Exploiting the phenomenotechnical potential from TAD: what is the price to pay ?

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On the use of the word ‘phenomenotechnical’ to describe engineering

The question of the relationship between research in didactics and the subsequent action on the learning system arises not in terms of innovation or research-action but in terms of testing the theoretical constructs developed by researchers in didactical realisations that essentially constitute, being research methodologies,

« the place of that crucial stage of the scientific activity to which Bachelard has given the parodic name of phenomenotechnique » (Chevallard, 82)

On the use of the word ‘phenomenotechnical’

An epistemological constructivism :

- Phenomena not as being observable items which an ontological reality would allow to see
- Applied rationalism (Bachelard): dialectical thinking between objects and concepts in light of a theory

On the use of the word ‘phenomenotechnical’

A theory is a priori « phenomenotechnical » in that it produces « phenomenotechniques »: that is

- experiences which allow to invalidate one or another of its hypotheses
- counterfactual situations for testing a theoretical hypothesis

Which transposition has to be performed when no longer dealing with « fragments of nature » ?

Engineering as phenomenotechnique

for the artificial generation of concepts at the cost of:

- looking at TSD as being a theory and not as an ideology
- tight analysis in order to distinguish between what relates to necessity and what to contingency: investigate « what, under the given hypotheses, is invalidated by the observed deviations » and not simply « propose modifications to the engineering so that these are reduced, without being really involved in a validation process » Artigue (1990)
- the analysis of the transposition in which the tasks proposed to students do take place

Engineering as phenomenotechnique

of denaturalisation (falsification) of institutional of « self-evidence » and « ready-to-think » :

- Adidactical situations, tool-object dialectic as invalidation of a deductive, unquestioned conception of teaching, which would pretend that an efficient learning should go from « general to specific », from « significant to signified » or from « object to tool »
- Existence of embryonic forms of knowledge which are distant from forms being socially standardized. But in which institutions, for what reasons and in the absence of which constraints ?

An institutional look at obstacles and situations

from preconstruction of knowledge to its scientific construction:

- An empiricist relation to « things » that can be qualified as primary experience, in the sense of Bachelard, where preconstructs are relative to gemetrical and physical quantities (Schneider, 1988)
- Obstacle that prevents an adequate relationship to definitions in their operation for deductive operations (Job, 2011)

But: social construction or private construction?

An institutional look at obstacles and situations

Obstacles with cultural roots:

- A positivistic vision interpreted to the light of our modern western society … servicing the development of the bourgeoisie
- A « middle of the road » ideology that hides the need to take into account the statistical variability
- « Primary experiences » of an institutional relationship that is changing with respect to an object of knowledge
- Learning practices which are relevant from ostention as origin of an empiricist relation of students to the world or as a matter of civilisation…

An institutional look at obstacles and situations

- Fundamental situations, in the broad sense, whose major stake is not that much in the construction of a specific knowledge by the students but more in them entering into a new institution where the relationship with an object of knowledge is changing
- The choice of distinguishing, like other researchers, fundamental situation and one of its declinations in adidactical situation
- But favour the first « cultural-mimetic » meetings with the help of « heuristic » speech in a sense that is adapted to the praxeological level: ’modelling’ or ‘deductive’

The question of articulating MO and DO and the role of REM

- Didactical Analysis of « Heuristic Approach of Calculus (Schneider, 2001):
- Constructivist references elaborated partially on the ideological mode, that resulted in non standard MO, in particular in their form of logos:

  « Which importance is to be given to constructivism ? Are non canonical mathematical praxeologies the price to pay […] ? »

The question of articulating MO and DO and the role of REM

- This analysis was raising the question of the articulation between praxeologies or mathematical organisations (MO) and praxeologies or didactical organisations (DO) at a time where TAD was more explicit on evaluation criteria for MO than for DO :
- *Beyond the description of mathematical and didactical praxeologies which are behind lectures or learning projects, it is their articulation that should put forward the internals of learning practices […] No doubt however that didacticians are eagerly waiting for the longest developments on the evaluation of a didactical organisation* (Schneider, 2001)

The question of articulating MO and DO and the role of REM

- A typology exists today for DO that are characterized by *« the fact that a great importance is given to a few moments in the learning process to the detriment of all others – which are then, in general, left to the sole responsibility of pupils or students »* (Bosch & Gascon, 2002)
- But whose usage supposes an external « reference »: *« […] if an DO can be described, as a first approximation, from his structuration in terms of moments, it nevertheless remains that the moments are not sufficient for such a description: the clarification of the various moments in learning will start, at first, from that given state that is the MO to be set in place, and that one must be capable of analysing in elements that are neither « too coarse » nor « too fine » in order not to suppress his « vital structure », while showing how his « re-composition » can or could be realised »* (Bosch & Gascon, 2002)

The question of articulating MO and DO and the role of REM

- Is the REM concept that reference ?
- Which phenomena does it allow to see at the level of articulation between MO and DO ?
- What is the price to pay for it to play a role of phenomenotechnique at that level?

Analysis of a poorly phenomenological usage of the REM concept

- A REM about Lagrange's theorem in order to study university learning processes in math and economy sections (Xhonneux & Henry)
- Composed of five local MO, constructed from three families of tasks of 'procedural' type and two of 'structural' type (Sfard)
- Structured according two levels, referring to processes: 'internalisation', 'condensation' and 'reification' (Sfard) and based on the usage of the TAD made by Winslow in order to express that theory can be transformed into tasks

Much information but no emerging phenomenon

Analysis of a poorly phenomenological usage of the REM concept

A few characteristics beyond criticism:

- Strong « theoretical » crossbreeding (Barbin, Douady, Duval, Poincaré, …) and many « criteria » that cannot avoid personal judgment from researchers, this being due to the lack of a sufficiently deep analysis
- A REM constructed around a theorem and not thought at the level of a domain, being here that of optimisation with constraints
- A REM « empirically constructed from praxeologies to be learned and from 'expert' texts such as mathematical papers, manuals or lecture notes from university level » with no analysis of what is relevant from 'expert' institutions' or from 'didactical' institutions or…

At the roots of the REM concept: the example of elementary algebra

An implicit model that is dominant among learning practices, that of generalised arithmetic (Chevallard, Gascon) :

- Emphasis put on algebraic symbolism, which ‘enlarges and generalises’ a supposed arithmetic language
- Disarticulation for the corpus of problems into equations, identities, application of formulas …
- Interpretation of learning difficulties too exclusively related to the arithmetic context

At the roots of the REM concept: the example of elementary algebra

Formulation of an alternative model by Gascon :

- Limits of the analysis/synthesis pattern:
  - In order to provide a general solution for isomorphic problems and to precise the conditions of existence
  - In order to solve some problems in arithmetic or in geometric constructions
- An epistemological thought on Viète's 'new algebra' and the 'method' from Descartes, where parameters and undetermined quantities play an equally important role

At the roots of the REM concept: the example of elementary algebra

Elementary algebra does not appear initially as MO at the same level as other organisations that are studied at school […]. [It is] a mathematical instrument for studying mathematical organisations: a didactical instrument […] To the question « what is elementary algebra ? » we do not answer in terms of MO, but in terms of processes which model MO via other MO […] it is the modelling process itself that is central, before leaving room for MO which are « totally algebrised » where the algebraic tool is studied as being an object (Bosch & Gascon, 2002)

A REM that performs as phenomenotechnique

Implicit empirical models that perform as systems of conditions and constraints:

- *One can consider that, in every didactical institution where math are taught, implicit models for the various domains of the taught mathematical knowledge do exist, from which an implicit model of the effective nature of the mathematical knowledge does emerge as an extension*
- This model performs as a system of conditions and constraints on practices, by *« allowing the existence of some of these and preventing the appearance of others »* (Gascon, 1993)

A REM that performs as phenomenotechnique

An alternative (theoretical) model, constructed from a question dealing with a mathematical domain in order to make clear the implicit empirical models

- This implicit model must be denaturalised and becomes the *object of a study, that is as being part of the didactical facts which constitute the 'empirical' base of the research*
- By stressing on the necessity for the researcher to have at his disposal an *alternative model of the mathematical domain of activity being taught, that he can use as a reference framework in order to interpret the model being dominant in the institution that he is studying* (Gascon, 1993)

The irreducible part of the researcher's choice

Another alternative model starting from the same sources and designed on the entirety of a mathematical curriculum at secondary school level (COJEREM & AHA):

- Geometrical constructions equipped not only with the method of two loci but also with the transformations of the plane (making the usage of algebra a lesser necessity)
- Algebraic knowledge subordinated to the study of functions, these being studied via parametric classes in connection with appropriate questions
- 3D Analytical geometry preparing to vector geometry

REM thus determine the phenomena they show

For the researcher, the construction of at least one reference model must show didactical phenomena that were hidden, allow their description and authorise tentative explanations.

But if a REM constitutes a phenomenotechnique, it considerably determines the phenomena that it allows to show and to study.

Hence the question of the spectrum through which the researcher considers his REM :

- the concept of fundamental situation remains here an unavoidable reference
- the invisibility of some institutions is a real obstacle when designing a REM

Underground institutions

- The institution from « formalists » in researches dealing with the concept of limit (Job, 2011)
- The institution from Platonic mathematicians, that may create an obstacle for heuristic-type speeches (Job & Schneider)
- MO which are ‘emblematic’ from university level and that teachers simplify : topics being too complex for secondary schools show holes that are filled using ostention mechanisms and momentary « breath of strictness » (Rouy, 2007, about MVT)
- Proponents of a certain « strictness » that leads to express the MVT « with a set of minimal hypotheses and demonstrate it in this framework », while its « realistic applications […] do only require a narrower set of validity conditions » (Bourgade, to be published)

To conclude : a « heuristic approach of calculus » being indicative of a phenomenon

- A constructivist DO
- Based on an explicit REM: two learning rings among which the first consists in modelling geometric and physical quantities and in the study of parametric classes of functions. The second is designed as a transition to modern calculus
- A particular logos with non canonical validations to justify the relevance of mathematical models in which common sense assigns particular properties
- An illustration of a double praxeological level

Two praxeological levels

Processes to describe two aspects of mathematical activity and the products of these processes in terms of mathematical organisations

- ✓ Praxeologies of type « modelling » : the goal is to model objects that are not mathematically defined but of which one has a certain knowledge (these are ‘preconstructs’ in Chevallard’s sense)
- ✓ Praxeologies of type « deduction » : one builds up a deductive organisation for the elements of the thereby constructed model, the objects being defined by the techniques that model themselves

The phenomenon being pointed out

- The absence of visibility of praxeologies of type ‘modelling’ at certain levels of teaching and for certain institutions
- Praxeologies of type ‘deduction’ remain a ‘beacon’ for mathematical work and hide the others
- We have here a type of articulation between MO and DO where it is the predominance of certain MO that impose constraints on DO and prevent certain forms of DO to appear

A phenomenon to be interpreted by going back to the roots of transposition

Go back to the very roots of the theory of didactic transposition

Give back to the text of knowledge an epistemological substance, by reconnecting the « theoretical-technological core of the work with its applications », well before that text might exist for the students, either by adidactic games and/or by a « heuristic-type » speech, this is violating a « bureaucratic school transmission » that Verret characterises by the processes of depersonalisation and desyncretisation for knowledge into partial knowledge that can be expressed in an autonomous language and is suitable for programmability of its acquisition.

To be continued… tomorrow !