Operational Modal Analysis using Second-Order Blind Identification

F. Poncelet[(1)], **G. Kerschen**[(1)], **J.C. Golinval**[(1)]

(1) Structural Dynamics Research Group
Aerospace & Mechanical Engineering Department
University of Liège, Liège, Belgium
E-mail: fponcelet,g.kerschen,jc.golinval@ulg.ac.be

ABSTRACT

For modal analysis of large structures, it is unpractical and expensive to use artificial excitation (e.g., shakers). However, engineering structures are most often subject to ambient loads (e.g., traffic and wind) that can be exploited for modal parameter estimation. One difficulty is that the actual loading conditions cannot generally be measured, and output-only measurements are available. This paper proposes to explore the utility of blind source separation (BSS) techniques for operational modal analysis. The basic idea of BSS is to recover unobserved source signals from their observed mixtures. The feasibility and practicality of the proposed method are demonstrated using an experimental application.

1 INTRODUCTION

For modal analysis of large structures, it is unpractical and expensive to use artificial excitation (e.g., shakers). However, engineering structures are most often subject to ambient loads (e.g., traffic and wind) that can be exploited for modal parameter estimation. One difficulty is that the actual loading conditions cannot generally be measured, and output-only measurements are available.

During the last few years, there have been several successful attempts to address this issue using operational modal analysis (OMA) techniques [1] [2]. Recently, signal processing techniques have been used to perform OMA through the estimation of the modal coordinates. For instance, Lardies et al. [3] exploit the wavelet transform to determine the response of each mode and to subsequently compute the modal parameters. In [4], output-only data are processed using the empirical mode decomposition (also known as Hilbert-Huang transform) to identify the different modal contributions. Digital band-pass filters are considered by Kim et al. [5] for the same purpose. Although attractive in principle, these signal processing-based methods present several drawbacks such as edge effects and difficulty in identifying closely spaced modes.

In this paper, we propose a new OMA method by borrowing one technique from the statistical literature. The technique, second-order blind identification (SOBI), decomposes measured signals in terms of elemental components. When SOBI is applied to the response of engineering structures, the elemental components are directly related to the modal coordinates.The feasibility and practicality of the proposed method are demonstrated using an experimental application and the results are compared with those of a well-established modal analysis method, the so-called Stochastic Subspace Identification [7].

2 FROM SIGNAL PROCESSING TO MODAL ANALYSIS

2.1 What is Blind Source Separation?

Blind Source Separation (BSS) techniques were initially developed for signal processing in the early 80's, but during the last decade the number of the application fields never stops increasing. This success certainly comes from two of their intrinsic characteristics. Firstly, the ambition of BSS (which is to recover unobserved source signals from their observed mixtures) is shared with many other research domains. Secondly, the small number of necessary assumptions allows to consider the application of the methodology to various kinds of data sets (resulting from fields as diverse as finance, image or speech processing, astrophysics, and even medicine).

However, if BSS techniques proved useful in numerous application domains, they were quite underused for many years in structural dynamics. Some applications were naturally carried out such as damage detection, condition monitoring and discrimination between pure tones and sharp-pointed resonances, but the modal parameter estimation remained quite marginal in these studies.

Recently, using the concept of virtual sources, a one-to-one relationship between the vibration modes and the BSS modes (i.e.. the mixing matrix) was demonstrated [6], allowing the use of BSS for modal analysis. Since then, two algorithms were tested, and one of them (namely the Second-Order Blind Identification) seemed to perform quite well [8].

2.2 Second-Order Blind Identification (SOBI)

The basic idea of BSS is to recover the unobservable inputs of a system, called the sources s_i, only from the measured outputs x_i even though very little, if anything, is known about the mixing system. The simplest BSS model assumes the existence of n sources signals $s_1(t), ..., s_n(t)$ and the observation of as many mixtures $x_1(t), ..., x_n(t)$. Note that we focus on systems with linear and static mixtures. Using matrix notations the noisy model can be expressed as

$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t) + \boldsymbol{\sigma}(t) \tag{1}$$

where $\boldsymbol{A}$ is referred to as the mixing matrix, and $\boldsymbol{\sigma}$ is the noise vector corrupting the data.

Most BSS approaches are based on a model in which the sources are independent and identically distributed variables. The objective of SOBI is to take advantage, whenever possible, of the temporal structure of the sources for facilitating their separation. The SOBI algorithm consists in constructing several time-lagged covariance matrices $\mathbf{R}(\tau)$ from the measured data and to find a matrix $\mathbf{U}$ which jointly diagonalizes all the covariance matrices. This matrix corresponds to the mixing matrix $\mathbf{A}$ of (1).

$$\mathbf{R}(\tau) = E[\mathbf{x}(t+\tau) \cdot \mathbf{x}^*(t)] \tag{2}$$

For further detail about the SOBI method, the reader can refer to [10].

2.3 Concept of Virtual Source

The dynamic response of mechanical systems which are considered in this study is described by the equation

$$\mathbf{M} \cdot \ddot{\mathbf{x}}(t) + \mathbf{C} \cdot \dot{\mathbf{x}}(t) + \mathbf{K} \cdot \mathbf{x}(t) = \mathbf{f}(t) \tag{3}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively. The vector $\mathbf{f}$ represents the real excitation sources applied to the structure. The system response $\mathbf{x}(t)$ may be expressed as a mixture of these **real sources** $\mathbf{f}(t)$. Unfortunately, this mixture is a convolutive product between the impulse response function, denoted $\mathbf{h}(t)$, and the sources $\mathbf{f}(t)$, and the separation of convolutive mixtures of sources is not yet completely solved.

An interesting alternative is to use the modal expansion. Indeed, the m normal modes $\mathbf{n}_{(i)}$ form a complete basis for the expansion of any m-dimensional vector (if m is the number of degrees of freedom). Then the response can be expressed using modal superposition

$$\mathbf{x}(t) = \sum_{i=1}^{m} \mathbf{n}_{(i)} \cdot \eta_i(t) = \mathbf{N} \cdot \boldsymbol{\eta}(t) \tag{4}$$

where the weight coefficients η_i are in fact the modal coordinates and represent the amplitude modulation of the corresponding normal modes $\mathbf{n}_{(i)}$. The similarity between equations (1) and (4) shows that the modal coordinates may act as **virtual sources** (which are statistically independent as proved in [?, ?]) regardless of the number and type of physical excitation forces. In addition, the time response can be interpreted as a static mixture of these virtual sources, which renders the application of the BSS techniques possible.

The SOBI algorithm (which requires sources with different spectral contents) is particulary appropriate for the separation of these sources. In the free response case of the system (3), the theoretical expression of the normal coordinates is an exponentially damped harmonic function

$$\eta_i(t) = Y \cdot exp(-\xi_i \cdot \omega_i \cdot t) \cdot cos(\sqrt{1-\xi_i^2} \cdot \omega_i \cdot t + \alpha_i) \quad (5)$$

where ω_i and ξ_i are the natural frequency and damping ratio of the i^{th} mode, respectively. The amplitude Y and the phase α are constants depending on the initial conditions. The modal coordinates are then monochromatic, with different spectral contents.

2.4 Procedure Details

In summary, a simple modal analysis procedure is proposed, using the modal coordinates as virtual sources. The procedure is as follows:

1. Perform experimental measurements of the structure response to obtain time series at different sensing position.
2. Apply SOBI directly to the measured time series to estimate the mixing matrix $\mathbf{A}$ and the sources $\mathbf{s}(t)$.
3. The mode shapes are simply contained in the mixing matrix $\mathbf{A}$.
4. In the case of random excitation, the identified (random) sources are transformed into free decaying responses using NExT (Natural Excitation Technique) algorithm [?].
5. The identification of the other modal parameters (frequencies and damping ratios) is carried out by fitting the time series of the sources $\mathbf{s}(t)$ with the theoretical expression (5).
6. The fitting error between the identified and fitted sources is then computed which allows to reject the non-reliable virtual sources easily.

The figure 1 describes this procedure.

3 EXPERIMENTAL DEMONSTRATION

To support the previous theoretical findings, the proposed OMA technique was applied to the response of the truss structure depicted in Figure 2. For the free response, a hammer provided a short impulse to the system. For the random response, the structure was mounted on a 26kN electrodynamic shaker, as shown in Figure 2. 16 accelerometers were distributed on the structure (two at each corner, eight on each storey), measuring its response in a horizontal plane. The results were also compared with another OMA technique, the (covariance-driven) stochastic subspace identification (SSI) technique [1].

3.1 Free Response

The free response was obtained using a hammer which provided a short impulse to the system. The sampling frequency was set to 5120 Hz, and the first 6000 samples of the measured time series were taken into account. The SOBI identification requires the definition of delays (for the construction of correlation matrices (2)). 20 delays were chosen uniformly distributed between 0.0025 and 0.1 seconds, which covers the whole frequency range of interest.

Because there are 16 measurement locations, a total of 16 virtual sources can be considered. The fitting error of each source is shown, in Figure 3(a). 11 sources have a fitting error below 7% and can be safely retained. The sum of their participation in the system response is above 97.7%. The identification results are listed in Table 1. Concerning the frequency and damping ratio

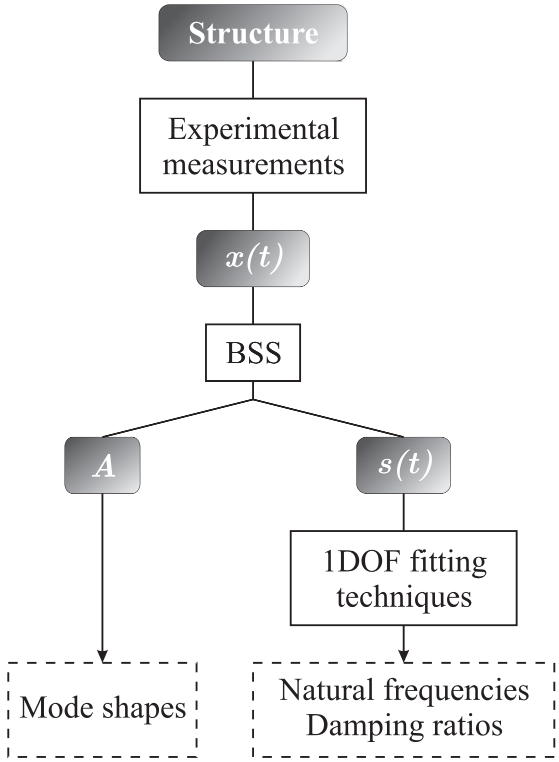


Figure 1: Process flowchart.

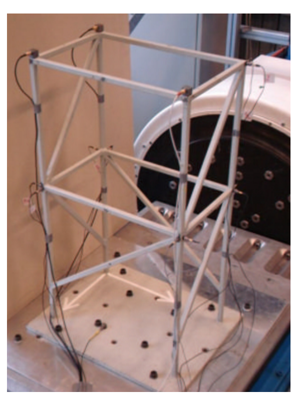
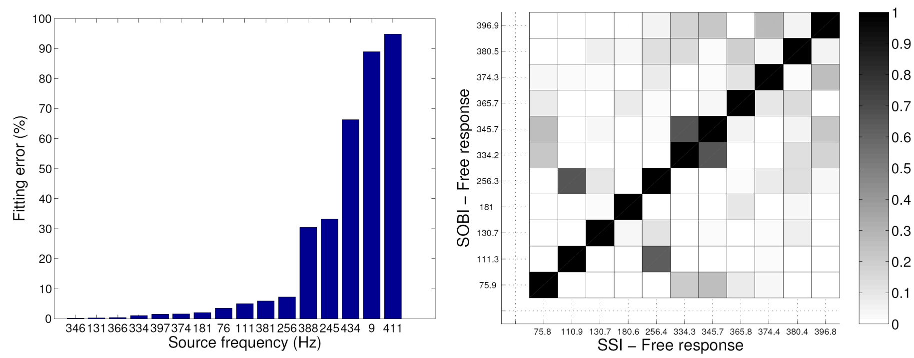


Figure 2: Experimental fixture mounted on a 26kN electrodynamic shaker.



(a) (b)

Figure 3: Fitting error of the 16 SOBI identified sources for the free response (a) and MAC comparison between SOBI and SSI modes (b)

identification, the SOBI results are totally similar to those of the SSI method. Note that the damping ratios of SSI are presented as intervals because the value changes according to the chosen model order. The comparison between the two methods for the mode shapes is performed using the Modal Assurance Criterion in the Figure 3(b). The closer the value to 1, the higher the correspondence. We can see a complete correlation between the modes identified using SSI and SOBI.

TABLE 1: Identified natural frequencies and damping ratios for the free response

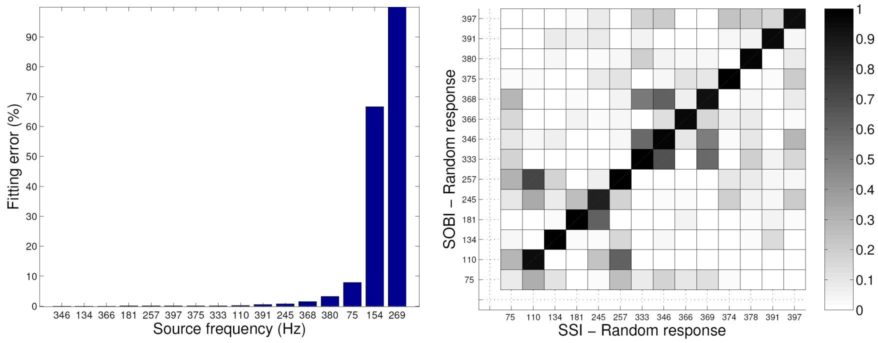
SOBI Freq. [Hz]	SSI Freq. [Hz]	SOBI Damping Ratio [%]	SSI Damping Ratio [%]
75.94	75.82	0.20	[0.05 - 0.12]
111.37	110.99	0.37	[0.40 - 0.60]
130.75	130.76	0.21	[0.20 - 0.28]
181.06	180.69	0.18	[0.20 - 0.28]
256.30	256.48	0.18	[0.10 - 0.15]
334.24	334.32	0.05	[0.02 - 0.05]
345.75	345.76	0.04	[0.04 - 0.05]
365.79	365.81	0.05	[0.05 - 0.06]
374.34	374.45	0.15	[0.10 - 0.30]
380.55	380.45	0.16	[0.20 - 0.40]
396.91	396.81	0.08	[0.07 - 0.10]

3.2 Forced Response

For the random response, the structure was mounted on a 26kN electrodynamic shaker (see Figure 2). The sampling frequency was set to 5120 Hz, and 160000 samples were considered for the measured time series. The same parameters as previously were chosen for the SSI and SOBI methods. The fitting error of each identified source was computed and is presented in Figure 4(a). This time, 14 sources have a fitting error below 8%.

Table 2 lists all the reliable identified results and Figure 4(b) compares the corresponding mode shapes. Once more the correspondence between both methods is remarkable, except for the mode at 75 Hz. If the results obtained using SSI in the free response case are taken as a reference we can note that none of the methods seems able to accurately estimate this mode. The MAC values SOBI random/SSI free and SSI random/SSI free are both lower than 0.65.

Finally, we note that the SSI method was able to identify 4 more modes in the frequency range considered (around 162, 189, 204



(a) (b)

Figure 4: Fitting error of the 16 SOBI identified sources for the random response (a) and MAC comparison between SOBI and SSI modes (b)

and 294 Hz). Nonetheless, because the participation in the system response of the 14 sources identified using SOBI amounts to 93%, these four modes have necessarily a very low participation in the system response.

TABLE 2: Identified natural frequencies and damping ratios for the random response

SOBI Freq. [Hz]	SSI Freq. [Hz]	SOBI Damping Ratio [%]	SSI Damping Ratio [%]
74.75	74.68	2.15	[1.70 - 2.00]
110.06	110.28	2.03	[1.50 - 2.00]
133.59	133.77	0.85	[0.60 - 0.80]
180.87	180.98	0.23	[0.20 - 0.30]
245.29	245.38	0.16	[0.01 - 0.05]
257.47	257.47	0.11	[0.09 - 0.11]
333.21	333.34	0.12	[0.05 - 0.10]
345.64	345.51	0.09	[0.10 - 0.12]
365.60	365.76	0.12	[0.07 - 0.15]
368.19	369.53	0.33	[0.15 - 0.30]
374.34	374.69	0.16	[0.20 - 0.40]
380.06	378.34	0.71	[0.50 - 0.70]
390.81	390.95	0.33	[0.45 - 0.50]
396.83	397.20	0.17	[0.15 - 0.25]

4 CONCLUSIONS

Based on the virtual source concept, a new application is developed for the BSS methods, and particulary for the SOBI algorithm, in the field of structural dynamics. An output-only modal analysis technique is proposed. The experimental application shows that the method holds promise for identification of mechanical system for free as well as for forced response.

- A truly simple identification scheme is proposed for the modal parameters, due to the straightforward application of SOBI to the measured data.
- A seemingly robust criterion has been developed for the selection of reliable sources. The use of stabilization charts, which always require a great deal of expertise, is therefore avoided. In addition, the selection of a model order, a common issue for conventional modal analysis techniques such as SSI, is not necessary.

- Compared to SSI, the computation load is very reduced, which makes the method a potential candidate for online modal analysis.

A possible limitation of the method is that sensors should always be chosen in number greater or equal to the number of active modes. This will be addressed in subsequent studies.

ACKNOWLEDGMENTS.

The author F. Poncelet is supported by a grant from the Walloon Government (Contract Nř 516108-VAMOSNL).

REFERENCES

[1] B. Peeters, G. De Roeck, *Stochastic system identification for OMA: A review*, Journal of Dynamic Systems Measurement and Control 123, 659-667, 2001.ţ

[2] R. Brincker, N. Moller, *Operational modal analysis - a new technique to explore*, Sound and Vibration 40, 5-6, 2006.

[3] J. Lardies, T.A. Minh Nghi, M. Berthillier, *Modal parameter estimation using the wavelet transform*, Archive of Applied Mechanics 73, 718-733, 2004.

[4] J.N. Yang, et al., *Identification of natural frequencies and dampings of in situ tall buildings*, Journal of Engineering Mechanics 130, 570-577, 2004

[5] B.H. Kim, N. Stubbs, T. Park, *A new method to extract modal parameters using output-only responses*, Journal of Sound and Vibration 282, 215-230, 2005.

[6] G. Kerschen, F. Poncelet, J.C. Golinval, *Physical interpretation of independent component analysis in structural dynamics*. Mechanical System and Signal Processing, Vol. 21 (2007) 1561-1575

[7] P. Van Overschee, B. De Moor, *Subspace Identification for Linear Systems: Theory, Implementation, Applications*. Kluwer Academic Publishers. Dordrecht (1996)

[8] F. Poncelet, G. Kerschen, J.C. Golinval, D. Verhelst *Output-only modal analysis using blind source separation techniques*. Mechanical Systems and Signal Processing. Volume 21, Issue 6, August 2007, Pages 2335-2358.

[9] P. Comon, *ICA: a new concept?* Signal Processing, Vol. 36 (1994) 287–314

[10] A. Belouchrani, K. Abed-Meraim, J.F. Cardoso, E. Molines *A BSS technique using 2nd-order statistics*. IEEE Trans. on Signal Processing, Vol. 45 (1997) 434–444