Wide range progressive inductor models in magnetic vector potential finite element formulations

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***Abstract*— Wide range progressive refinements of the inductors in magnetic vector potential finite element formulations are done with a subproblem method. Their current sources are first considered via magnetomotive force or Biot-Savart models up to their volume finite element models, from statics to dynamics. A novel way to define the associated source fields is proposed to lighten the computational efforts, via the conversion of the common volume sources to surface sources, with no need of any pre-resolution. Accuracy improvements are then efficiently obtained for local currents and fields, and global quantities, i.e. inductances, resistances, Joule losses and forces.**

I. INTRODUCTION

The inductors in finite element (FE) magnetodynamic problems can generally be considered via Biot-Savart (BS) models, possibly giving the conductors some simplified geometries, with, e.g., filament, circular or rectangular cross sections [1]. In the slots of magnetic core regions, an alternative to the explicit definition of such simplified geometries is to consider the slot-core interfaces as perfect magnetic walls via a zero flux boundary condition (BC). The slot regions are thus omitted from the calculation domain, neglecting the slot leakage flux instead of calculating other inaccurate distributions. The associated sources are magnetomotive forces (MMFs) [2].

MMF sources are inherently associated with surfaces whereas BS models define source fields (SFs) that are originally volume sources (VSs). A novel procedure is here proposed to convert the BS SFs into surface sources (SSs) as well, to lighten the computational efforts, mainly by reducing the number of BS evaluations. It is based on interface conditions (ICs) that define field discontinuities fixed from surface BS fields. The developments are performed in the frame of the magnetic vector potential $\boldsymbol{a}$ formulations. Accuracy improvements up to volume FE representations of the conductors, that improve the local field distributions, and from static to dynamic excitations, that accurately render skin and proximity effects, can be done at a second step via the subproblem (SP) method (SPM) [3], [4], which defines a general frame for the whole modeling procedure.

II. PROGRESSIVE INDUCTOR MODELS

A. Magnetomotive force (MMF) model

Boundary Γ_m of some magnetic (conducting or not) regions Ω_m, possibly extended with air gaps $\Omega_g \subset \Omega_m$, can first be considered as a perfect magnetic wall (Fig. 1), thus with no leakage flux. For the so-defined SP $p \equiv$ MMF, the calculation domain is thus limited to Ω_m, called a flux tube, with a BC on Γ_m fixing a zero normal trace of the magnetic flux density $\boldsymbol{b}_p$. In terms of a magnetic vector potential $\boldsymbol{a}_p$, with $\boldsymbol{b}_p$ = curl $\boldsymbol{a}_p$, one has the equivalent essential BCs

$$\boldsymbol{n} \cdot \boldsymbol{b}_p|_{\Gamma_m} = 0 \quad \Leftrightarrow \quad \boldsymbol{n} \times \boldsymbol{a}_p|_{\Gamma_m} = \boldsymbol{n} \times \operatorname{grad} w_p|_{\Gamma_m}, \quad \text{(1a-b)}$$

where $\boldsymbol{n}$ is the exterior unit normal and w_p is a multivalued unknown surface scalar potential. The required gauge condition on $\boldsymbol{a}$ allows to particularize the distribution of w_p.

Through such a process, the actual current source regions Ω_s are idealized as perfect solenoids winded all along Γ_m, i.e., around the flux tubes. In 3-D, scalar potential w_p in BC (1b) can be reduced to a constant jump through each of the cut lines making Γ_m simply connected. In 2-D, such constant jumps come down to the definition of a constant $\boldsymbol{a}_p$ (a kind of floating potential) on each non-connected portion $\Gamma_{m,i}$ (with i the portion index) of Γ_m (Fig. 1, *left*). The constant jumps are directly (strongly) related to the unknown magnetic fluxes flowing in Ω_m and are related to the MMFs via the weak formulation, tested with the non-local jump test functions [2]. An example of result is given in Fig. 3.

Each $\Gamma_{m,i}$ can be considered as the boundary of a slot in a device. The related MMF gathers all the current sources in the slot, for all coils, e.g., coils with different phases in a machine or primary/secondary coils in a transformer. A slot can be generalized to represent the exterior region, including coils as well.

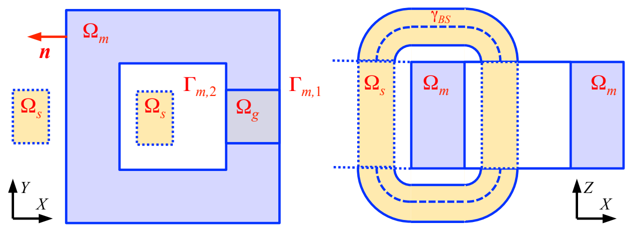


Fig. 1. Example of a magnetic region Ω_m, including an air gap Ω_g, first considered without leakage flux, via perfect magnetic walls BCs on $\Gamma_{m,1}$ and $\Gamma_{m,2}$ in channel slots (XY-plane, *left*) coupled to end windings coil γ_{BS} via BS-SF model (XZ-plane, *right*).

B. Biot-Savart – source field (BS-SF) model

With the SPM, the BS SF evaluations can be limited to the material regions Ω_m via local VSs [3], [4], instead of the whole domain with the common method [1]. Such a support reduction already allows to lighten the BS calculations. Then, for accurate combinations with the reaction fields, the BS SFs gain at being projected onto similar function spaces (edge FEs for both source and reaction fields). Also, instead of volume projections of the SFs in the mesh of Ω_m, the SFs gain at being calculated there via an FE problem with their boundary values as BCs on $\partial\Omega_m$, thus already limiting the BS evaluations to surfaces.

To go one step further, such a preliminary FE problem can be avoided through its inclusion in the main SP $p \equiv$ BS-SF. The key is to think of two successive SPs pa and pb actually solved together (Fig. 2). SP pa first prevents the field to enter Ω_m, thus with a reaction field in Ω_m opposing the BS field (direct solution of SP q), keeping unchanged the field out of Ω_m. This constraint is simply expressed via both tangential and normal field trace discontinuities of magnetic field $\boldsymbol{h}_{pa}$ and magnetic flux density $\boldsymbol{b}_{pa}$ through Γ_m, i.e., with ICs with SSs

$$[\boldsymbol{n} \times \boldsymbol{h}_{pa}]_{\Gamma_m} = \boldsymbol{n} \times \boldsymbol{h}_q|_{\Gamma_m}\,, \quad [\boldsymbol{n} \cdot \boldsymbol{b}_{pa}]_{\Gamma_m} = \boldsymbol{n} \cdot \boldsymbol{b}_q|_{\Gamma_m}\,. \qquad \text{(2a-b)}$$

In terms of $\boldsymbol{a}_p$, IC (2b) leads to

$$[\boldsymbol{n} \times \boldsymbol{a}_{pa}]_{\Gamma_m} = \boldsymbol{n} \times \boldsymbol{a}_q|_{\Gamma_m}\,. \qquad (3)$$

The result is an exact zero field in Ω_m, with no need of volume calculation. Then, SP pb considers the actual physical properties in Ω_m, with no more VSs, which is a great advantage. Combining SPs pa and pb thus gives a single SP p that considers the physical properties of Ω_m and with the trace discontinuities of SP pa (2a-b) still being valid for its solutions $\boldsymbol{h}_p$ and $\boldsymbol{b}_p$ (because SP pb introduces no discontinuities).

BS SF evaluations are thus only required on Γ_m, which is a significant advantage. At the discrete level, the IC-SSs in (2a-b)-(3) can be obtained from a mesh projection of only the $\boldsymbol{a}$ BS SF in a layer of FEs along the boundary of Ω_m. Details will be given. An example of result is given in Fig 4.

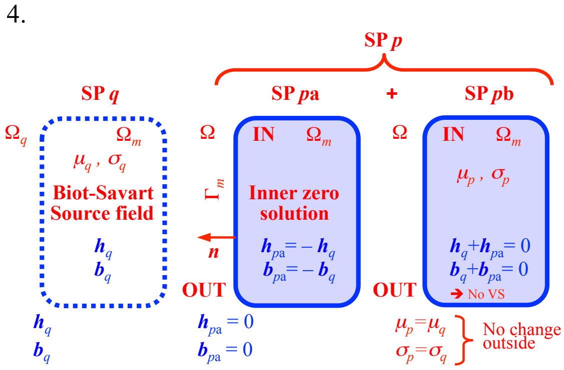


Fig. 2. BS SF (SP q) for a material region Ω_m (SP p): SP p is split into SPs pa and pb, simultaneously solved, SP pa removing the volume BS solution q inside Ω_m and SP pb considering the actual properties of Ω_m, with no need of VSs for change of properties, but with IC-SSs for unified SP p.

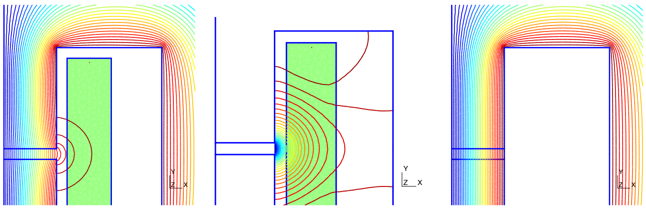
C. Coupling between MMF and BS-SF models

In 3-D, the slots are generally of two types: 1) a channel surrounded by magnetic regions and possible air gaps (usually considered in 2-D models), coupled through interfaces to 2) an open exterior region for the end windings (3-D effects). It is here proposed to define the channel slots via the MMF model and the end winding regions via the actual consideration of the winding, e.g., via a BS model (Fig. 1, *right*). With such a coupling between MMF and BS-SF models, the flux wall surface has to be extended to these interfaces. Practical details will be given.

D. Volume FE models

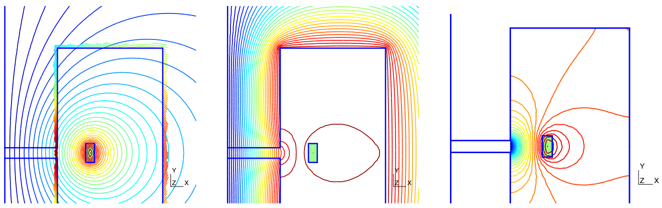
Additional SPs for volume FE models of the source conductors, for both static and dynamic excitations, can follow (Fig. 5) [3], [4], for accurate determination of their characteristics (impedances, losses, forces).

The proposed methodology has first been validated on 2-D test problems. It offers tools that allow to reduce the computational effort of the classical approach. When applied up to 3-D problems, its advantages will be shown to be numerous and significant, with quantification of the benefits.



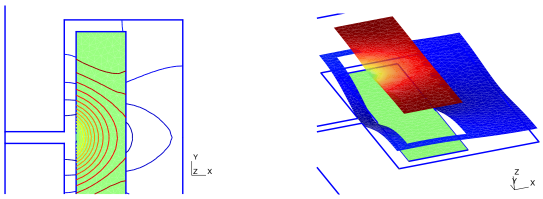
(a) total solution (b) total sol., in-slot zoom (c) MMF model sol.

Fig. 3. Current source in a slot with air gap (portion of a device): field lines with (a)-(b) classical full model and (c) MMF model.



(a) BS SF, FE layer (b) BS-SF model sol. (c) in-slot BS-SF sol.

Fig. 4. Current source in a slot with air gap: field lines with BS-SF model; the BS SF is only needed in an FE layer along core boundary Γ_m; field discontinuities appear (b) through Γ_m because the total field is obtained in Ω_m whereas the reaction field is obtained outside; due to the simplified shape of the coil, the field distribution in the slot (c) is far from the actual one (Fig. 3b).



(a) volume correction (b) elevation of volume correction

Fig. 5. (a) Field lines of volume correction of coil and its surrounding from BS-SF solution, (b) with its elevation (z-component of $\boldsymbol{a}$) pointing out the field trace discontinuities; the field distribution (a) is now the correct one: the total field in the actual coil and the reaction field elsewhere.

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