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A two-step optimization approach for the optimal design of composite structures, including geometric non-linear behavior, design rules and manufacturing constraints

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Abstract

A two-step solution procedure for the optimal design of aircraft composite structures including conventional plies oriented at 0°, 45°, 90° and -45° is described. The structure is divided in several zones of possible different thickness and stacking sequences. In the first step of the optimization procedure, the optimal thickness of the plies oriented at 0°, 90° and ±45° is obtained, assuming that the laminates are homogeneous in each zone. Here, buckling and post-buckling finite element analyses can be conducted in the optimization loop, knowing that working with a geometric non-linear behavior allows providing more accurate safety margins and even allows decreasing the structural weight. This first step is solved with a gradient-based optimization algorithm working with continuous design variables. Once the optimal thickness and percentages of plies at 0°, 90°, 45° and -45° are obtained in each zone, a specific integer programming algorithm is used. This algorithm allows determining the optimal stacking sequence in each zone of the structure, while satisfying the ply continuity constraints across the regions (i.e. the blending of the plies). This constraint must be satisfied in order to produce a composite part which can be manufactured. Moreover, the stacking sequence in each zone satisfies specific design rules. The methodology is demonstrated on an academic example, as well as on the industrial case of a fuselage section made of a curved panel including several hat stiffeners limiting zones of possible different material properties.

1. Introduction

Composite structure optimization is a very complicated task. Let's consider the wing skin illustrated in Fig. 1. The plies are laid down on the structure, defining zones of different thickness. Usually, conventional orientations are used (0°, 45°, -45°, 90° plies). In each zone the optimal stacking sequence must be defined. This sequence must satisfy some design rules. The usual design rules require that the laminate must be balanced (i.e. the number of plies at -45° is equal to the number of plies at 45°), the laminate must be symmetric, there must be no more than Nmax successive plies with the same orientation in the laminate (Nmax is often equal to 3 or 4), the transition between two plies must be at most of 45°, that is [0/90] and [45/-45] sequences are forbidden, and finally, minimum and maximum percentages of each possible orientation must exist. As the plies cover different zones, these optimization problems are not local, and the ply continuity constraint across the zones must be taken into account in order to produce a composite structure with ply drops between the regions that can be manufactured (Fig. 2).

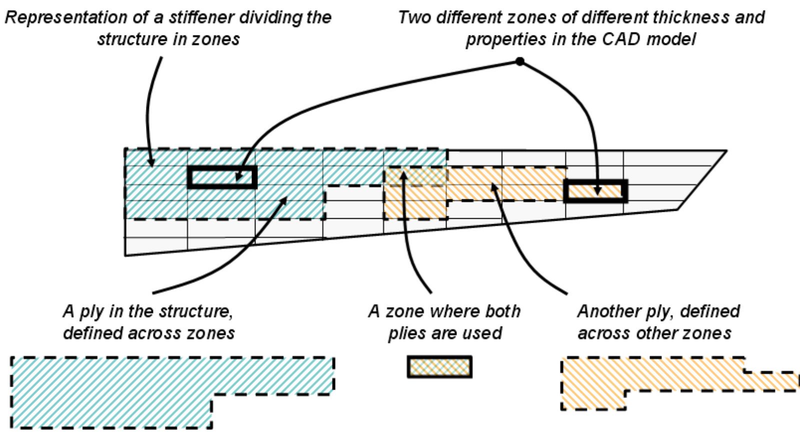

Figure 1. Definition of plies on the wing structure

Different approaches have been proposed over the last 20 years to solve, often partially, this complicated problem. In [1-8], genetic algorithms are use, with penalty methods. In [7,8], a sub-laminate approach is used to guarantee the continuity of the plies (the blending) in all the regions. In [9-12], the continuity of the plies is satisfied by a guide-based design. It gives blended structures but it does not provide a lot of flexibility in the design of the panel: the stacking sequence of the thickest region imposes the stacking sequences of the all other regions.

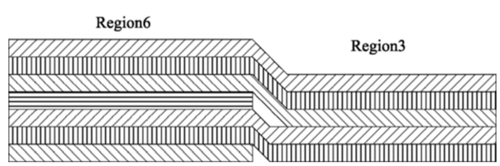

Figure 2. Ply drops between the regions

In [13], a penalty approach is used to overcome this problem, but according to the authors, this is not the most efficient approach. Another blending approach is the one described in [14-15], where the sequences of the regions are arranged into sets of plies which satisfy the blending principles. The approach in these two papers have the advantage of using the lamination parameters to compute the buckling instead of running expensive finite elements analyses. In [16], the stacking sequence optimization problem is formulated as a linear integer programming problem where the orientation of each ply is modeled with four binary variables, used to derive a mathematical expression of the manufacturing constraints. The proposed approach cannot handle the general case of a panel with regions of different thicknesses. The same drawbacks have been found with the topology optimization approach proposed in [17-19], which is able to optimize the buckling load with the manufacturing constraints but for a fixed blending scheme. In [20], a combinatorial method to optimize a buckling load with respect to the stacking sequence guide and the ply drop-offs is proposed, but the thicknesses were constant. In [21], the design of a composite panel is formulated as a bi-level integer programing where the weight of the panel is minimized subject to the buckling load higher than a safety threshold. The thicknesses of the regions are expressed in number of plies and they are updated together with the stacking sequence guide and the ply drop-off. The manufacturing and design rules are satisfied using the backtracking approach proposed in [20]. This approach can be generalized to a reserve factor of any type.

2. Solution procedure

In this work, a bi-level solution procedure is proposed to solve the composite structure optimization problem. As depicted in Fig. 3, the first level addresses the optimization problem with a gradient-based optimizer and continuous design variables. At that stage, the structure is divided in different regions, and the goal is to determine in each region the optimal thickness of the plies at 0°, 90°, and 45°, assuming that the laminate is balanced and homogenized. Before entering step 2, a rounding-off is done, in order to translate the optimal thicknesses to an equivalent number of plies. In step 2, a specific integer programming approach is used, together with a particular parameterization of the optimization problem, and provides in each region the optimal stacking sequence satisfying some design rules and the ply continuity constraint. The ingredients of the two steps are described in the next sections.

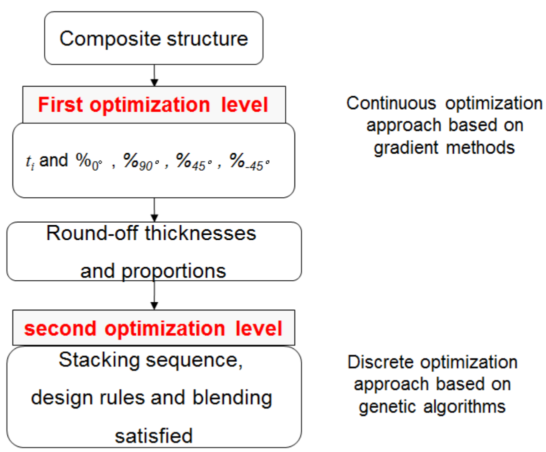

Figure 3. Principle of the bi-level optimization scheme

3. Step 1: gradient-based optimization with continuous design variables

3.1. Parameterization of the optimization problem

In the first step of the solution procedure, the optimal thickness of the plies oriented at 0°, 90° and ±45° is obtained assuming that the laminates in each region are homogeneous and balanced, as expressed by equations (1) where t is the total laminate thickness. Therefore, there are 3 design variables per region. Working with such an assumption allows decreasing the number of design variables and getting rid of the notion of stacking sequence. However, the resulting simplification in the definition of the out-of-plane stiffness (1) may result in an approximation of the buckling behaviour of the composite structure, especially since the bending-torsion coupling coefficients of the **D** matrix may have an influence on the stability behavior.

$$A_{16} = A_{26} = 0 \qquad D_{ij} = {A_{ij}t^2}/{12} \qquad B_{ij} = 0 \tag{1}$$

3.2 Problem formulation

At that stage, the optimization problem consists in minimizing the weight of the whole structure, with restrictions on the buckling load factors. These factors are determined with a linear buckling analysis (eigen-value problem). It is also possible to include a non-linear static analysis (based on the arc-length method) in the optimization loop, in order to control the collapse load. It was demonstrated in [22] that using such a non-linear analysis is beneficial for weight saving, as the stability behaviour of the structure is better represented compared to a solution where only linear buckling is considered. A large number of buckling loads are computed and used in the optimization, as this will limit some numerical instabilities appearing in the optimization problem when the weight is minimized [23]. The optimization problem is given in (2), where n is the number of regions:

$$\begin{gathered}
\min_{\mathbf{t}} w(\mathbf{t}) \\
\lambda_j(\mathbf{t}) \geq \underline{\lambda} \qquad j = 1,\ldots,m \\
\lambda_{collapse}(\mathbf{t}) \geq \underline{\lambda}_{collapse} \\
\underline{t}_i \leq t_i \leq \bar{t}_i \qquad i = 1,\ldots,3n \\
\mathbf{t} = \left\{ t_i^{\theta}, i = 1,\ldots,n\,; \theta = 0°, 90°, 45° \right\}
\end{gathered} \tag{2}$$

where w is the structural weight to be minimized, λ_j is the j^{th} buckling load, $\lambda_{collapse}$ is the collapse load, and **t** is the set of ply thicknesses, which must satisfy the side constraints. At the optimum, the buckling and collapse loads must be larger than the prescribed values $\underline{\lambda}$ and $\underline{\lambda}_{collapse}$, respectively.

3.3. Optimization algorithm

The gradient-based methods used in the paper belong to the Sequential Convex Programming methods, SCP [24]. These are not purely Mathematical Programming methods, which would require too many iterations (and therefore structural analyses) to obtain the solution, but rather an approach where the solution of the initial non-linear optimization problem is replaced by the solution of successive convex approximated problems, based on Taylor-series expansions in terms of specific intermediate variables (Fig. 4).

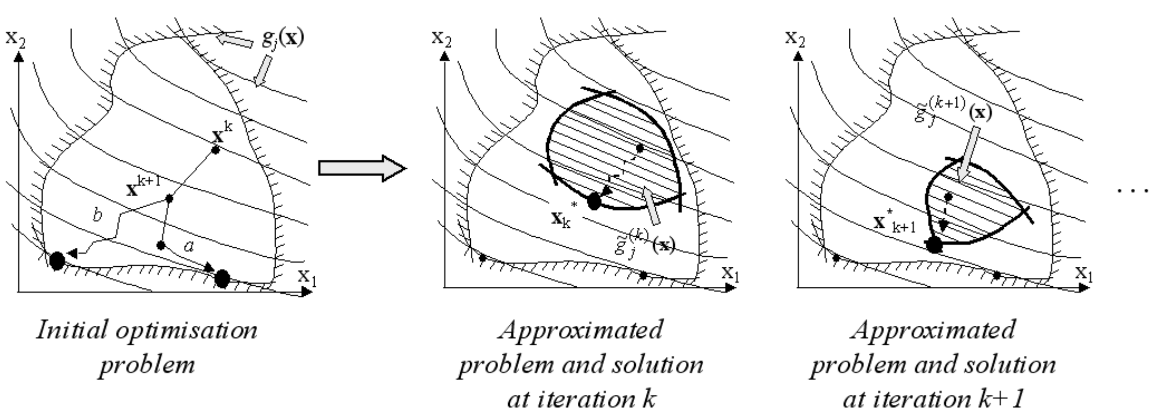

Figure 4. Principle of Sequential Convex Programming approach

Our own implementation of an adaptation of the GCMMA gradient-based optimizer is used [25-27]. This mixed approximation is based on the monotonous MMA and on an adaptation of the GCMMA non-monotonous approximation.

3.4. Sensitivity analysis

The first order derivative of the buckling load factor is well known. It is given by (3), where x_i is the considered design variable. This expression is based on the eigen-modes $\mathbf{\Phi}_j$ obtained when solving (1), and on the derivatives of the stiffness and initial stress matrices, **K** and **S**:

$$\frac{\partial \lambda_j}{\partial x_i} = \mathbf{\Phi}_j^T \left(\frac{\partial \mathbf{K}}{\partial x_i} - \lambda_j \frac{\partial \mathbf{S}}{\partial x_i} \right) \mathbf{\Phi}_j \tag{3}$$

In an industrial finite element code, the sensitivity of **K** and **S** is carried out at the element level with a finite difference scheme in order to provide a general procedure applicable to the whole library of finite elements. The resulting approach is then called semi-analytical sensitivity analysis, since it is based on the analytical expression (3) including derivatives obtained from finite differences. The sensitivity analysis of the collapse load is much more original and complicated. It is a semi-analytical sensitivity analysis computed based on the system of non-linear equations, including the additional equations from the arc-length method, at each converged step of the non-linear solution. It is based on the tangent stiffness matrix, and on the derivatives of the forces, which are computed by a central finite difference. All the details and the equations are given in [22].

4. Step 2: integer programming approach

Finding admissible sequences is not a trivial task given the combinatorial nature of the constraints. Most of the time, one cannot guess intuitively such sequences and computer-based algorithms must be used to perform this task. The easiest but not the most efficient way to find sequences which are admissible for a given ply drop-off is the so-called brute-force enumeration. It consists in enumerating all the sequence candidates and checking for each one its admissibility. The main disadvantage of this method is that its computational cost grows exponentially with the number of plies. For example, for 16 plies there are 4^{16}= 4294967296 candidates to be checked and for N=32 plies there are 4^32 ~1.844×10^{19} possibilities. A more sophisticated technique has to be used in order to decrease the number of candidates to be checked. Enumerating all possible sequences consists in building an enumeration tree like in Fig. 5. Each level of the tree represents a ply and each node has four children which are the four possible angle values of the next ply. The enumeration tree must have the number of plies +1 levels. A stacking sequence is a branch of the tree connecting the root to a leaf (the lowest node). One can see that the size of the tree grows exponentially with the number of plies and spanning the whole tree becomes quickly unfeasible. The idea of the backtracking is to span the entire tree and to check at each node the admissibility of the partial stacking sequence constituted by the branch going from the root to the current node. If the partial sequence violates the constraint, then all the sub-tree derived from the current node is eliminated from the enumeration tree. This pruning technique reduces considerably the size of the tree and makes the enumeration efficient. For example in Fig. 5, all the sub-sequences starting with (-45, 45), (0, 90), (45, -45) and (90, 0) are eliminated from the tree because they violate the 90° gap rule. The leaves of the tree are only the admissible sequences. The optimization algorithm is based on the backtracking procedure with a local search one. For more information, see [20].

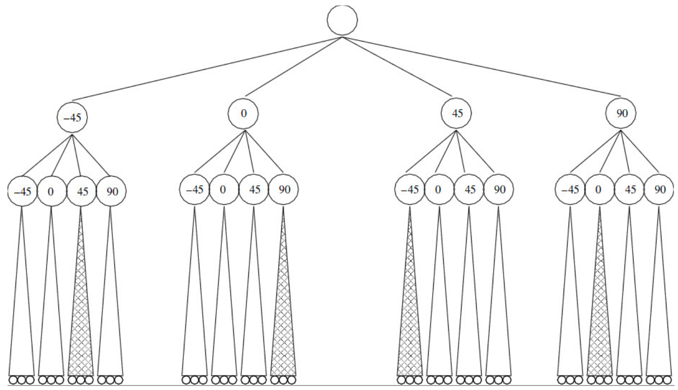

Figure 5. Principle of back-tracking algorithm

In Fig. 6, the general parameterization used to define the stacking sequence table is described. At each line corresponds a ply and a given orientation. Each column corresponds to a total number of plies and the associated stacking sequence. The orientation associated to each ply is variable and takes its value in the set {0°,45°,-45°,90°}. The lines can be permuted, so that the initial pyramidal solution is transformed into a general blending scheme of the plies. Design variables are associated to the position of each line. By permuting the order of the lines and changing the value of the ply orientation, one may identify optimal stacking sequences in zones of pre-defined thickness, as it is the case in Fig. 6 for zones made up of 24, 22 and 16 plies.

	24	22	20	18	16	14	12	10	8	6	4	2
45	1											
45	1	1										
45	1	1	1									
45	1	1	1	1								
90	1	1	1	1	1							
90	1	1	1	1	1	1						
90	1	1	1	1	1	1	1					
90	1	1	1	1	1	1	1	1				
-45	1	1	1	1	1	1	1	1	1			
-45	1	1	1	1	1	1	1	1	1	1		
-45	1	1	1	1	1	1	1	1	1	1	1	
-45	1	1	1	1	1	1	1	1	1	1	1	1

Sym.

	24	22	20	18	16	14	12	10	8	6	4	2
0	1	1	1	1	1							
0	1	1										
0	1	1	1	1	1	1	1	1	1			
45	1	1	1	1	1	1	1	1	1	1		
0	1	1	1									
0	1	1	1	1								
0	1											
45	1	1	1	1	1	1	1	1	1	1	1	
90	1	1	1	1	1	1	1	1	1	1	1	1
-45	1	1	1	1	1	1						
90	1	1	1	1	1	1	1					
-45	1	1	1	1	1	1	1	1				

Sym.

Figure 6. Design variables

5. Academic application

In this section, we consider the simple problem of a cantilever beam divided in 3 regions. The goal is to determine for the structure with a minimum weight the optimal stacking sequences in each region, while satisfying the design rules, the ply continuity constraint, as well as some mechanical restrictions on buckling, compliance and ply strength (Tsai-Wu criterion). The structure is submitted to compression and bending. The corresponding finite element mesh made of multi-layer shell elements, is depicted in Fig. 7.

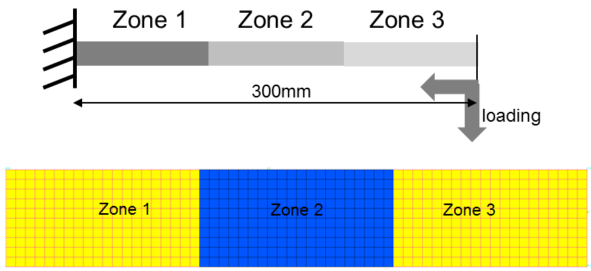

Figure 7. Academic use case

In the first step of the optimization problem, 3 design variables are defined in each zone, for a total of 9 design variables. The weight is minimized, and the first 20 buckling loads are required to be larger than 1.5, with an additional limitation on the displacement at the tip (stiffness constraint). The design variables take their value between 0.8mm and 4mm, and so all the orientations will be present at the solution, even if intuitively only 0° plies are relevant in the structure. The evolution of the weight and the buckling loads over the iterative process is illustrated in Fig. 8.

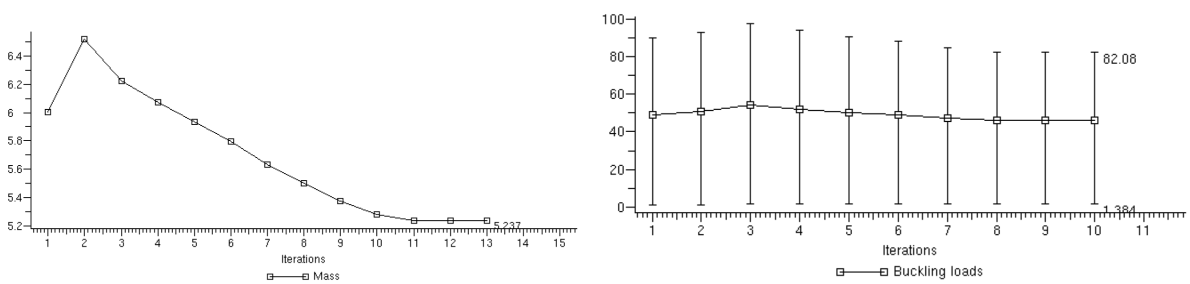

Figure 8. Convergence history for step 1

The optimal thicknesses and the corresponding number of plies in each region are reported in Table 1. It is assumed that the ply thickness is equal to 0.2mm. From the results, it is seen that there is a variation of the total thickness along the beam which is in agreement with what was expected: larger thickness at the clamping zone, and lowest value at the tip. The thickness for the plies at 90°, 45° and -45° reaches its lower bound.

Thickness (mm)/Number of plies	**Zone 1**	**Zone 2**	**Zone 3**
t_0/N_0	2.19/12	1.47/10	0.8/4
t_45/N_45	0.8/4	0.8/4	0.8/4
t_135/N_135	0.8/4	0.8/4	0.8/4
t_90/N_90	0.8/4	0.8/4	0.8/4
Total number of plies	24	22	16

Table 1. Solution of the first step of the optimization procedure

In the second step of the optimization process, some reserve factors are maximized. They impact the ply strength, the stiffness and the buckling loads. The weight is fixed, as determined in the first step (after the rounding-off of the continuous ply thicknesses). The reserve factors to maximize are written as follows:

$$2 - TW > 1 \qquad \frac{\bar{\delta}}{|\delta|} > 1 \qquad \frac{\lambda_k}{1.5} > 1$$

The sum of the reserve factors is maximized. It is checked that their value is larger than 1 at the solution. The initial lay-up for each region is illustrated in Fig. 9. The laminate is symmetric, and the reference lay-up includes 24 plies. The pyramidal scheme is used for the stacking sequence table, and the initial orientations (in the first column) take initial arbitrary values in the set of conventional orientations. The goal of the optimization will be to permute the lines of the stacking sequence table, and determine for each line the optimal fiber orientation.

	clamping		load
Pli number	Zone 1	Zone 2	Zone 3
1	45		
2	45	45	
3	45	45	
4	45	45	
5	90	90	90
6	90	90	90
7	90	90	90
8	90	90	90
9	-45	-45	-45
10	-45	-45	-45
11	-45	-45	-45
12	-45	-45	-45
13	-45	-45	-45
14	-45	-45	-45
15	-45	-45	-45
16	-45	-45	-45
17	90	90	90
18	90	90	90
19	90	90	90
20	90	90	90
21	45	45	
22	45	45	
23	45	45	
24	45		

Pli number	Zone 1	Zone 2	Zone 3
1	45		
2	45	45	
3	45	45	
4	45	45	
5	90	90	90
6	90	90	90
7	90	90	90
8	90	90	90
9	-45	-45	-45
10	-45	-45	-45
11	-45	-45	-45
12	-45	-45	-45
13	-45	-45	-45
14	-45	-45	-45
15	-45	-45	-45
16	-45	-45	-45
17	90	90	90
18	90	90	90
19	90	90	90
20	90	90	90
21	45	45	
22	45	45	
23	45	45	
24	45		

	24	22	20	18	16	14	12	10	8	6	4	2
45	1											
45	1	1										
45	1	1	1									
45	1	1	1	1								
90	1	1	1	1	1							
90	1	1	1	1	1	1						
90	1	1	1	1	1	1	1					
90	1	1	1	1	1	1	1	1				
-45	1	1	1	1	1	1	1	1	1			
-45	1	1	1	1	1	1	1	1	1	1		
-45	1	1	1	1	1	1	1	1	1	1	1	
-45	1	1	1	1	1	1	1	1	1	1	1	1

Sym.

Figure 9. Initial solution for step 2

The optimal solution is given in Fig. 10. It can be checked that the regions with 24, 22 and 16 plies, which are relevant in the problem, have stacking sequences satisfying the design rules.

	clamping		load
Pli number	Zone 1	Zone 2	Zone 3
1	0	0	0
2	0	0	
3	0	0	0
4	45	45	45
5	0	0	
6	0	0	
7	0		
8	45	45	45
9	90	90	90
10	-45	-45	-45
11	90	90	90
12	-45	-45	-45
13	-45	-45	-45
14	90	90	90
15	-45	-45	-45
16	90	90	90
17	45	45	45
18	0		
19	0	0	
20	0	0	
21	45	45	45
22	0	0	0
23	0	0	
24	0	0	0

Pli number	Zone 1	Zone 2	Zone 3
1	0		
2	0	0	
3	0	0	
4	45	0	
5	0	45	0
6	0	0	0
7	0	0	45
8	45	45	45
9	90	90	90
10	-45	-45	-45
11	90	90	90
12	-45	-45	-45
13	-45	-45	-45
14	90	90	90
15	-45	-45	-45
16	90	90	90
17	45	45	45
18	0	0	45
19	0	0	0
20	0	45	0
21	45	0	
22	0	0	
23	0	0	
24	0		

	24	22	20	18	16	14	12	10	8	6	4	2
0	1	1	1	1	1							
0	1	1										
0	1	1	1	1	1	1	1	1	1			
45	1	1	1	1	1	1	1	1	1	1		
0	1	1	1									
0	1	1	1	1								
0	1											
45	1	1	1	1	1	1	1	1	1	1	1	
90	1	1	1	1	1	1	1	1	1	1	1	1
-45	1	1	1	1	1	1						
90	1	1	1	1	1	1	1					
-45	1	1	1	1	1	1	1	1				

Sym.

Figure 10. Optimal solution for step 2

The iteration history is provided in Figure 11. It is seen that about 100 function evaluations are computed, which is quite low for a zero order algorithm. At the solution, the Tsai-Wu criterion is satisfied in each ply, what was not the case in the initial design of Fig. 9.

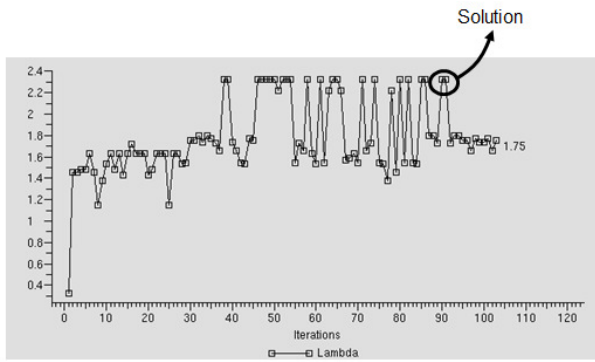

Figure 11. Convergence history for step 2

6. Industrial application

The structure depicted in Fig. 12 is studied. It is a portion a curved composite fuselage, made of 6 super-stiffeners. Each super-stiffener is built with a portion of panel and its corresponding hat stiffener. The structure is submitted to shear and compression, and is therefore sensitive to geometrical instabilities. Figure 12 illustrates the application of the two-step optimization procedure. In the first step, the optimal proportions of plies at 0°, 90° and 45° is determined in each super-stiffener (panel and stiffeners). In the second step, the backtracking algorithm is used and the optimal stacking sequences are obtained (in the panels only).

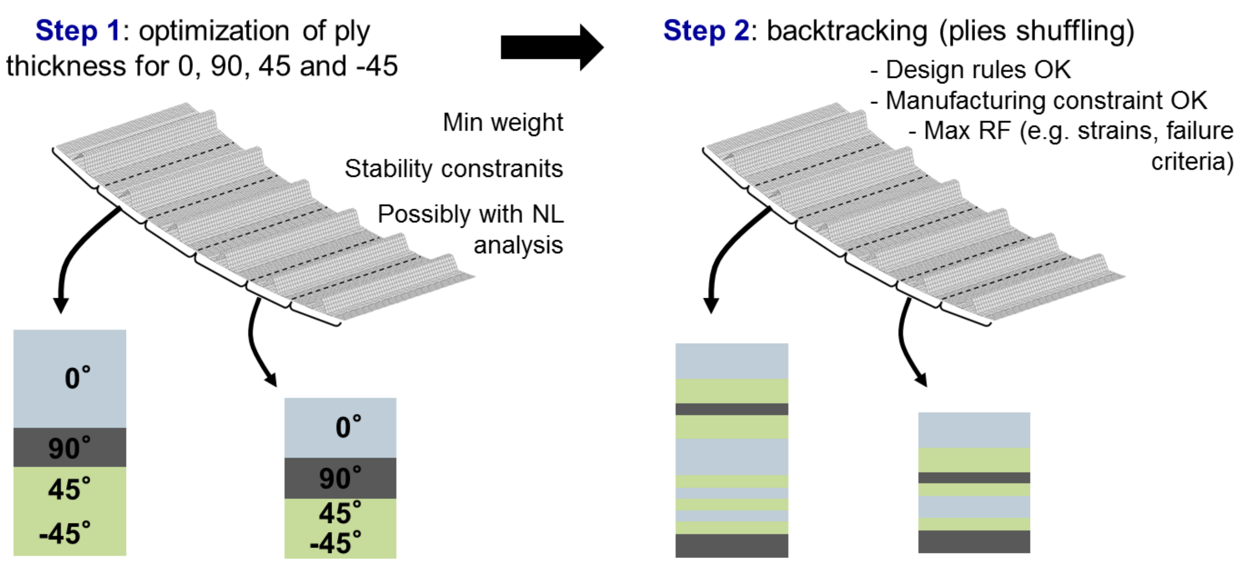

Figure 12. Application of the two-step approach to the industrial use case: principle of the approach

In Fig. 13, the convergence history for the step 1 is provided. The weight is minimized, and constraints are defined on the buckling and collapse loads. For linear buckling, an eigen-value analysis is conducted, while a non-linear static analysis is run with the arc-length method to determine the full non-linear equilibrium path, including post-buckling and collapse. The semi-analytical sensitivies for linear and non-linear responses are used. The solution is obtained in 9 iterations, and the non-linear equilibrium path is tuned thanks to optimization, meaning that the prescribed values of the buckling and collapse loads are reached at the solution. The solution is translated to a discrete number of plies.

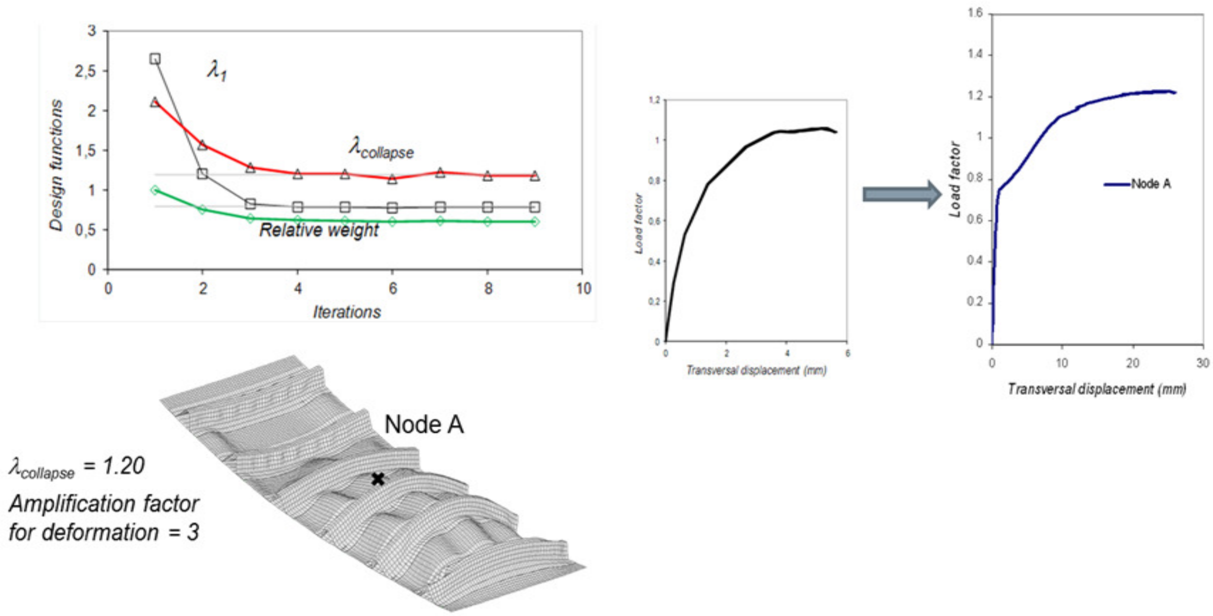

Figure 13. Solution of step 1

In step 2, the backtracking algorithm is used. The optimal stacking sequences satisfying the design rules and the ply continuity constraints are illustrated in Fig. 14.

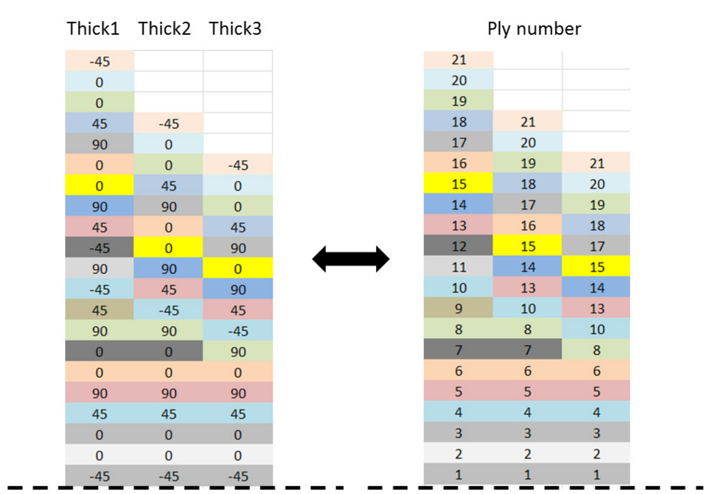

Figure 14. Solution of step 2

Conclusions

In this paper, a two-step optimization procedure for composite structures optimization was presented. It is based on the chaining of a continuous optimization step and a specific integer programming approach. It provides, in each region of the structure, optimal stacking sequences satisfying the design rules, and a solution that can be manufactured. The methodology was demonstrated on academic and industrial use cases.

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