ANALYTICAL MODEL FOR THE EVALUATION OF TORSIONAL CONNECTION RESTRAINTS OF STEEL BEAMS IN BENDING

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ABSTRACT

One of the most commonly employed general formulae to estimate the elastic critical moment Mcr is the so-called 3-factor formula. This formula is applicable only for extreme end support conditions such as fixed or pinned supports. However, some particular cases met in practice are currently not covered, most notably the case of beams with partial end restraints against minor bending axis, and partial end restraints against twisting and fin-plate connections are a good example. The scope of the present work is to attempt to fill in one of the insufficiencies identified in the so-called 3-factor formula used by the Eurocode 3, by proposing a mathematical model for computing the value of the connection torsional restraint coefficient. In order to take into account the real restraint conditions of the supports, the proposed coefficient will be introduced in the formulae of the elastic critical moment Mcr given in the Eurocode 3. A comparison between the elastic critical moments for various beam cross-sections, lengths, loading conditions and various end restraints, obtained from the finite elements software LTBeam, and those derived from EC3 ENV formulae, in which the proposed coefficient is introduced, confirms the reliability of this coefficient that models satisfactorily the end support conditions.

1. INTRODUCTION

A slender beam subjected to the action of bending loads in the plane of maximum flexural rigidity can buckle by combined twist and lateral bending of the cross-section, unless it has continuous lateral support. This phenomenon, which was first investigated theoretically and experimentally during the nineteenth century, is known as lateral buckling. The low torsional rigidity is an important factor, so thin-walled open section beams such as channels or zeds are also susceptible to this form of instability. The elastic buckling stress is also influenced by the conditions of support at the ends of the beam, and by the type and position of the applied loads that cause bending.
Research developments on lateral torsional buckling of steel members have been accompanied by the realization of updated design codes and standards. Modern steel codes for structures, such as AISC LRFD [1,2], BS 5950-1 [3] and EC3 [4, 5], provide, on the basis of the limit state concept, design procedures to compute the lateral-torsional buckling

resistance of beams. Initial imperfections, residual stresses and inelastic buckling are taken into account through the use of buckling curves [6].
The elastic critical moment is directly dependent on the following factors [7]: material properties such as the modulus of elasticity and shear modulus; geometric properties of the cross-section such as the torsion constant, warping constant, and moment of inertia about the minor axis; properties of the beam such as length, and lateral bending and warping conditions at supports; and finally loading, since lateral-torsional buckling is greatly dependent on moment diagram and loading position with respect to the section shear centre.
Many simple connections met in practice have only partial lateral bending restraint and are generally assumed to provide full torsional restraint. However, some connections such as long fin-plate connections provide both partial lateral bending and torsional restraints. Therefore, the beams connected with fin-plates are prone to undergo some twisting about the longitudinal axis at the supports, in addition to lateral bending. EC3 ENV* takes into account the effect of the lateral bending restraint of the end support in the evaluation of the elastic critical moment M_{cr} by means of a coefficient k_z. However, it is assumed that full torsional restraint is provided by the connection.
The scope of this work is to study the effects torsional end restraints on the lateral torsional buckling moment of the beam. An analytical model has been developed in order to evaluate the torsional restraint coefficient k_θ. The model also allows the evaluation of the percentage of reduction of the lateral torsional bending moment M_{cr} against M_{0cr} for full restraint. On the basis of the value of the percentage reduction of M_{cr} for a particular connection, its classification as simple, partial restraint or full restraint can be made. Therefore, using the analytical model developed, it is possible to determine the required torsional restraint of the connection to ensure full lateral restraint.
A variety of connections with different end restraints are investigated using the finite element software LTBeam [8] in order to determine their influence on the lateral torsional buckling critical moment. It is done for two simple load cases. A uniform distributed load, acting on the beam in the vertical direction at the shear centre and a concentrated load at mid-span acting at the shear centre. A comparison between the elastic critical moments for various beam lengths and various end restraints, obtained from LTBeam, and those derived from the EC3 ENV formula in which the coefficient k_θ computed from the proposed formula is introduced, confirms the reliability of this coefficient that models end support conditions.

2. LATERAL TORSIONAL BUCKLING AND ELASTIC CRITICAL MOMENT

Under increasing loading (see Fig. 1), the beam first bends strictly in the plane of loading. Once the moment reaches a certain magnitude M_{cr}, called elastic critical moment, the beam may deflect suddenly out of the plane of bending. This instability phenomenon is known as lateral torsional buckling. Lateral torsional buckling is said to occur by bifurcation of equilibrium. The beam simultaneously exhibits lateral displacements v in the

* Reference is made to EC3 ENV [5] and not to EC3 EN [4] as the formulae for M_{cr} have been removed from [4] in the so-called "conversion period"

y direction (bending about the minor axis of the cross-section) and twist rotation θ about its longitudinal axis x.
It is clear that lateral-torsional buckling is resisted by a combination of lateral bending resistance $EI_z d^2v / dx^2$ and torsional resistances $GI_t\, d^2\theta / dx^2$ and $EI_w\, d^3\theta / dx^3$. Thus, a member is especially prone to lateral torsional buckling when it has low lateral flexural stiffness EI_z and its torsional stiffness GI_t and warping stiffness EI_w / L^2 are low compared to its stiffness in the plane of loading.
With the nomenclature used in Eurocode 3 [4], where (x-x) is the axis along the member, (y-y) is the major axis of cross-section and (z-z) is the minor axis of the cross-section, the governing differential equation for the lateral torsional buckling is [7]:

$$EI_w \frac{d^4\phi}{dx^4} - GI_t \frac{d^2\phi}{dx^2} - \frac{1}{EI_z} M_y^2 \phi + \frac{1}{EI_z} M_y M_z = 0 \tag{1}$$

with

$$\frac{dM_y}{dx} = V_z \;;\; \frac{dV_z}{dx} = -q_z \;;\; \frac{dM_z}{dx} = -V_y \;;\; \frac{dV_y}{dx} = 0 \tag{2}$$

where q_z is the distributed load acting on the beam, V_y and V_z are the shear forces, M_y and M_z are the bending moments, and ϕ is the torsion deformation. In order to be able to impose appropriate boundary conditions at supports, the internal shear forces and the bending moment components in Eqs. (1) and (2) are referred to the axis in the undeformed configuration.
Exact solutions for Eq. (1) are obtained for a doubly symmetrical beam with simply supported conditions, free warping and subjected to a uniform moment diagram. The elastic critical moment for this basic case is:

$$M_{cr} = \frac{\pi^2 EI_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z}} \tag{3}$$

The elastic critical moment obtained for the basic situation by formula (3), is multiplied by the equivalent uniform moment factor C_1 which takes into account the actual bending moment diagram. Thus, the value of M_{cr} may be computed by the expression:

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(k_z L)^2} \sqrt{\left(\frac{k_z}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(k_z L)^2 GI_t}{\pi^2 EI_z}} \tag{4}$$

Where the lateral bending coefficient k_z and the warping coefficient k_w are introduced in order to take into account support conditions other than simply supported.

2.1 Eurocode 3 (ENV) Approach and its Limitations

The assessment of the stability behaviour of steel beams based on simplified calculations, as described in the standards of most countries, is not always a realistic evaluation. The assumption that member end connections behave as either pinned or completely rigid is a highly simplified approach because experimental investigations show that true joint behaviour has characteristics between these two simplified extremes.

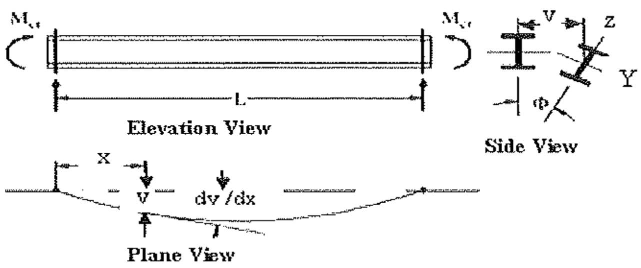


Fig .1: Buckling of a simply supported I-beam.

The design proposals for the buckling of beams assume that the end supports should completely prevent end twisting. If the supports have only limited elastic torsional restraint stiffness, the beam will buckle at a lower load than that estimated from the idealised case (Fig.1).

One of the most commonly employed general formulae to estimate elastic critical moment M_{cr} is the so-called 3-factor formula, which was included in the ENV version of EC3 [5]. In theory, this formula should be applicable to beams subjected to major axis bending having doubly or singly symmetrical cross-sections and arbitrary support and loading conditions.

It can be seen from the M_{cr} expression Eq.(4), that the effect of the end twisting at the supports of the beam which is supposed to be introduced by a coefficient k_θ has not been taken into account. This means that the beam is assumed to be completely prevented from twisting about the longitudinal axis at the end supports. However, many real situations met in practice are not in compliance with these standard conditions. Therefore the effect of the partial end torsional restraint should be considered by introducing a coefficient k_θ in the expression of M_{cr}.

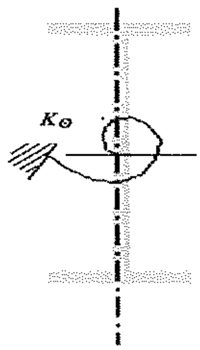


Fig. 2 : Torsional end restraint.

This paper attempts to fill in one of the two insufficiencies identified in the above expression of M_{cr} used by the Eurocode 3, by proposing a formula for computing the value of the torsional restraint coefficient k_θ of the end support (Fig.2) which depends on both the torsional stiffness of the support K_Θ and of the torsional rigidity of the beam GI_t / L.
In the EC3 ENV [5], it is suggested to take k_w=1,00 unless special provision for warping fixity is made. Therefore, in this paper, the warping coefficient k_w is set to be equal to 1,00.

2.2 Theoretical Background on Torsional Restraints at Supports

Flint [9] presented an analysis for a member with end connections providing only limited torsional stiffness. For a beam under single point load or two symmetrical point loads, Flint derived the following relationship between the critical load and the support torsional restraint stiffness.

$$m = 1 - \frac{4}{3} R_T \qquad (5)$$

In Eq. (5) m is the ratio between buckling loads for beams with finite and infinite support torsional stiffness. R_T is the ratio of the torsional stiffness of the beam to its supports and is given by $\frac{GJ / L}{T / \theta}$ where θ is the rotation of a support.
A theoretical study was carried out by Schmidt [10] to determine the effect of elastic end torsional restraint on the critical load of a beam. It shows that the beam is incapable of supporting any load if the end supports offer no resistance to end twisting. It further shows that the critical load increases little as the end torsional restraint stiffness parameter e increases beyond 20. The parameter e is defined as $e = 1/R_T$. According to Flint's equation (5), Bose [11] found that the critical load for $e = 20$ will be 93% of the load for rigid torsional end support.
Bennetts et al [12] and Grundy et al [13] have attempted to investigate the value of the torsional stiffness K of an end restraint and in particular, the torsional stiffness K_s of the connection component. They investigated the behaviour of fin-plates and noted that the torsional stiffness varied almost continuously with the applied moment.

3. PARAMETRICAL STUDY ON BEAM ELASTIC STABILITY

The elastic critical moment expression considers the beam as completely prevented from twisting at the end supports. If the supports have only limited elastic torsional restraint stiffness, as it is the case of some practical connections such as fin-plates, the beam will buckle at a lower load than that estimated from the idealised case. Therefore, the aim of this work is to derive an analytical expression for the torsional end restraint k_θ to be introduced in the expression of the elastic critical moment M_{cr}.

3.1 Proposed Expression of the Torsional Restraint Coefficient at Supports

The behaviour of beams is dependent on their end support conditions and possibly on their intermediate supports. These conditions depend not only on major axis bending (primary bending) but also on minor axis bending, uniform torsion and warping torsion. The latter three types of support conditions influence deeply the *LTB* resistance. In the existing codes, the support conditions are accounted for by means of so-called effective length factors k_z and end warping factor k_w. Each of these two factors varies from 0,5 for full fixity to 1,00 for no fixity at all, and takes the value of about 0,7 for one end fixed and one end free. In the EC3 ENV [5], it is suggested to take k_w=1,0 unless special provision for warping fixity is made.

Different expressions for computing the torsional restraint k_θ of the end support have been considered, and the following one Eq. (6) is finally selected, even though further research might provide a more exact formulation.

$$k_\theta = \sqrt{1 + 5\frac{GI_t / L}{K_\Theta}} \tag{6}$$

where: L is the unbraced length, G is the shear modulus, I_t is the torsional constant and K_Θ is the torsional restraint of the support .

Eq. (6) shows that if no torsional restraint is provided by the support ($K_\Theta = 0$), then the torsional restraint coefficient k_θ is infinity, and for full torsional restraint of the support (K_Θ=infinity), the torsional restraint coefficient k_θ is 1,00.

For doubly symmetrical cross-section and for end moment loading or transverse loads applied at the shear centre, the elastic critical moment to be considered as the critical value of the maximum moment in the beam may be assessed by the new proposed formula:

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(k_\theta k_z L)^2} \sqrt{(\frac{k_\theta k_z}{k_w})^2 \frac{I_w}{I_z} + \frac{(k_\theta k_z L)^2 GI_t}{\pi^2 EI_z}} \tag{7}$$

where k_θ is the torsional restraint coefficient which varies from 1,00 (for support prevented from twisting about longitudinal axis) to infinity (for free twisting of the support about longitudinal axis).

Eq. (7) may be written in its simplified form as:

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{k_\theta (k_z L)^2} \sqrt{(\frac{k_z}{k_w})^2 \frac{I_w}{I_z} + \frac{(k_z L)^2 GI_t}{\pi^2 EI_z}} \tag{8}$$

It can be seen that Eq. (8) is the same as the one given by EC3 ENV [5], only for a beam fully prevented from twist rotation at the supports ($k_\theta = 1$). It can also be seen from Eq. (8) that for a support connection providing no torsional restraint (k_θ= infinity), the elastic critical moment M_{cr} tends towards zero and therefore the beam will be in the state of instability.

Finally Eq. (8) can be expressed as:

$$M_{cr} = \frac{1}{k_\theta} M_{0cr} \tag{9}$$

It can be seen from Eq. (9), that the critical moment M_{cr} can be obtained by multiplying the critical moment M_{0cr} obtained for a beam with full torsional restraint at supports ($k_\theta = 1$) by $1/k_\theta$.

4. ANALYTICAL EVALUATION OF THE ELASTIC STABILITY

4.1 Influence of End Torsional Restraint

In order to perform a comparative study, numerical analysis was conducted using the LTBEAM software which was developed by CTICM [8] within the framework of a European project and based on the finite element method. In this study, the effect of the variation of support torsional restraint k_θ on the elastic critical moment M_{cr} is investigated numerically and analytically with the application of the EC3 ENV formulation. Therefore it is recommended that the effect of torsional restraint provided by end supports of the beam should be taken into account in the EC3 ENV formula by means of a coefficient k_θ. The values of k_θ may be obtained using the proposed formula (6).

The lateral-torsional buckling of four IPE profiles (IPE 300, IPE 360, IPE 400 and IPE 500) with three different lengths (L=6m, L=10m and L=12m) have been studied for two load cases, uniformly distributed vertical loads applied to the shear centre and a concentrated vertical load at mid-span applied to the shear centre. In this analysis the end supports of the beam are assumed to be fixed for out of plane deflection ($v = 0$) and for lateral bending rotation ($k_z = 1$) but not restrained against warping ($k_w = 1$). The torsional restraint of the support is modelled by a spring of torsional restraint value K_Θ. If no torsional restraint is provided by the support (K_Θ=0) then $1/k_\theta$ =0 and for full torsional restraint of the support (K_Θ= infinity) then $1/k_\theta$ =1. For partial torsional restraints of the end supports, which correspond to the values of $1/k_\theta$ varying from 0 to 1,00, the corresponding values of the spring torsional restraints K_Θ can be calculated from Eq. (6).

Figs.3 to 6 show the numerical and analytical results of the variation of M_{cr} / M_{0cr} against variation of $1/k_\theta$ for the case of uniformly distributed vertical loads. According to Eq. (9), the graphs of M_{cr} / M_{0cr} versus $1/k_\theta$ for all IPE cross-sections and lengths obtained from the analytical expression through the application of the new proposed formula are represented by a single straight line. It can be seen from the figures that the numerical results of M_{cr} / M_{0cr} against $1/k_\theta$ obtained through LTBEAM software in which the values of the spring torsional restraints K_Θ are computed from the proposed analytical formula (6), are represented by curved lines.

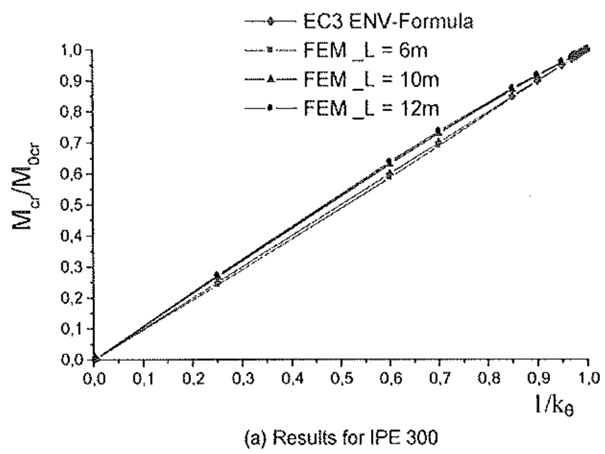


Fig. 3: Numerical and analytical results for IPE 300 and a uniformly distributed load

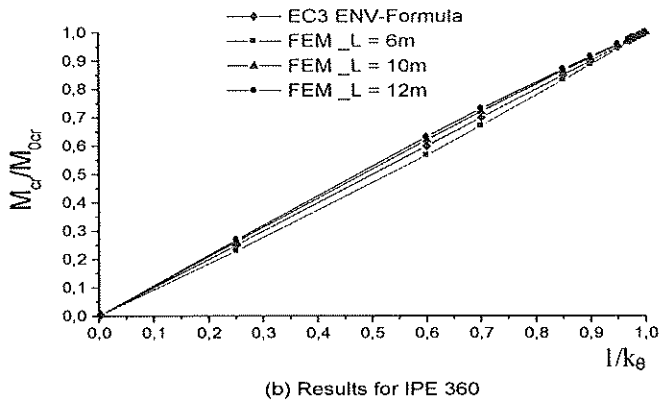


Fig. 4: Numerical and analytical results for IPE 360 and a uniformly distributed load

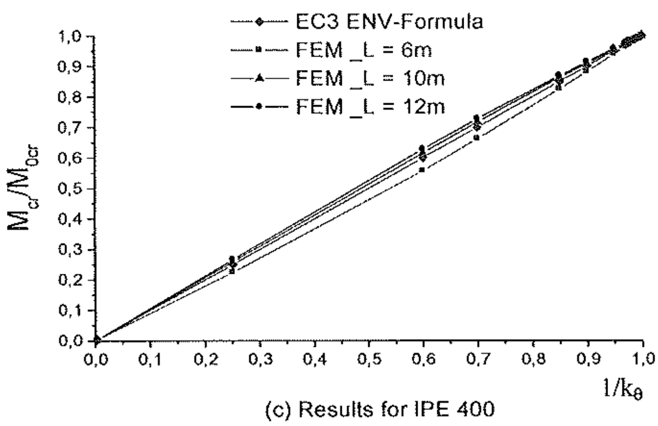
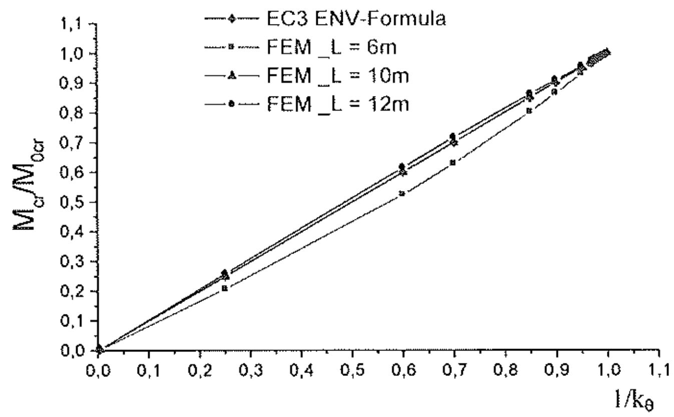


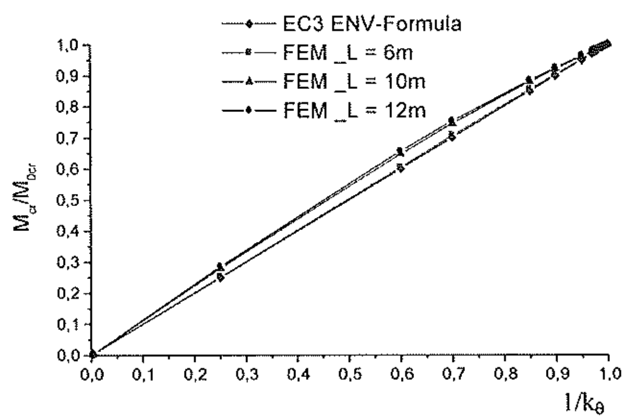
Fig. 5: Numerical and analytical results for IPE 400 and a uniformly distributed load



(d) Results for IPE 500

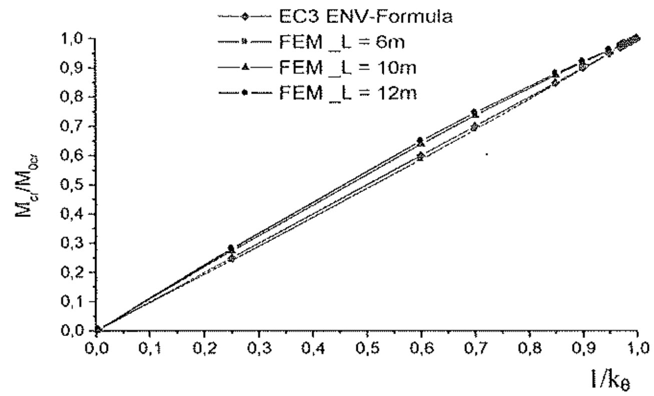
Fig. 6: Numerical and analytical results for IPE 500 and a uniformly distributed load

Figs.7 to 10 show the numerical and analytical results of the variation of M_{cr} / M_{0cr} against variation of $1/k_\theta$ for the case of a concentrated vertical load at the mid-span acting at the shear centre. Again it also show that the numerical results of M_{cr} / M_{0cr} against $1/k_\theta$ obtained through LTBEAM software in which the values of the spring torsional restraints k_Θ are computed from the proposed analytical formula (6) are represented by curved lines. It is worth noting from Figs.3 to 10 that, the closer the M_{cr} / M_{0cr} graphs to the straight line, the more accurate the formula (6). Therefore it can be seen from the figures that, for both load cases and for all IPE cross-sections and lengths considered in this study, the graphs of M_{cr} / M_{0cr} obtained from FEM analysis are particularly close to the ones obtained from the proposed analytical expression (eq.9), in which the torsional restraint coefficient k_θ has been introduced. In other words, comparison of the cases performed in this analysis revealed that an acceptable small difference exists between analytical and numerical results (errors are within about 10%). Therefore, there is quite good agreement between the results given by the proposed analytical formula and the numerical results of the FEM approaches.



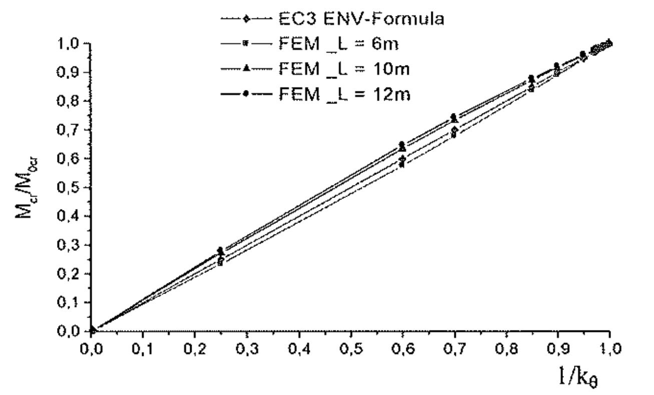
(a) Results for IPE 300

Fig. 7: Numerical and analytical results for IPE 300 and a concentrated vertical load



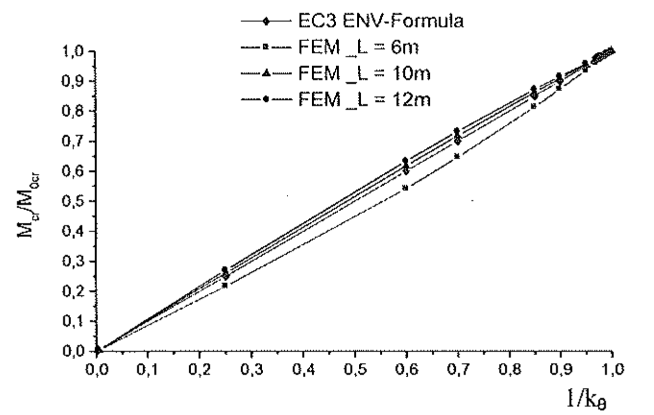
(b) Results for IPE 360

Fig. 8: Numerical and analytical results for IPE 360 and a concentrated vertical load



(c) Results for IPE 400

Fig. 9: Numerical and analytical results for IPE 400 and a concentrated vertical load



(d) Results for IPE 500

Fig. 10: Numerical and analytical results for IPE 500 and a concentrated vertical load

5. REQUIRED CONNECTION TO ENSURE SUFFICIENT RESTRAINTLEVEL

Using the analytical model developed in section 3.1, it is now possible to evaluate the torsional restraints for any beam end connection, therefore its classification can be made.
If a connection torsional restraint results in less than 10% drop of M_{cr} from M_{0cr}, then full torsional restraint connection can be assumed, otherwise, it is considered as partial tortional restraint connection.
Finite elements analyses have been performed to verify the accuracy of Eq. (6).
The analyses show that for beams with IPE profiles subjected to uniformly distributed loads or, a concentrated point load at the mid span acting at the shear centre and for k_z=1, Eq. (6) can be satisfactory applied to simulate the effect of the torsional restraint of the end support k_θ.
According to the results obtained from the proposed expression of k_θ given in section 3.1, it is recommended that if the percentage drop in the value of the elastic critical moment M_{cr} against M_{0cr} for full restraint remains within 10%, then full restraint may be assumed.
Figs. 3 to 10 show the values of $\frac{1}{k_\theta}$ for percentage drops in the value of the elastic critical moment M_{cr}, against M_{0cr} for full torsional restraint.
For all IPE cross-sections, beam lengths and load cases performed in this analysis, 10% reduction in the value of $\frac{1}{k_\theta}$, results in 10% drop in the value of M_{cr} against M_{0cr} for full torsional restraint.
According to results in section 3.1, for IPE cross-sections under uniformly distributed loads or a concentrated point load at mid span acting at the shear centre, Eq. (6) provides values of end torsional restraint coefficients k_θ that are in very good agreement with the FEM results.
It can be seen from Figs. 3 to Fig. 10 and Eq. (6) that for 10% drop in the value of the elastic critical moment M_{cr} against M_{0cr} for full torsional restraint, it results in a value of $\frac{1}{k_\theta}$=0,9, which corresponds to a value of the torsional stiffness of the support K_Θ=21,3 GI_t / L. Thus, for $\frac{K_\Theta}{GI_t / L}$=21,3 (ratio of the torsional stiffness of the supports to its beam), the value of the elastic critical moment is $M_{cr} = 0,9 M_{0cr}$.
From these results, it is recommended to assume that the torsional stiffness of the connection is acceptable and may be considered as full torsional restraint, if it results in no more than 10% drop in the value of M_{cr} for full restraint. Therefore, if it is proved that the ratio of the torsional stiffness of a connection K_Θ to its beam GI_t / L is at least equal to 21,3, then it is recommended to assume full restraint connection, as M_{cr} will be 90% of M_{0cr} for rigid torsional end support.

6. CONCLUSIONS

When dealing with lateral torsional buckling, modern design standards require the computation of the elastic critical moment, which mainly depends on the moment distribution along the beam and on the end supports restraints.

This paper presents a review of EC3 ENV approach and its limitations with regards to lateral bending and torsional restraints of the end supports. Based on these limitations, the paper has presented a new expression for estimating the actual degree of torsional restraint k_θ of the end supports. The value of the coefficient k_θ obtained from the proposed expression, is introduced in the general formulae that estimates the elastic critical moment. The influence of the torsional restraint on the lateral-torsional buckling of IPE beams with various cross-sections, different loading conditions and lengths has been investigated using analytical and FEM approach. Comparison between these two approaches allows to show the accuracy of the proposed expression of k_θ.

The results of variation of M_{cr} / M_{0cr} versus $\frac{1}{k_\theta}$ computed from FEM are quite close to those obtained from EC3 (ENV) formula. Finally the following can be concluded from this study:

(i) Eq. (6) can be reasonably applied to evaluate torsional restraints of end supports.

(ii) Full torsional restraint of end supports may be assumed if it results in less than 10% drop in the value of elastic critical moment M_{cr} against full torsional restraint moment M_{0cr}.

(iii) To assume full torsional restraint of end supports, it is necessary that the ratio between the torsional stiffness of the supports and of the beam $\frac{K_\Theta}{GI_t / L}$ be at least 21,3.

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