Supplementary Material for Paper: Probabilistic Reliability Management Approach and Criteria for Power System Real-time Operation

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Nomenclature

The main mathematical symbols used in this document are defined as follows. Others may be defined as needed within the text.

Indices

c Index of contingencies.

d Index of demands.

g Index of generating units.

ℓ Index of transmission elements (i.e. of lines and phase shifting transformers).

ℓ_p Index of phase shifting transformers.

n Index of nodes.

Sets

$\mathcal{C}_c \subseteq \mathcal{C}$ Subset of contingencies covered by the SCOPF formulation, including the pseudo-contingency of no outage.

$\mathcal{D}$ Set of demands.

$\mathcal{D}_n \subseteq \mathcal{D}$ Subset of demands connected at node n.

$\mathcal{G}$ Set of generating units.

$\mathcal{G}_n \subseteq \mathcal{G}$ Subset of generating units connected at node n.

$\mathcal{L}$ Set of transmission elements (i.e. of lines and phase shifting transformers).

$\mathcal{L}_p \subseteq \mathcal{L}$ Subset of phase shifting transformers.

$\mathcal{N}$ Set of nodes.

Parameters

w_0 Prevailing weather (and market) conditions.

c_g Marginal generation cost of generating unit g.

$P_g^M(w_0)$ Dispatch of generating unit g as per the market clearing.

P_g^{max} Capacity of generating unit g.

P_g^{min} Minimum stable generation of unit g.

ΔP_g^- Ramp-down limit of generating unit g in corrective mode.

ΔP_g^+ Ramp-up limit of generating unit g in corrective mode.

$P_d(w_0)$ Load active power of demand d.

$v_d(w_0)$ Value of lost load of demand d.

f_ℓ^{max} Long-term thermal rating of transmission element ℓ.

r_ℓ Ratio of the short-term thermal rating to the long-term thermal rating of transmission element ℓ ($r_\ell \geq 1$).

X_ℓ Reactance of transmission element ℓ.

$\beta_{n,\ell}$ Element of the flow incidence matrix, taking a value of one if node n is the sending node of element ℓ, a value of minus one if node n is the receiving node of element ℓ, and a zero value otherwise.

$\pi_c(w_0)$ Probability of occurrence of contingency c.

$a_{\ell,c}$ Binary parameter taking a zero value if element $\{\ell \in \mathcal{L}\}$ is unavailable under contingency $c \geq 1$.

$\Delta\theta_{\ell_p}^{\min}$ Lower limit on angle of phase shifting transformer ℓ_p.

$\Delta\theta_{\ell_p}^{\max}$ Upper limit on angle of phase shifting transformer ℓ_p.

$\pi_g^{fail}(w_0)$ Probability of failure of an elementary corrective control operation of generating unit g.

$\pi_{\ell_p}^{fail}(w_0)$ Probability of failure of an elementary corrective control operation of phase shifting transformer ℓ_p.

ε_{RT} Tolerance level of the reliability target.

M A large constant.

Continuous Variables

$P_{g,0}^+$ Preventive ramp-up of generating unit g.

$P_{g,0}^-$ Preventive ramp-down of generating unit g.

$P_{g,c}^+$ Corrective ramp-up of generating unit g following contingency c.

$P_{g,c}^-$ Corrective ramp-down of generating unit g following contingency c.

$\Delta\theta_{\ell_p,c}$ Corrective phase shift of transformer ℓ_p following contingency c.

$f_{\ell,0}$ Power flowing through transmission element ℓ under the pre-contingency state.

$f_{\ell,c}^{ST}$ Power flowing through transmission element ℓ following contingency c and prior to the application of corrective control.

$f_{\ell,c}$ Power flowing through transmission element ℓ following contingency c and the successful application of corrective control.

$\theta_{n,0}$ Voltage angle at node n under the pre-contingency state.

$\theta_{n,c}^{ST}$ Voltage angle at node n following contingency c and prior to the application of corrective control.

$\theta_{n,c}$ Voltage angle at node n following contingency c and the succesful application of corrective control.

Nb: All continuous variables are non-negative with the exception of the transmission element flow variables, and angle variables.

Binary Variables

$\gamma_{g,c}$ Auxiliary variable, taking a value of 1 if the scheduled corrective action concerning contingency c involves an elementary operation of generator g.

$\lambda_{\ell_p,c}$ Auxiliary variable, taking a value of 1 if the scheduled corrective action concerning contingency c involves an elementary operation of a phase shifting transformer ℓ_p.

ζ_c Auxiliary variable, taking a value of 1 if, upon occurrence of contingency c, operational limits would be violated in the short-term post-contingency state prior to the application of corrective actions.

1 Introduction

This document provides supplementary material regarding the *Security Constrained Optimal Power Flow* (SCOPF) formulation implemented for the purposes of the analysis presented in section IV of [1]. More specifically, section 2 recalls the compact statement of the probabilistic RMAC proposed in [1] and presents an overview of this implementation. Section 3 presents the corresponding detailed mathematical formulation while section 4 concludes by presenting additional data on the RTS-96 [2] system used in the case studies.

2 Mathematical modeling overview

Let us recall from section II.C of [1] the following compact statement of the proposed RT-RMAC.

$$\min_{u\in\mathcal{U}(x_0)} \left\{ CP\,(x_0,u_0,w_0) + \sum_{c\in\mathcal{C}_c} \pi_c(w_0)\cdot \Big[CC\,(x_c,u_c,w_0) + \sum_{b\in\mathcal{B}} \pi_b(w_0)\cdot S(x_c^b(u_0,u_c),w_0) \Big] \right\} \tag{6}$$

$$\text{s.t. } \mathbb{P}\left\{ (x_0,x_c,x_c^b)\in X_a | (c,b)\in \mathcal{C}_c\times\mathcal{B} \right\} \geq (1-\varepsilon_{RT}) \tag{7}$$

while

$$R_{\mathcal{C}\setminus\mathcal{C}_c}(u) \leq \Delta E_{RT}. \tag{8}$$

where,

x_0	Pre-contingency operating state.
x_c	Short-term steady-state reached after the occurrence of any contingency $c\in\mathcal{C}_c$ and before the application of the respective corrective control action (if any).
x_c^b	Steady-state reached after the occurrence of any contingency $c\in\mathcal{C}_c$ and the realization of corrective control behavior $b\in\mathcal{B}$.
X_a	Set of acceptable trajectories, *i.e.* implying an acceptable level of service to the system end-users.
$u\in\mathcal{U}(x_0)$	A joint preventive/corrective control strategy consisting of a preventive control decision $u_0\in\mathcal{U}_0(x_0)$ and a set of corrective control decisions $\{u_c\in\mathcal{U}_c(x_0,u_0,c),\forall c\in\mathcal{C}\}$.
$CP\,(x_0,u_0,w_0)$	Preventive control cost function.
$CC\,(x_c,u_c,w_0)$	Corrective control cost function.
$S(x_c^b(u_0,u_c),w_0)$	Severity function.
$R_{\mathcal{C}\setminus\mathcal{C}_c}(u)$	Residual risk associated to the contingencies $c\in\mathcal{C}\setminus\mathcal{C}_c$ for reliability management strategy $u\in\mathcal{U}(x_0)$.
ΔE_{RT}	Accuracy threshold.

We focus here on the mathematical model used for the implementation of the SCOPF problem (6-7) in the case studies presented in section IV of [1]. Prior to presenting the detailed mathematical formulation and relevant data, we briefly introduce the following approximations used in our implementation:

a.) We employ the DC power flow approximation [3] to express all network constraints.

b.) We model an "acceptable system trajectory", denoted as $(x_0, x_c, x_c^b) \in X_a$ in (7), as the existence of a steady-state equilibrium *with no loss of load* throughout: (i) the pre-contingency operation, (ii) the short-term interval after the occurrence of any contingency $c \in \mathcal{C}_c$ and before the application of the respective corrective control actions, and, (iii) the final state reached by following the application of corrective control actions while taking into account their possible failures. It follows that any case under which the mathematical constraints expressing the system operational limits would be violated for at least one of these three regimes is regarded in our implementation as an "unaccaptable system trajectory".

c.) To model the possible failure of corrective control actions, we assume that any elementary control operation may either work or fail. Moreover, we also assume that the failure of any such elementary operation also implies the complete failure of the respective action. Exploiting the fact that, in a cost-minimization setting corrective actions of non-negative cost would only be employed if needed to enforce a post-contingency operational limit, we deduce that any failure of corrective control results in an "unacceptable system trajectory".

d.) Rather than modeling, within the limited scope of the static DC power flow approximation, the potential evolution of the system following any such violation of operational limits, we employ the pessimistic, yet conservative approximation that the violation of the aforementioned limits results in the loss of the total system load. Accordingly, we consider that an "unacceptable system trajectory" implies a severity value equal to the scalar product of the value of lost load and load demand, *i.e.* the maximum socio-economic cost of service interruption for the time period in question.

3 Detailed mathematical formulation

This section presents the detailed mathematical formulation of the proposed RT-RMAC, as approximated in the framework of a mixed integer linear programming (MILP) problem. In order to gradually build our implementation, we begin in subsection 3.1 with the model of the pre-contingency operation of the system. Subsection 3.2 refers to the short-term post-contingency state before the application of any corrective action, while subsection 3.3 concerns the application of corrective control. We derive the chance constraint expressing the reliability target (7) in subsection 3.4 while subsection 3.5 completes the mathematical statement of the problem in question by presenting the analytical statement of objective function (6).

In the following sections, all numbered equations from (9) to (38) are composing the joint SCOPF problem formulation; auxiliary equations used for explanatory purposes are therefore not numbered.

3.1 Preventive operation

The pre-contingency operation is mathematically modeled in our implementation by means of constraints (9-14). Bounds on preventive active power generation rescheduling are shown in

(9-10) while (11 - 14) are standard expressions of the DC power flow approximation. The nodal power balance is expressed as in (11), while (12) expresses the power flowing through each transmission element of the network. Finally (13 - 14) enforce the long-term thermal ratings of transmission elements.

$$\forall g \in \mathcal{G}:$$

$$0 \leq P^{-}_{g,0} \leq P^{M}_{g}(w_0) - P^{\min}_{g} \tag{9}$$

$$0 \leq P^{+}_{g,0} \leq P^{max}_{g} - P^{M}_{g}(w_0). \tag{10}$$

$$\forall n \in \mathcal{N}:$$

$$\sum_{g\in\mathcal{G}_n} \left[P^{M}_{g}(w_0) + \left(P^{+}_{g,0} - P^{-}_{g,0}\right)\right] - \sum_{\ell\in\mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,0} = \sum_{d\in\mathcal{D}_n} P_d(w_0). \tag{11}$$

$$\forall \ell \in \mathcal{L}:$$

$$f_{\ell,0} - \frac{1}{X_\ell} \sum_{n\in\mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,0} = 0 \tag{12}$$

$$f_{\ell,0} \leq f^{\max}_{\ell} \tag{13}$$

$$- f_{\ell,0} \leq f^{\max}_{\ell}. \tag{14}$$

3.2 Short-term post-contingency states

We express the constraints corresponding to the viability of the short-term steady-state reached after the occurrence of any contingency $c \in \mathcal{C}_c$ and before the application of the respective corrective control action (if any) as shown in (15 - 20).

Prior to explaining these constraints, let us recall that in our formulation short-term post-contingency steady-states are not required to be viable for each and every contingency, but rather that the reliability target (7) needs to be satisfied globally over the whole set of covered contingencies $\mathcal{C}_c$. Hence, the formulation allows to relax short-term post-contingency constraints for some contingencies provided that this is at the same time compliant with (7) and beneficial from the point of view of the objective function. In order to enable this feature, we use in our implementation, for each $c \in \mathcal{C}_c$, an auxiliary binary variable (ζ_c) taking a value of one if for that contingency the respective short-term operational limits are chosen to be relaxed (and such a relaxation will be accounted in (7) and in the cost function (6) as implying indeed an "unacceptable system trajectory" for these contingencies).

Therefore, inequalities (16 - 17) jointly enforce the nodal power balance constraint only for the value of $\zeta_c = 0$. Likewise, the short-term thermal ratings of transmission elements (19 - 20) are only restrictive when $\zeta_c = 0$.

$$\forall c \in \mathcal{C}_c:$$

$$\zeta_c \in \{0; 1\}. \tag{15}$$

$$\forall c \in \mathcal{C}_c, \forall n \in \mathcal{N}:$$

$$\sum_{g\in\mathcal{G}_n} \left[P^{M}_{g}(w_0) + \left(P^{+}_{g,0} - P^{-}_{g,0}\right)\right] - \sum_{\ell\in\mathcal{L}} \beta_{n,\ell} \cdot f^{ST}_{\ell,c} - \zeta_c \cdot M \leq \sum_{d\in\mathcal{D}_n} P_d(w_0) \tag{16}$$

$$- \sum_{g\in\mathcal{G}_n} \left[P^{M}_{g}(w_0) + \left(P^{+}_{g,0} - P^{-}_{g,0}\right)\right] + \sum_{\ell\in\mathcal{L}} \beta_{n,\ell} \cdot f^{ST}_{\ell,c} - \zeta_c \cdot M \leq - \sum_{d\in\mathcal{D}_n} P_d(w_0). \tag{17}$$

$\forall c \in \mathcal{C}_c, \forall \ell \in \mathcal{L}:$

$$f_{\ell,c}^{ST} - a_{\ell,c} \cdot \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,c}^{ST} = 0 \tag{18}$$

$$f_{\ell,c}^{ST} - a_{\ell,c} \cdot (r_\ell \cdot f_\ell^{\max} + \zeta_c \cdot M) \leq 0 \tag{19}$$

$$- f_{\ell,c}^{ST} - a_{\ell,c} \cdot (r_\ell \cdot f_\ell^{\max} + \zeta_c \cdot M) \leq 0. \tag{20}$$

3.3 Corrective control and its possible failure

We consider the application of corrective actions, only under the condition that the short-term post-contingency state meets the respective limits. To mathematically express such a requirement, in our implementation we use the set of constraints (21-30). Firstly, notice that from (15, 21 - 22) and (15, 27 - 28) a value of $\zeta_c = 1$ would restrict auxiliary binary variables $(\gamma_{g,c}\ \forall g \in \mathcal{G})$ and $\left(\lambda_{\ell_p,c}\ \forall \ell_p \in \mathcal{L}_p\right)$ respectively to zero. The corrective control of generating units is restricted by (23-26) within applicable limits and only under the condition that $\gamma_{g,c} = 1$. Likewise, (29-30) restrict the angle of phase shifting transformers within the respective minimum and maximum limits while $\lambda_{\ell_p,c} = 1$, conversely this angle is also bounded to zero, thus restricting the corrective control of phase-shifting transformers.

$\forall c \in \mathcal{C}_c, \forall g \in \mathcal{G}:$

$$\gamma_{g,c} \in \{0; 1\} \tag{21}$$

$$\gamma_{g,c} + \zeta_c \leq 1 \tag{22}$$

$$P_{g,0}^{-} + P_{g,c}^{-} \leq P_g^M(w_0) - P_g^{\min} \tag{23}$$

$$P_{g,0}^{+} + P_{g,c}^{+} \leq P_g^{\max} - P_g^M(w_0) \tag{24}$$

$$0 \leq P_{g,c}^{+} \leq \gamma_{g,c} \cdot \Delta P_g^{+} \tag{25}$$

$$0 \leq P_{g,c}^{-} \leq \gamma_{g,c} \cdot \Delta P_g^{-}. \tag{26}$$

$\forall c \in \mathcal{C}_c, \forall \ell_p \in \mathcal{L}_p:$

$$\lambda_{\ell_p,c} \in \{0; 1\} \tag{27}$$

$$\lambda_{\ell_p,c} + \zeta_c \leq 1 \tag{28}$$

$$- \Delta\theta_{\ell_p,c} + \lambda_{\ell_p,c} \cdot \Delta\theta_{\ell_p}^{\min} \leq 0 \tag{29}$$

$$\Delta\theta_{\ell_p,c} - \lambda_{\ell_p,c} \cdot \Delta\theta_{\ell_p}^{\max} \leq 0. \tag{30}$$

Similarly, to express the equilibrium state following the *successful* application of corrective control, we use the constraints of the DC power flow approximation only under the condition that the value of auxiliary binary variable ζ_c is equal to zero. Indeed, if this value is non-zero, there is no feasible transition from the pre-contingency operating state to a state following the successful application of corrective control. In mathematical terms, the corresponding set of constraints is (31-36).

$\forall c \in \mathcal{C}_c, \forall n \in \mathcal{N}:$

$$\sum_{g \in \mathcal{G}_n} \left[P_g^M(w_0) + \left(P_{g,0}^{+} - P_{g,0}^{-}\right) + \left(P_{g,c}^{+} - P_{g,c}^{-}\right)\right] - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c} - \zeta_c \cdot M \leq \sum_{d \in \mathcal{D}_n} P_d(w_0) \tag{31}$$

$$- \sum_{g \in \mathcal{G}_n} \left[P_g^M(w_0) + \left(P_{g,0}^{+} - P_{g,0}^{-}\right) + \left(P_{g,c}^{+} - P_{g,c}^{-}\right)\right] + \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c} - \zeta_c \cdot M \leq - \sum_{d \in \mathcal{D}_n} P_d(w_0). \tag{32}$$

$\forall c \in \mathcal{C}_c, \forall \ell_p \in \mathcal{L}_p :$

$$f_{\ell_p,c} - a_{\ell_p,c} \cdot \frac{1}{X_{\ell_p}} \left(\sum_{n \in \mathcal{N}_n} \beta_{n,\ell_p} \cdot \theta_{n,c} + \Delta\theta_{\ell_p,c} \right) = 0. \tag{33}$$

$\forall c \in \mathcal{C}_c, \forall \ell \in \mathcal{L} \setminus \mathcal{L}_p :$

$$f_{\ell,c} - a_{\ell,c} \cdot \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,c} = 0. \tag{34}$$

$\forall c \in \mathcal{C}_c, \forall \ell \in \mathcal{L} :$

$$f_{\ell,c} - a_{\ell,c} \cdot (1 - \zeta_c) \cdot f_\ell^{\max} \leq 0 \tag{35}$$

$$- f_{\ell,c} - a_{\ell,c} \cdot (1 - \zeta_c) \cdot f_\ell^{\max} \leq 0. \tag{36}$$

Let us finally underline here that, as explained in section 2 of this document, in our implementation the failure of any elementary control operation implies that constraints (31 - 36) would be violated for the respective contingency. Indeed, let us first notice that all such constraints are expressed per contingency. Furthermore, recalling that any corrective action implies (i) a non-negative cost of application and (ii) a non-negative socio-economic cost attached to it's possible failure, in a cost minimization framework, contingency specific corrective actions would optimally be selected so as to make at least one of (31 - 36) binding. Hence, under the assumption of failure of any such corrective control action, at least one of the constraints (31 - 36) would be violated and the respective system trajectory should be indeed regarded as "unacceptable".

3.4 The reliability target (7)

Let us firstly recall from subsection 3.2 that, for any contingency $c \in \mathcal{C}_c$ a value of $\zeta_c = 1$ implies an unacceptable system trajectory due to the inability to reach a short-term post-contingency equilibrium. It follows that the probability of realizing such a kind of unacceptable trajectory, due to the chosen preventive actions and given that $c \in \mathcal{C}_c$ can be expressed by the following expression,

$$\sum_{c \in \mathcal{C}_c} \tilde{\pi}_c(w_0) \cdot \zeta_c,$$

where $\tilde{\pi}_c(w_0) = \pi_c(w_0) / \sum_{c \in \mathcal{C}_c} \pi_c(w_0)$.

On the other hand, as explained in subsection 3.3 any potential failure of corrective control also implies an "unacceptable system trajectory". In our implementation, we assume that the failure probability of any corrective control action is equal to the sum of the failure probabilities of all concerned elementary control operations. Recalling, that for any contingency $c \in \mathcal{C}_c$ elementary control operations on generating units ($g \in \mathcal{G}$) and phase shifting transformers ($\ell_p \in \mathcal{L}_p$) would be indicated by auxiliary binary variables $\gamma_{g,c} = 1$ and $\lambda_{\ell_p,c} = 1$ respectively, the conditional probability of corrective control failure is hence mathematically expressed as follows,

$$\sum_{g \in \mathcal{G}} \pi_g^{fail}(w_0) \cdot \gamma_{g,c} + \sum_{\ell_p \in \mathcal{L}_p} \pi_{\ell_p}^{fail}(w_0) \cdot \lambda_{\ell_p,c} \quad \forall c \in \mathcal{C}_c.$$

Combining the two precedent expressions, we mathematically formulate the reliability target

(7) by

$$\left[\sum_{c\in\mathcal{C}_c}\tilde{\pi}_c(w_0)\cdot\zeta_c+\sum_{c\in\mathcal{C}_c}\tilde{\pi}_c(w_0)\cdot(1-\zeta_c)\cdot\left(\sum_{g\in\mathcal{G}}\pi_g^{fail}(w_0)\cdot\gamma_{g,c}+\sum_{\ell_p\in\mathcal{L}_p}\pi_{\ell_p}^{fail}(w_0)\cdot\lambda_{\ell_p,c}\right)\right]\leq\varepsilon_{RT}.$$

Noticing that whenever $\zeta_c = 1$ we also have $\forall g \in \mathcal{G} : \gamma_{g,c} = 0$ (given (22)) and $\forall \ell_p \in \mathcal{L}_p : \lambda_{\ell_p,c} = 0$ (given (28)), this latter equation is equivalent to the following one:

$$\left[\sum_{c\in\mathcal{C}_c}\tilde{\pi}_c(w_0)\cdot\zeta_c+\sum_{c\in\mathcal{C}_c}\tilde{\pi}_c(w_0)\cdot\left(\sum_{g\in\mathcal{G}}\pi_g^{fail}(w_0)\cdot\gamma_{g,c}+\sum_{\ell_p\in\mathcal{L}_p}\pi_{\ell_p}^{fail}(w_0)\cdot\lambda_{\ell_p,c}\right)\right]\leq\varepsilon_{RT}. \tag{37}$$

3.5 Objective function (6)

Concluding, the objective function of the proposed RT-RMAC is implemented as shown in (38). In the first row of (38) the first term corresponds to the costs of the preventive generation re-dispatch with respect to the market clearing outcome, while the second term expresses the expected cost of corrective control. The latter is computed here as the probability weighted summation of the ramp-up costs with respect to the pre-contingency generation dispatch. In the second row of this function, we multiply the probability of violating the system operational limits by the scalar product of the value of lost load and the load demand. As already introduced, under the conservative assumption that any violation of the system operational limits would lead to the loss of the whole system load, and since with our model any acceptable trajectory always allows to cover the total demand, the latter quantity is used as a severity value in monetary terms.

$$\begin{aligned}\min\sum_{g\in\mathcal{G}}&\left[c_g\cdot\left(P_{g,0}^{+}-P_{g,0}^{-}\right)+\sum_{c\in\mathcal{C}_c}\pi_c(w_0)\cdot\left(c_g\cdot P_{g,c}^{+}\right)\right]\\&+\left[\sum_{c\in\mathcal{C}_c}\pi_c(w_0)\cdot\zeta_c+\sum_{c\in\mathcal{C}_c}\pi_c(w_0)\cdot\left(\sum_{g\in\mathcal{G}}\pi_g^{fail}(w_0)\cdot\gamma_{g,c}+\sum_{\ell_p\in\mathcal{L}_p}\pi_{\ell_p}^{fail}(w_0)\cdot\lambda_{\ell_p,c}\right)\right]\cdot\sum_{d\in\mathcal{D}}v_d(w_0)\cdot P_d(w_0).\end{aligned} \tag{38}$$

4 Data used in the case studies

This section provides additional detailed data used in the case studies presented in section IV of [1]. Notice that the data not presented here can be found in [1] and/or [2].

4.1 Generation Data

Table 1 provides an overview of the generation data. The first four columns of this table show, for each generating unit $g \in \mathcal{G}$, the maximum active power generation for each one of the four segments of the assumed piece-wise linear generation cost function. Notice that this data are also listed on the fifth column of table 9 in [2]. The last four columns of table 1 list the marginal generation cost per segment of the assumed piecewise linear generation cost function. This data have been computed as in [4], *i.e.* using the incremental heat rates presented in the last column of table 9 in [2] and the fuel cost data found in [5].

$g\backslash k$	$P_g^{\max,k}$ (MW) 1	2	3	4	c_g^k $(\$/MWh)$ 1	2	3	4
1 − 4	15.8	0.2	3.8	0.2	47.705	49.060	69.058	69.808
5 − 8	15.8	0.2	3.8	0.2	47.705	49.060	69.058	69.808
9 − 11	25	25	30	20	30.008	32.304	34.945	36.640
12 − 14	68.95	49.25	39.4	39.4	30.968	32.768	34.222	35.687
15	0	0	0	0	0	0	0	0
16 − 20	2.4	3.6	3.6	2.4	37.761	38.321	43.285	49.038
21 − 22	54.25	38.75	31	31	15.997	16.531	17.226	18.157
23 − 24	100	100	120	80	8.563	8.676	8.913	9.134
25 − 30	50	0	0	0	0	0	0	0
31 − 32	54.25	38.75	31	31	15.997	16.531	17.226	18.157
33	140	87.5	52.5	70	16.262	17.218	17.892	18.906

Table 1: Generation data

4.2 Demand data

The demand data are summarized in table 2. The first two columns of this table show the active power load of each demand $d \in \mathcal{D}$. As mentioned in [1], the listed values for *Cases A and B* correspond to to the 1-hour interval [12:00;13:00) on the Monday of week 23 (summer) and week 46 (winter) respectively for an annual peak load of 3000MW, as per tables 2,3,4 and 5 of [2]. The last two columns of table 2 present the assumed value of lost load for any demand $d \in \mathcal{D}$. The data referring to *Case A* have been found in [6]. Concerning *Case B* the listed data have been computed under the assumption that, due to adverse weather conditions, the voll of each demand is increased by 15%.

4.3 Outage probability data

Table 3 shows the outage probability data. **Notice that for the sake of readability all values listed in this table have been multiplied by** 10^5. The values corresponding to *Case A* have been computed according to the permanent outage rates shown in table 12 of [2] for a time interval of 1 hour, as per the methodology developed in [7]. As suggested in [2], we used a frequency of 7.5 % to compute the outage rates for common mode outages on circuits that share a common right of way only. Concerning *Case B*, we have arbitrarily assumed that, due to adverse weather condition, forced outage rates are increased by 10% on all branches, with the exception of the cables [A1;A10]. Finally, in both cases, the probability of no outage (denoted as c_0) is computed as $1-\sum_{c\in\mathcal{C}\backslash c_0}\pi_c(w_0)$, *i.e.* by assuming that the set of mutually exclusive and collectively exhaustive events for the time interval of interest includes only the outages listed in table 3 and the pseudo-contingency of no outage. Accordingly, the probability of realizing no outage is found to be equal to [0.9985,0.9981] in *Case A,B* respectively.

d	$P_d(w_0)$ (MW)		$v_d(w_0)$ $(\$/MWh)$	
	Case A	*Case B*	*Case A*	*Case B*
1	95.418	96.476	6200	7130
2	85.374	86.320	4890	5623.5
3	158.193	159.947	5300	6095
4	65.286	66.010	5620	6463
5	62.775	63.471	6110	7026.5
6	120.528	121.864	5500	6325
7	110.484	111.709	5410	6221.5
8	150.660	152.330	5400	6210
9	153.171	154.869	2300	2645
10	170.748	172.641	4140	4761
11	233.523	236.112	5390	6198.5
12	170.748	172.641	3410	3921.5
13	276.210	279.272	3010	3461.5
14	87.885	88.859	3540	4071
15	293.787	297.044	3750	4312.5
16	160.704	162.486	2290	2633.5
17	112.995	114.248	3640	4186

Table 2: Demand data

4.4 Phase shifting transformers

We have modified the standard version of the RTS-96 by assuming that the four transformers linking buses 9,10,11 and 12 (*i.e.* branches A14 – A17) are replaced by phase shifting transformers. The upper/lower bounds on the phase angle of all such devices is set to $\pm 10°$ in the case studies.

Outage	*Case A*	*Case B*	Outage	*Case A*	*Case B*
A1	2.74	2.74	A24	3.76	4.70
A2	5.81	7.27	A25-1	4.67	5.84
A3	3.76	4.70	A25-2	4.67	5.84
A4	4.45	5.56	A26	4.67	5.84
A5	5.47	6.84	A27	3.99	4.99
A6	4.33	5.41	A28	3.88	4.84
A7	0.23	0.28	A29	3.65	4.56
A8	4.10	5.13	A30	6.16	7.69
A9	3.88	4.84	A31-1	3.99	4.99
A10	3.76	3.76	A31-2	3.99	4.99
A11	3.42	4.27	A32-1	4.33	5.41
A12-1	5.02	6.27	A32-2	4.33	5.41
A13-2	5.02	6.27	A33-1	3.88	4.84
A14	0.23	0.28	A33-2	3.88	4.84
A15	0.23	0.28	A34	5.13	6.41
A16	0.23	0.28	A12-1,A12-2	0.38	0.47
A17	0.23	0.28	A18,A20	0.34	0.43
A18	4.56	5.70	A25-1,A25-2	0.35	0.44
A19	4.45	5.56	A30,A34	0.46	0.58
A20	4.56	5.70	A31-1,A31-2	0.30	0.37
A21	5.93	7.41	A32-1,A32-2	0.32	0.41
A22	5.59	6.98	A33-1,A33-2	0.29	0.36
A23	4.33	5.41			

Table 3: Outage probability data ($\times \mathbf{10^5}$)

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