DE^3: Yet another high-order integration scheme for linear structural dynamics

Alexandre Depouhon, Emmanuel Detournay, Vincent Denoël

Abstract

Since Newmark's landmark paper [1], a vast number of integration schemes have been developed to advance in time the canonical equation of motion of linear structural dynamics

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t).$$

These are typically derived from the mathematical treatment of the equation of motion and continuity assumptions of the field to be discretized, *e.g.*, Taylor expansion, finite element approaches based on weak formulations, or finite difference approximations. For linear problems, what matters, though, are the stability and accuracy properties of the derived schemes, their structure-preserving nature (energy conservation for instance), if any, as well as their intrinsic computational cost.

The DE^3 scheme [2] belongs to the class of two-level one-step implicit integration schemes enjoying unconditional stability. It is a reformulation of the scheme proposed by Bottasso and Trainelli [3] by condensation of the internal variables of the original scheme. As such, it exhibits the same properties. Namely, it preserves the linear momentum of the mechanical system and offers control on the numerical dissipation of mechanical energy, in the high-frequency range, via a single parameter. It is fourth-order accurate when used in its conservative setting; the accuracy drops to third order when numerical damping is present, however. This makes it an ideal candidate for structural dynamics simulations [4] or, even, wave propagation ones in combination with the finite element method [5], as the numerical dissipation enables control of high-frequency Gibbs oscillations. More generally, although this integration scheme could in principle be efficiently applied to any linear structural dynamics problem, it is of interest where and when small timesteps are required for sufficient accuracy, *e.g.* elastodynamics. Another possibility concerns piecewise linear dynamical systems where any saving on the solution of the linear set of equations on a domain could be invested in an accurate event detection procedure.

The focus of this contribution is set on the derivation of the scheme and the illustration of its superior features by application to a number of structural dynamics problems.

References

[1] N.M. Newmark. A method of computation for structural dynamics. *Journal of Engineering Mechanics, ASCE*, 85(EM3):67–94, 1959.

[2] A. Depouhon, E. Detournay, and V. Denoël. Accuracy of one-step integration schemes for damped/forced linear structural dynamics. *International Journal for Numerical Methods in Engineering*, 99(5):333–353, August 2014.

[3] C.L. Bottasso and L. Trainelli. An Attempt at the Classification of Energy Decaying Schemes for Structural and Multibody Dynamics. *Multibody System Dynamics*, 12:173–185, 2004.

[4] A. Depouhon, E. Detournay, and V. Denoël. Event-driven integration of linear structural dynamics models under unilateral elastic constraints. *Computer Methods in Applied Mechanics and Engineering*, 276(0):312–340, July 2014.

[5] A. Depouhon, V. Denoël, and E. Detournay. Numerical simulation of percussive drilling. *International Journal for Numerical and Analytical Methods in Geomechanics*, Accepted.