Using a vector Jiles-Atherton hysteresis model for isotropic magnetic materials with the FEM, Newton-Raphson method and relaxation procedure

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Abstract — This paper deals with the use of a vector Jiles-Atherton hysteresis model included in 2D finite element modelling. The Newton-Raphson algorithm is used with a relaxation procedure, which ensures the convergence in most of the cases. We have simulated a T-shaped magnetic circuit with rotating fields and then a three-phase transformer model.

I. INTRODUCTION

In this paper, the 2D magnetic vector potential formulation is used with the finite element (FE) method, the Newton-Raphson (NR) method and the implicit Euler scheme for time stepping. Loss computation accounting for magnetic hysteresis and eddy currents in the lamination stacks of electrical devices, such as transformers or rotating machines, is often performed with a posteriori loss models. However, when a circuit coupling exists, this approach is not valid anymore and the hysteresis model must be included in the FE equations. The Jiles-Atherton (JA) model is widely employed, because of the small number of coefficients required, usually denoted by \( m_a, a, k, c \) and \( \alpha \). The main governing equation of the model is:

\[
\frac{dm}{dh} = [I_d - \alpha \chi]^{-1} \cdot \chi \quad \text{with} \quad \chi = (1 - c) \frac{dm_{an}}{dh_e} + c \frac{dm_{irr}}{dh_e},
\]

where \( \mathbf{m} \), \( \mathbf{m}_{an} \) and \( \mathbf{m}_{irr} \) are the total, anhysteretic and irreversible magnetization respectively, \( \mathbf{h} \) is the effective field (\( \mathbf{h} = \mathbf{h} + \mathbf{cm} \)), and \( I_d \) the unit tensor. See [3][4] for the details of the computation of \( \frac{dm_{an}}{dh_e} \) and \( \frac{dm_{irr}}{dh_e} \). The differential permeability tensor is then computed with:

\[
\frac{db}{dh} = \mu_0 \left( I_d + \frac{dm}{dh} \right).
\]

It is inverted to get the differential reluctivity tensor \( dh/db \) and the field \( \mathbf{h} \) at the new time step can then be obtained. This model has been developed in the Flux® software [7].

C. Newton-Raphson method with relaxation

The NR method is applied to solve the non-linear FE system: the magnetic field \( \mathbf{h} \) and a differential reluctivity tensor \( dh/db \) are computed by the JA model at the previous iteration of the NR algorithm [3][4]. So as to ensure the convergence of the NR method, a relaxation factor is employed, which is calculated with the method described in [8]. This relaxation factor is determined at each NR iteration so as to minimize the norm of the residual of the linearized system of equations.

D. Low frequency lamination model

In the 2D finite element model, it is possible to take into account the eddy current losses due to the magnetic flux density flowing in the lamination plane of a magnetic circuit [1]. For the sake of simplicity, we assume a stacking factor of one. The skin effect is assumed to be negligible. So we use the following relation:

\[
\mathbf{h}_s(t) = \mathbf{h}_a(t) + \frac{c \sigma d^2}{12} \frac{dh_a}{dt},
\]

where \( \mathbf{a} \) is a test function, \( \mathbf{h} \) the magnetic field, \( \mathbf{j} \) the current density.
where \( \mathbf{h} \) is the magnetic field at the surface of a lamination, \( \mathbf{b}_a \) and \( \mathbf{b}_b \) the average magnetic field and flux density respectively (linked by the hysteresis model presented in subsection B, with \( \mathbf{h} = \mathbf{b}_a = \mathbf{b}_b = \mathbf{b} \)), \( d \) the thickness of the laminations and \( \sigma \) their conductivity.

III. NUMERICAL EXAMPLES

A. Square region example

We have first tested the method on a simple case: a square domain with pulsating or rotating field described in [4]. We have verified that in the case of a pulsating field, the vector JA model gives the same results as the scalar one, according to the theory, as stated in [2]. In the rotational case, the b and h loci are circular as expected.

B. T-joint and three-phase transformer

We have then performed simulations of two test cases concerning a three-phase transformer operating at 50 Hz: 1) a T-joint with imposed shifted currents in two coils with \( I_{\text{max}} = 0.01 \text{A} \) or \( I_{\text{max}} = 0.2 \text{A} \) and 2) the whole three-phase transformer described in [1] operating at no load at 100 Vrms or 230 Vrms (cf. fig. 1). The JA coefficients we have used have been found in [9] corresponding to non-oriented M330-50A steel sheet: \( m_\alpha = 1.28 \times 10^6 \text{A/m}, a = 26.1 \text{A/m}, k = 52.3 \text{A/m}, c = 0.13 \) and \( \alpha = 7.45 \times 10^{-5} \). 50 periods have been simulated with 200 time steps per period. The amplitude of the currents or voltages are smoothly increased from 0 s until 0.8 s by a sine step function \( sf(t) \) so as to reduce the simulation time to reach the steady state.

Both test cases have been simulated with and without the lamination model (cases 1 and 2 respectively). When used, a lamination is 0.5 mm thick and has a conductivity of 2.03 \( 10^6 \text{S/m} \). In the T-joint case, we have also taken into account eddy currents in the z-direction by considering that the magnetic circuit is solid (not laminated), with a 500 S/m conductivity and zero net current (case 3). With the relaxation procedure, the NR algorithm converges in all cases presented in this paper. With a current of 0.01A in the coils of the T-joint and with the transformer at 100 V, a drift of flux density and also a drift of magnetic field, however to a lesser extent, are observed after 0.7 s in case 1 or 2. It is much reduced in case 3 (with conductivity) of the T-joint. Notice that this simulation gives different results as the magnetic circuit is not any more considered laminated. With a current of 0.2A in the coils of the T-joint and with the transformer at 230 V, the simulation results do not exhibit any drift of flux density in time. An explanation can be that, in these cases, the saturation is reached at most of the points of the magnetic circuit.

REFERENCES


