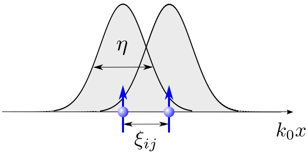
RESULTS :

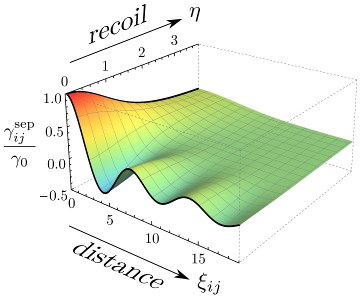
$$\left.\begin{array}{l}\gamma_{ii} = \gamma_0 \quad \forall i = 1, \ldots, N \\ |\gamma_{ij}| \leq \gamma_0 \quad \forall i, j = 1, \ldots, N\end{array}\right\} \textbf{ for any motional state}$$

- γ_{ij} and Δ_{ij} depend on the motional state $\hat{\rho}_A^{\mathrm{ex}}$ through $\mathcal{C}_{ij}^{\mathrm{ex}}(\mathbf{k}) = \langle e^{i\mathbf{k}\cdot\hat{\mathbf{r}}_{ij}} \rangle_{\hat{\rho}_A^{\mathrm{ex}}}$
- Analytical expressions of γ_{ij} found for
  - Gaussian states
  - Fock states
  - thermal states
- Numerical computations of Δ_{ij} for Gaussian states

Motional Gaussian states - decay rates γ_{ij}

Distinguishable atoms



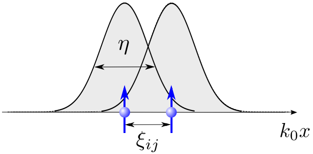


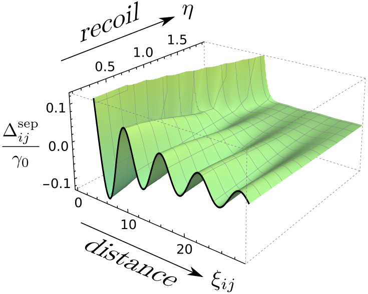
Analytical expression **valid for any** ξ_{ij} **and** η

$$\gamma_{ij}^{\rm sep}(\boldsymbol{\xi}_{ij},\eta) = \frac{3\gamma_0}{16\eta^5}\left(\frac{\sqrt{\pi}}{6}e^{-\frac{\xi_{ij}^2}{4\eta^2}}\left[16\eta^4 - q_{ij}\left(4\eta^4 + 3\xi_{ij}^2 - 6\eta^2\right)\right]\mathrm{Re}\left\{\mathrm{erf}\left(\eta + \frac{i\xi_{ij}}{2\eta}\right)\right\}\right.$$
$$\left. - q_{ij}\eta e^{-\eta_0^2}\left[2\eta^2\cos\xi_{ij} - \xi_{ij}\sin\xi_{ij}\right]\right)$$

Motional Gaussian states - dipole-dipole shifts Δ_{ij}

Distinguishable atoms



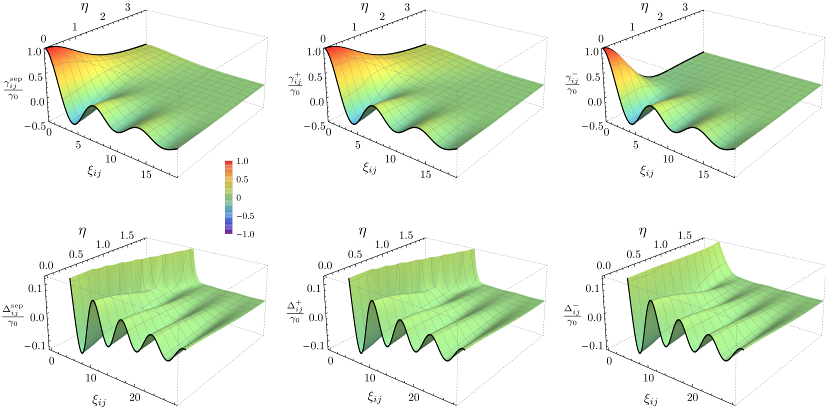


$$\Delta_{ij} = \left[\int_{-\infty}^{-\epsilon} + \int_{\epsilon}^{\infty}\right] \Delta^{\rm cl}(x)\, \mathcal{F}_x^{-1}\left[\mathcal{C}_{ij}^{\rm ex}(k_x)\right] dx$$

- Δ_{ij} diverges when $\mathcal{F}_{x=0}^{-1}\left[\mathcal{C}_{ij}^{\rm ex}(k_x)\right] \neq 0$
- A cutoff ϵ must be introduced ($\epsilon \simeq a_0 =$ Bohr radius)

Motional Gaussian states - γ_{ij} and Δ_{ij}

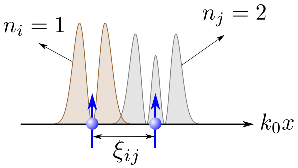
Distinguishable and indistinguishable atoms

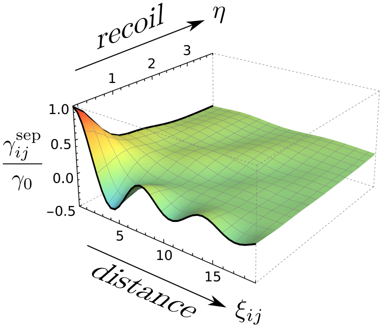


separable	**symmetric**	**antisymmetric**
motional state	motional state	motional state

Motional Fock states - decay rates γ_{ij}

Distinguishable atoms





Application

Spatial Pauli-blocking of spontaneous emission : $N = 2$ indistinguishable atoms

$t = 0$: initial state

antisymmetric internal state

$$|\psi_A^{\text{in}}\rangle = \frac{|eg\rangle - |ge\rangle}{\sqrt{2}}$$

fermions →

symmetric motional state

$$|\psi_A^{\text{ex}}\rangle = \frac{|0_1 0_2\rangle + |0_2 0_1\rangle}{\sqrt{2}}$$

bosons →

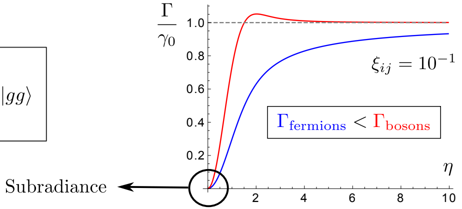
antisymmetric motional state

$$|\psi_A^{\text{ex}}\rangle = \frac{|0_1 0_2\rangle - |0_2 0_1\rangle}{\sqrt{2}}$$

$|0_j\rangle$: gaussian state at $\mathbf{r}_j$

$t > 0$

$$\frac{|eg\rangle - |ge\rangle}{\sqrt{2}} \xrightarrow{e^{-\Gamma t}} |gg\rangle$$



Conclusion and applications

Conclusion

Derivation of a general **master equation** for internal dynamics including quantized atomic motion

- General expressions of **decay rates** and **dipole-dipole shifts** valid for any motional state
- Analytical expressions obtained for relevant motional states

Applications and outlook

Quantum programmed internal dynamics through motional state engineering

- spatial Pauli-blocked spontaneous emission
- control of cooperative processes (super and subradiance)
- tuning of the dipole-dipole couplings between atoms (Rydberg atoms)

Thanks for your attention !

More info : Phys. Rev. A **93**, 022124 (2016)