Identification of Elastic Orthotropic Material Parameters Based on ESPI Measurements

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ABSTRACT

Parameter identification methods, which integrate optimization techniques and numerical methods such as the finite element method (FEM), offer an alternative tool for material characterisation. The most common approach is to determine the optimal estimates of the model parameters by minimizing a selected measure-of-fit between the responses of the system and the model. The possibility is studied of retrieving the four independent elastic engineering constants for an orthotropic medium, based on the measurement of a heterogeneous displacement field. In the present case a tensile test is performed on a perforated specimen. The responses of the system, i.e. the surface displacements are measured with an Electronic Speckle Pattern Interferometer. Strains are subsequently calculated, based on the measured displacement field. A finite element model of the perforated specimen is made. The difference between the experimental and numerical strains is minimized in a least squares sense by updating the values of the parameters. The obtained material (or model) parameters are very well in agreement with the traditionally determined ones.

INTRODUCTION

Material properties determined from measurements based on the use of test samples with a well-defined standardized geometry and loading in a laboratory may be strongly different from those of actual structural components manufactured in factory or used in engineering. In some cases, it is unacceptable to take out a test sample from a larger structure because of the disruption of the internal coherence of the material [1]. Therefore, material parameter identification methods, which integrate optimization techniques and numerical methods such as the finite element method (FEM), offer an alternative tool. The most common approach is to determine the optimal estimates of the model parameters by minimizing a selected measure-of-fit between the responses of the system and the model. The elastic material parameters corresponding to an orthotropic material can be identified from laboratory tests by classical and already well-known experiments [2]. In the present study a method is proposed for the identification of the four independent elastic in-plane material parameters (E_1, E_2, G_{12} and ν_{12}) for an orthotropic medium based on surface measurements of a specimen subjected to heterogeneous deformation.

In the present case a tensile test is performed on a perforated specimen. The responses of the system, i.e. the surface displacements are measured with an Electronic Speckle Pattern Interferometer. Strains are subsequently calculated, based on the measured displacement field, which is subjected to noise. The calculated strains are not only influenced by the experimental noise on the displacements, but also by the chosen gauge length, i.e. the length over which the strains are calculated and the location in which they are determined. A method for the calculation of strain, dealing with these problems, is presented.

A finite element model of the perforated specimen serves as numerical counterpart for the experimental set-up. The difference between the experimental and numerical strains (ε_x, ε_y and j'ai mis un autre gamma γ_{xy}) is minimized in a least squares sense by updating the values of the four independent elastic moduli. The sensitivities used to obtain the parameter updates are determined analytically, using the stresses calculated during the previously converged step. This choice allows a convergence rate that is three times faster than a routine based on finite difference sensitivities. The optimization routine used, is based on a simple Newton-Raphson algorithm. A more sophisticated routine is not necessary since the problem has a good convergence rate.

ELECTRONIC SPECKLE PATTERN INTERFEROMETER (ESPI)

The ESPI technique is based on the scattered reflection of incident light on a rough surface which in the present case is the surface of the specimen illuminated by a monochromatic laser [3]. The experimental set-up is shown in figure1.

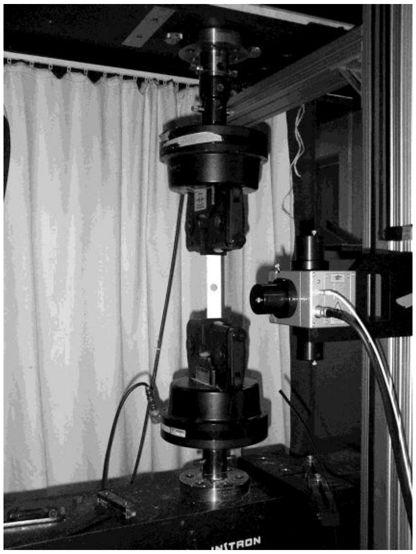

Figure 1: set-up for tension test measured by ESPI

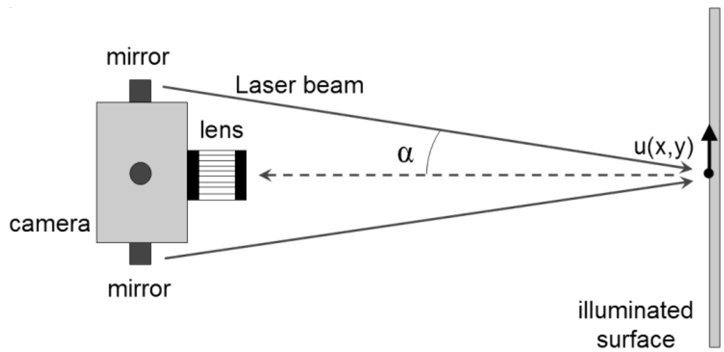

Figure 2: Principle of in-plane displacement measurement

The scattered light creates a so-called speckle pattern as shown in figure 3a and 3b. When using two identical wave's symmetrically incident on an object surface, a camera aligned perpendicular to the reflecting surface will visualize an interferential image due to the combination of the two speckle patterns. The correlation fringes are obtained by subtracting the image of the reference speckle and the speckle image obtained after the displacement or the deformation of the object (figure 3c).

In order to understand the formation of fringes, consider the intensities of the beams I_{before} before deformation and I_{after}, the intensity after deformation in each point of the image plane.

$$I_{before}(x,y) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi \tag{1}$$

$$I_{after}(x,y) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\varphi + \Delta\varphi) \tag{2}$$

I_1: scattered intensity of first beam
I_2: scattered intensity of second beam
φ: phase difference between both beams
$\Delta\varphi$: phase change caused by current displacement

The intensity at any position (x,y) in the image plane may be approximated [4] by the following expression:

$$I_{final}(x,y) = 4\sqrt{I_1 I_2}\sin\left(\varphi + \frac{1}{2}\Delta\varphi\right)\sin\left(\frac{1}{2}\Delta\varphi\right) \tag{3}$$

A high pass filter is used to eliminate the part of expression 3 containing φ, which can be considered as noise when displacements are to be measured. Hereafter, the corresponding wrapped phase map is obtained (figure 3d).
The relation between $\Delta\varphi$ and the displacement component u(x,y) is given by expression 4:

$$\Delta\varphi = \frac{4\pi}{\lambda}u(x,y)\sin\alpha \tag{4}$$

In equation (4) u(x,y) is the resolved component of the surface displacement in the x or y direction, depending on the orientation of the incident laser beams. λ is the wavelength of the used light source and α represents the angle of incidence of the laser beams as shown in figure 2.

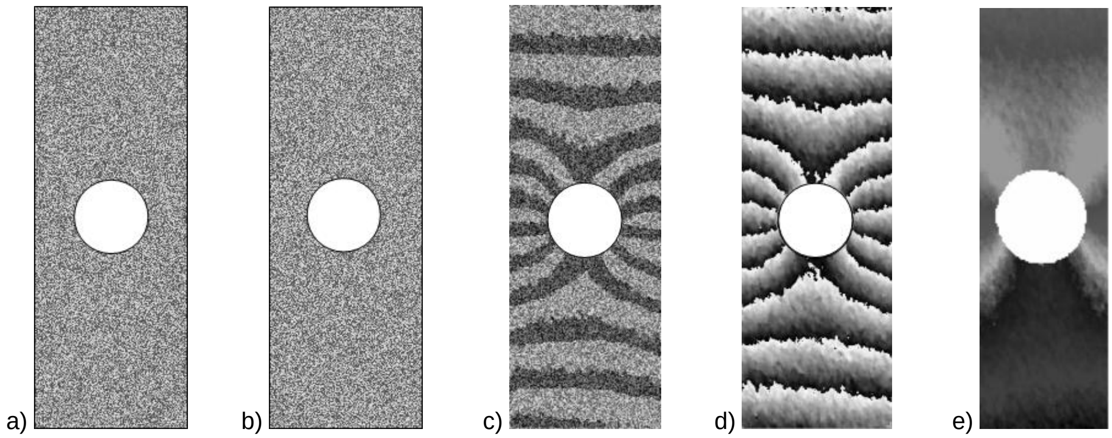

Figure 3: (a) Speckle before deformation; (b) Speckle after deformation; (c) resulting fringe pattern by subtraction; (d) wrapped phase map; (e) unwrapped phase map describing the vertical displacement field

In the current set-up a CCD camera with 752 x 582 pixels is used and the wave length of the laser diodes is 780nm. The sensitivity of the system with respect to the measured displacements lies between 0.05µm and 1µm.

STRAIN CALCULATION

In the framework of the Mixed Numerical Experimental Methods, an adequate procedure is required to determine the experimental strains and compare them with the strains based on a Finite element Analysis of the experiment. In this paragraph a method for strain calculation based on a measured displacement field is presented. It permits to deal with the corrupted displacement data and calculates the experimental strains immediately at the position of the integration points in the FE model. A direct matching of experimental and numerical strains is possible because they are calculated using the same routine. An additional advantage is that the rigid body motions, induced by the tension machine, can be omitted.

The displacement data obtained by ESPI is linked to a well structured and dense grid of data points (figure 4).

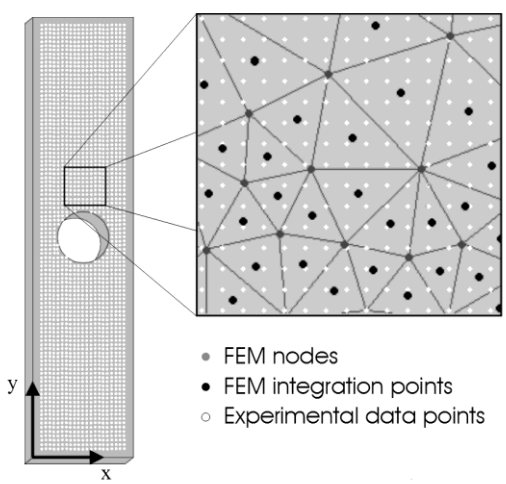

Figure 4: Comparison between experimental data and FEM mesh

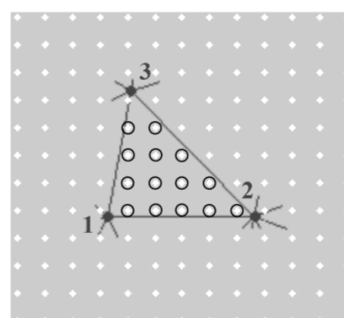

Figure 5: Triangular 3 node element in an experimental displacement field

The idea of the present method for strain calculation is based on the smoothening of the experimental displacement field with the entire FE mesh. The aim is to determine the displacement values at the nodes of the finite element mesh, based on the measured displacements. Once these nodal displacement values are determined, the strains can be calculated using the FEM-routine.

The numerical displacement values in an arbitrary point within a given element (figure 5) can be calculated using the three interpolation functions related to the element nodes:

$$u_i^{num} = \sum_{j=1}^{3} N_j(\xi_i,\eta_i)\, u_j^{node} \tag{5}$$

$$v_i^{num} = \sum_{j=1}^{3} N_j(\xi_i,\eta_i)\, v_j^{node} \tag{6}$$

u_j^{node} and v_j^{node}: horizontal and vertical displacement of node j

u_i^{num} and v_i^{num}: horizontal and vertical displacement of a point i within the element

ξ_i and η_i: reduced coordinates of point I within the element

$N_j(\xi_i,\eta_i)$: Interpolation function corresponding to node j, expressed in point i

First of all one has to determine the element in which every single data point is positioned (figure 5). Once this is performed, it is possible to calculate its reduced coordinates within the element and therefore the value of the three nodal interpolation functions. The next step is the Formulation of the smoothing of the displacement field in every element as a least squares problem. Therefore the following expression has to be minimised per every element with respect to the nodal displacement components:

$$C = \sum_{i=1}^{n} \left(u_i^{num} - u_i^{exp}\right)^2 + \sum_{i=1}^{n} \left(v_i^{num} - v_i^{exp}\right)^2 \tag{7}$$

u_i^{exp} and v_i^{exp}: experimental displacement components of data point i

n: total number of data points within an element

In function of the nodal displacement components and interpolation functions expression (7) becomes

$$C = \sum_{i=1}^{n} (N_1(\xi_i,\eta_i)u_1^{node} + N_2(\xi_i,\eta_i)u_2^{node} + N_3(\xi_i,\eta_i)u_3^{node} - u_i^{exp})^2 + \sum_{i=1}^{n} (N_1(\xi_i,\eta_i)v_1^{node} + N_2(\xi_i,\eta_i)v_2^{node} + N_3(\xi_i,\eta_i)v_3^{node} - v_i^{exp})^2 \tag{8}$$

The necessary condition for C to attain a minimum in u_i is expressed by:

$$\frac{\partial C}{\partial u_1^{node}} = 2\sum_{i=1}^{n} N_1(\xi_i,\eta_i)\left[N_1(\xi_i,\eta_i)u_1^{node} + N_2(\xi_i,\eta_i)u_2^{node} + N_3(\xi_i,\eta_i)u_3^{node} - u_i^{exp}\right] = 0 \tag{9}$$

Equation (9) has to be formulated for every nodal displacement component and every element in the finite element mesh. The solution of the global system of equations is nothing else than the fitted experimental nodal displacement components. Subsequently, the strains are calculated using the FEM routine

INVERSE MODELLING AND SENSITIVITY CALCULATION

A direct problem is the classical problem where a given experiment is simulated in order to obtain the final geometry of the specimen, the stresses and the strains.
Inverse problems on the other hand are concerned with the determination of the unknown state of a mechanical system considered as a black box , using information gathered from the response to stimuli on the system [6]. The inverse problem is a problem where certain input data of the direct problem is deduced from the comparison between the experimental results and the numerical FE-simulation of that same problem. Not only the boundary information is used, but relevant information coming from local or full-field surface measurements is also integrated in the evaluation of the behavior of a given material.

The inverse method described here can actually be narrowed to parameter identification, as the only item of interest to this study is actually the determination of the physically relevant constitutive parameters. The values of the material parameters cannot be derived immediately from the experiment. A numerical analysis is necessary to simulate the actual experiment. However, this requires that the material parameters are known. The identification problem can then be formulated as an optimization problem where the function to be minimized is some error function that expresses the difference between numerical simulation results and experimental data. In the present case the strains are used as output data. This choice offers two advantages: the exact knowledge of the boundary conditions is less important and the sensitivity calculation as a part of the optimization procedure is much faster. Figure 6 represents the flow-chart of the present inverse modeling problem.

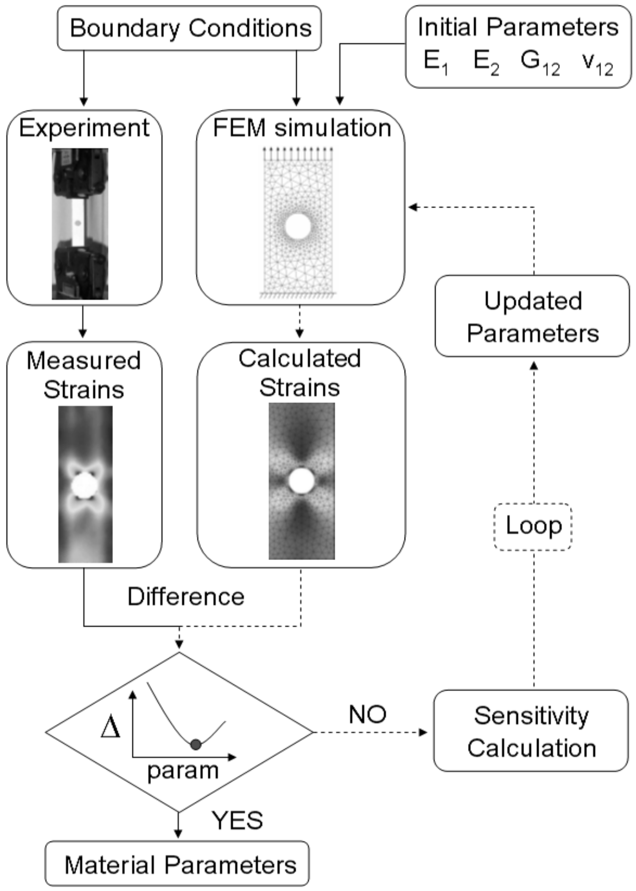

Figure 6: Flow-chart of the parameter identification problem

The optimization of the parameters is done by a Gauss-Newton method, which can be written as follows:

$$\underline{\Delta p} = \left(\underline{\underline{S}}^t \underline{\underline{S}}\right)^{-1} \underline{\underline{S}}^t \left(\underline{\varepsilon}^{exp} - \underline{\varepsilon}^{num}(\underline{p}^k)\right) \tag{10}$$

$\underline{\Delta p}$:	column vector of the parameter updates of E_1, E_2, G_{12} and v_{12}
$\underline{\varepsilon}^{exp}$:	column vector of the experimental strains
$\underline{\varepsilon}^{num}(\underline{p}^k)$:	column vector of the finite element strains as a function of the parameters
$\underline{p}^k$:	the four parameters at iteration step k

$$\underline{\underline{S}} = \begin{bmatrix} \frac{\partial \varepsilon_x}{\partial E_1} & \frac{\partial \varepsilon_y}{\partial E_1} & \frac{\partial \gamma_{xy}}{\partial E_1} \\ \frac{\partial \varepsilon_x}{\partial E_2} & \frac{\partial \varepsilon_y}{\partial E_2} & \frac{\partial \gamma_{xy}}{\partial E_2} \\ \frac{\partial \varepsilon_x}{\partial G_{12}} & \frac{\partial \varepsilon_y}{\partial G_{12}} & \frac{\partial \gamma_{xy}}{\partial G_{12}} \\ \frac{\partial \varepsilon_x}{\partial v_{12}} & \frac{\partial \varepsilon_y}{\partial v_{12}} & \frac{\partial \gamma_{xy}}{\partial v_{12}} \end{bmatrix} : \quad \text{sensitivity matrix} \tag{11}$$

The components of this sensitivity matrix can be derived analytically from the constitutive relation between stress and strain, which is given by expression (12) in the case of a plane stress problem. The stresses that are used in the calculation of the derivatives are taken from the previously converged simulation. The selection of the strain field as the experimental output in the inverse method, allows to perform material identification without having to access into the source code that is used for the finite element simulation.

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{12}$$

NUMERICAL TESTS AND EXPERIMENTAL VALIDATION

In order to determine the elastic properties (E and v) of an isotropic material, tests have proven that a tensile test in a single direction of a perforated specimen contains enough information [7]. For orthotropic materials, this is not longer true. When an orthotropic material is loaded in one of its principal directions, the characteristics in the perpendicular direction can not be revealed. For that reason two tension tests are performed parallel to both principal material axes. The measured strains are combined into one single vector when used in the optimization routine. In order to test the method, a virtual experiment is set up. The reference values for the four parameters used for the direct problem and the starting values for the optimization routine are shown in table1.

Table 1: Reference and starting values for numerical validation

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
Reference values	210	190	70	0.30
Starting values	300	100	20	0.15

The results of the calculations are shown in figure 7.

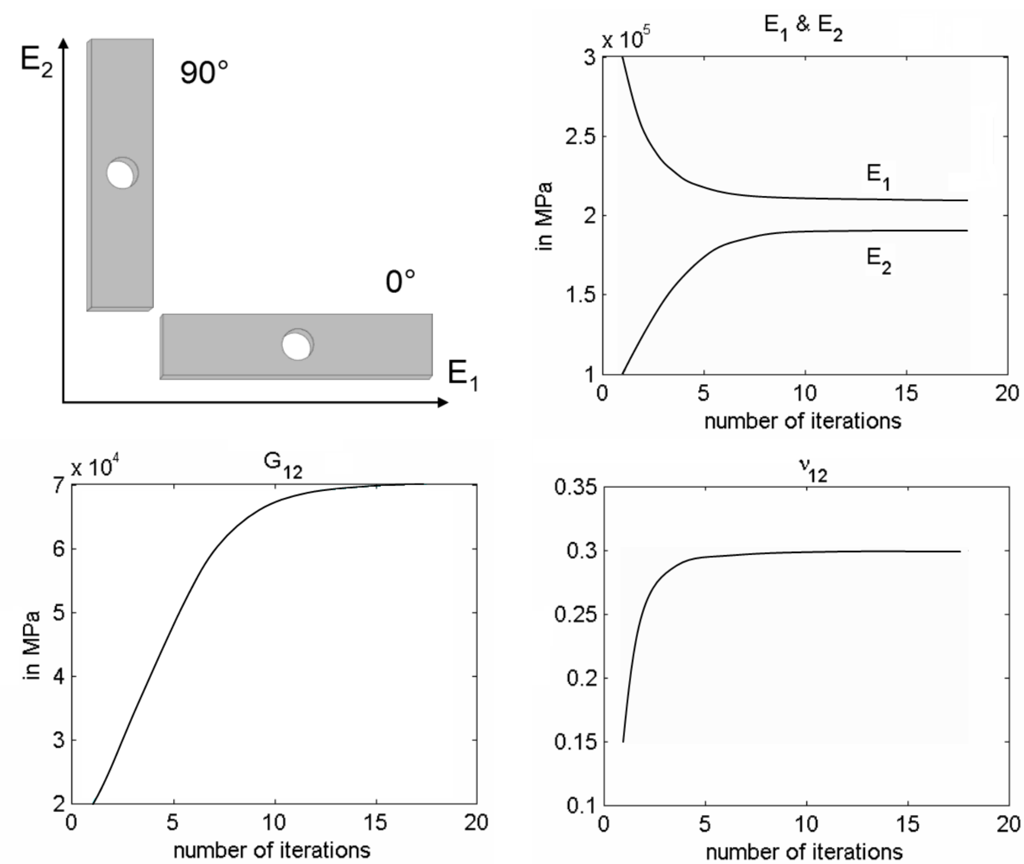

Figure 7: Specimen principal directions (top left) and results of the numerical validation (top right and bottom)

Figure 7 shows that after 15 iterations the material parameters are exactly identical to the reference values. Different starting values were chosen. They all lead to the same result needing more or less iteration steps depending on the difference between the starting values and the reference values.

For the actual experiments a Ti6Al4V alloy is used. Two specimens are cut out of a hot rolled plate, one in the rolling direction (0°) and one in the transverse direction (90°). The dimensions of the specimen are the following: 180mm x 25mm x 1mm, with a hole in the center of 10mm diameter. The length of the specimen is such that the heterogeneous force distribution at the clamps has no influence on the strain distribution around the hole. White removable chalk-spray is used on the surface of the specimen in order to avoid abundant light reflection. The test apparatus is a load controlled Instron 4505 tensile bench. The specimen is analyzed in the mechanical elastic region. To make sure that the deformation remains below the plastic limit, pictures are taken at various load levels for both specimens. The parameters are identified based on the measured strain field corresponding to five different load steps. The elastic limit is exceeded at a given load step, when a sudden drop in the obtained parameters is observed. In that case the corresponding load steps are removed from the measurement series.

In the present study, five different load steps in both principal material axes are used for the identification of the four independent orthotropic material parameters. All load steps yielded a deformation below the elastic limit. The mean value and the standard deviation for E_1, E_2, G_{12} and ν_{12} are calculated based on the corresponding measured strain fields. The results are summarized in table 2 and compared to the values obtained with the traditional tests.

Table 2: Results obtained based on five different load steps and traditional tests for the Ti6Al4V specimens

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
Starting values	140	120	50	0.25
Results by traditional tests	112.16	109.76	39.76	0.385
Results by identification	110.2 ± 3.5	111.6 ± 3.8	41.6 ± 2.7	0.36 ± 0.02

The results show that the present technique is able to identify all the parameters simultaneously with acceptable accuracy. However, its precision does not allow the detection of the orthotropic nature of the Ti6Al4V alloy, due to the fact that the behavior in both principal material directions presents only a small difference. Too small for the identification technique to be discovered. It is suspected that for materials, presenting a distinctive orthotropic behavior like fiber-reinforced composite materials, the technique is accurate enough to detect the difference between E_1 and E_2.

CONCLUSION

An inverse method has been proposed to determine the elastic parameters (E_1, E_2, G_{12} and ν_{12}) of an orthotropic material. The method is based on a Finite element calculated strain field of a perforated specimen under tension and the measured displacement field by an Electronic Speckle Pattern Interferometer. A method for strain calculation and mapping based on the measured displacement fields is presented. A least-squares formulation of the difference between the experimental and the numerical strains is used along with a Gauss-Newton algorithm in the optimization process. Virtual experiments, in which the material parameters are known, are used to analyze the influence of the starting values and the noise in the measured displacement field on the obtained parameter estimates. A tension test on a perforated Ti6Al4V specimen was performed and measured by ESPI. The obtained material parameters were in agreement with the values obtained by traditional testing methods.

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REFERENCES

[1] Meuwissen, M., An Inverse Method for the Mechanical Characterization of Metals, Ph.D. thesis, Eindhoven University of Technology, 1998
[2] ASTM, section 3 Metals Test Methods and Analytical Procedures, Volume 03.01, 1997
[3] Gasvik, K.J., Optical metrology, John Wiley & Sons Ltd., New York, 1995
[4] Moore, A.J., Lucas, M. and Tyrer J.R., An electronic speckle pattern interferometer for two-dimensional strain measurement, Measurement Science and Technology, 1740-1747, 1996
[6] Bui, H.D., Inverse Problems in the Mechanics of Materials: An Introduction, CRC Press, Inc., Florida, 1994
[7] Lecompte, D., Sol, H. and Vantomme, J., Identification of Elastic Isotropic Material Parameters Based on ESPI Measurements, Optical Measurement Techniques for Structures and Systems, Optimess workshop, 2004