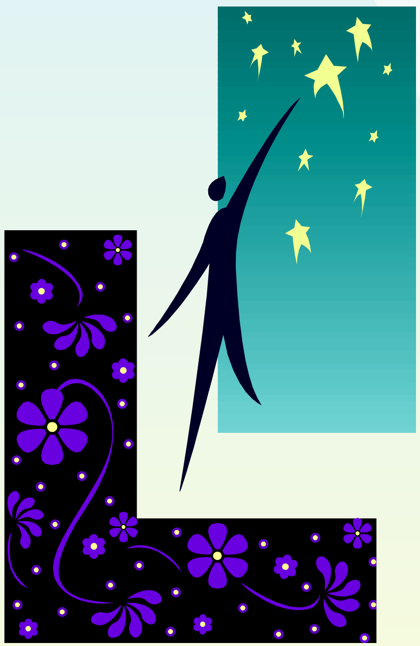
“La vraie faute est celle qu’on ne corrige pas …”

Confucius

An introduction to optical/IR interferometry

Brief summary of main results obtained during the last lecture:



$$V = \left|\gamma_{12}(0,u,v)\right| = \left|\iint_S I'(\zeta,\eta)\exp\left\{-i2\Pi\left(u\zeta + v\eta\right)\right\}d\zeta\, d\eta\right|$$

$$I'(\zeta,\eta) = \iint \gamma_{12}(0,u,v)\exp\left\{i2\Pi\left(\zeta u + \eta v\right)\right\}d(u)d(v)$$

- For the case of a 1D uniformly brightening star whose angular diameter is ϕ = b/z', we found that the visibility of the fringes is zero when λ/B = b/z' = ϕ where B is the baseline of the interferometer

- For the case of a double star with an angular separation ϕ = b/z', we found that the visibility of the fringes is zero when λ/2B = b/z' = ϕ

An introduction to optical/IR interferometry

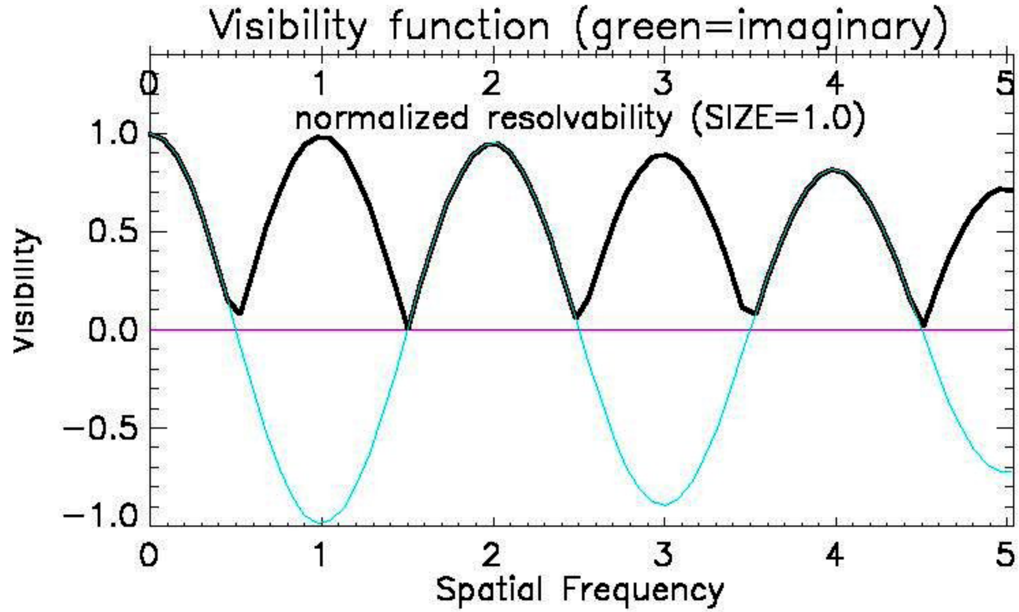
- 5 Light coherence
- **5.5 Aperture synthesis**

Exercises:

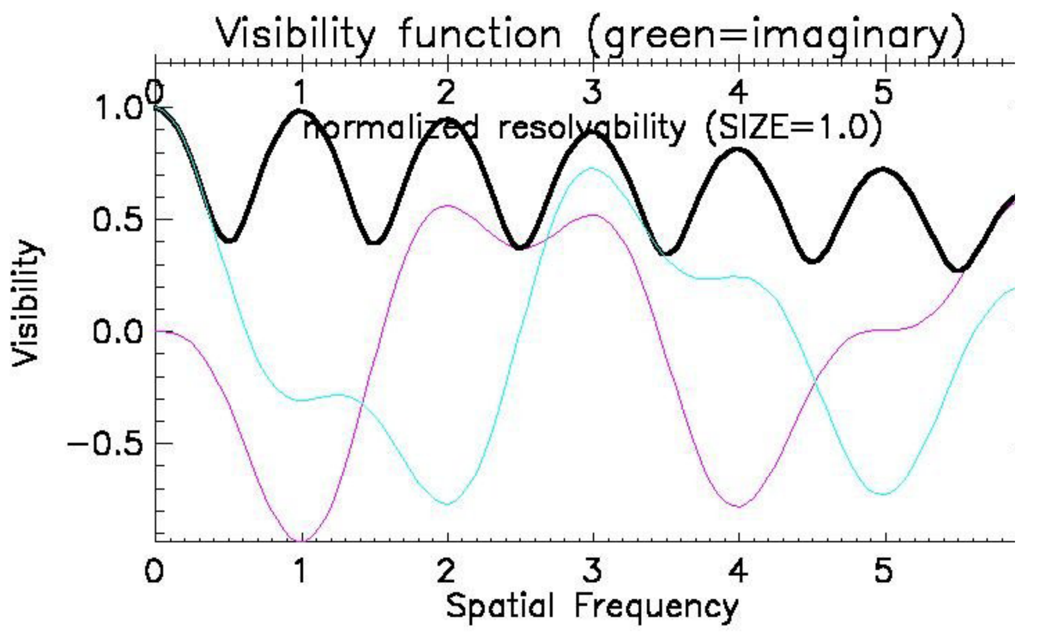
- the case of a gaussian-like source?

- let us assume that the observed visibility $|\Upsilon_{12}(0,u)|$ of a celestial object is $|\cos(\pi u\theta)|$, please retrieve the intensity distribution I' of the source

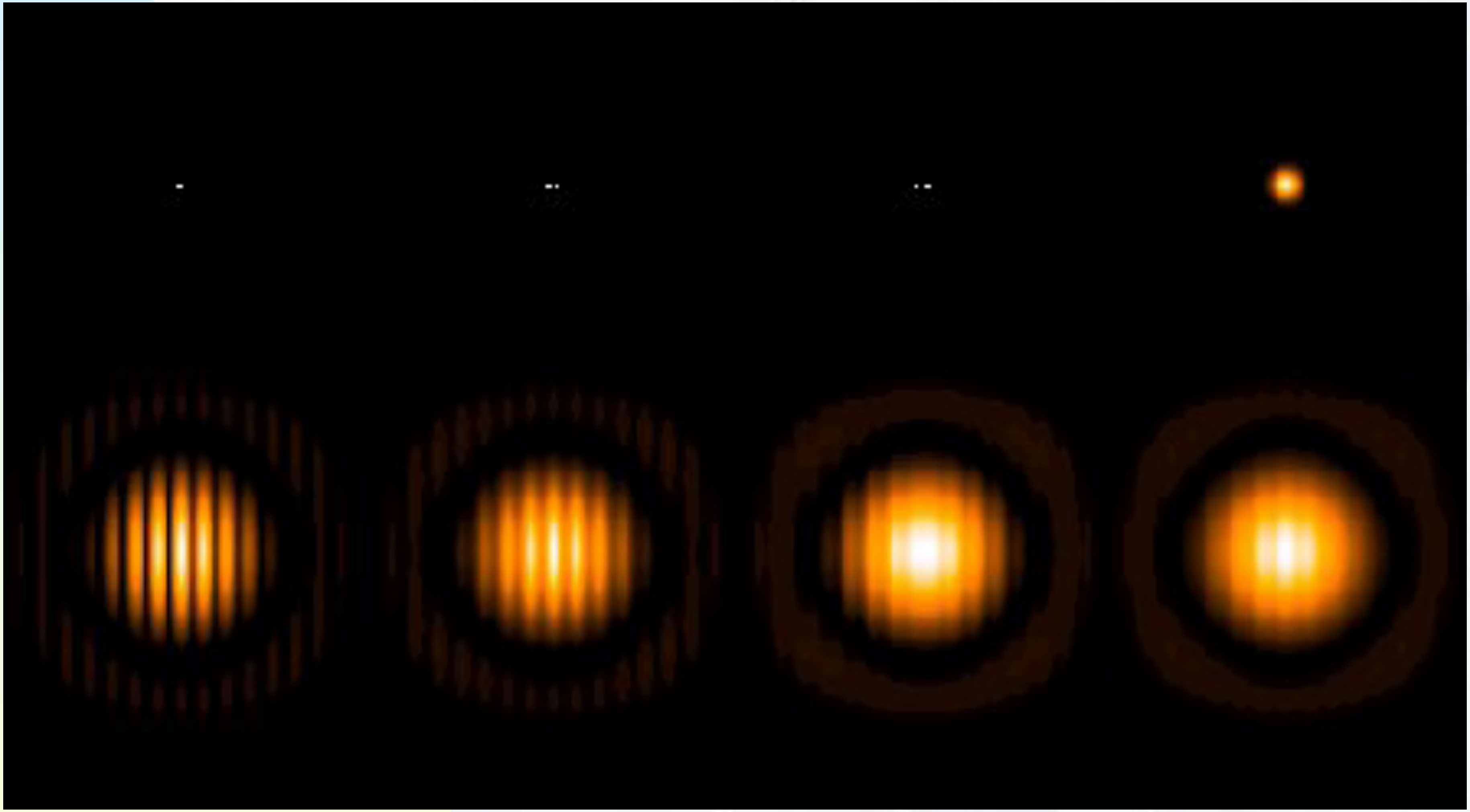
Case of a double point-like source with a flux ratio = 1



Case of a double point-like source with a flux ratio 0.7/0.3

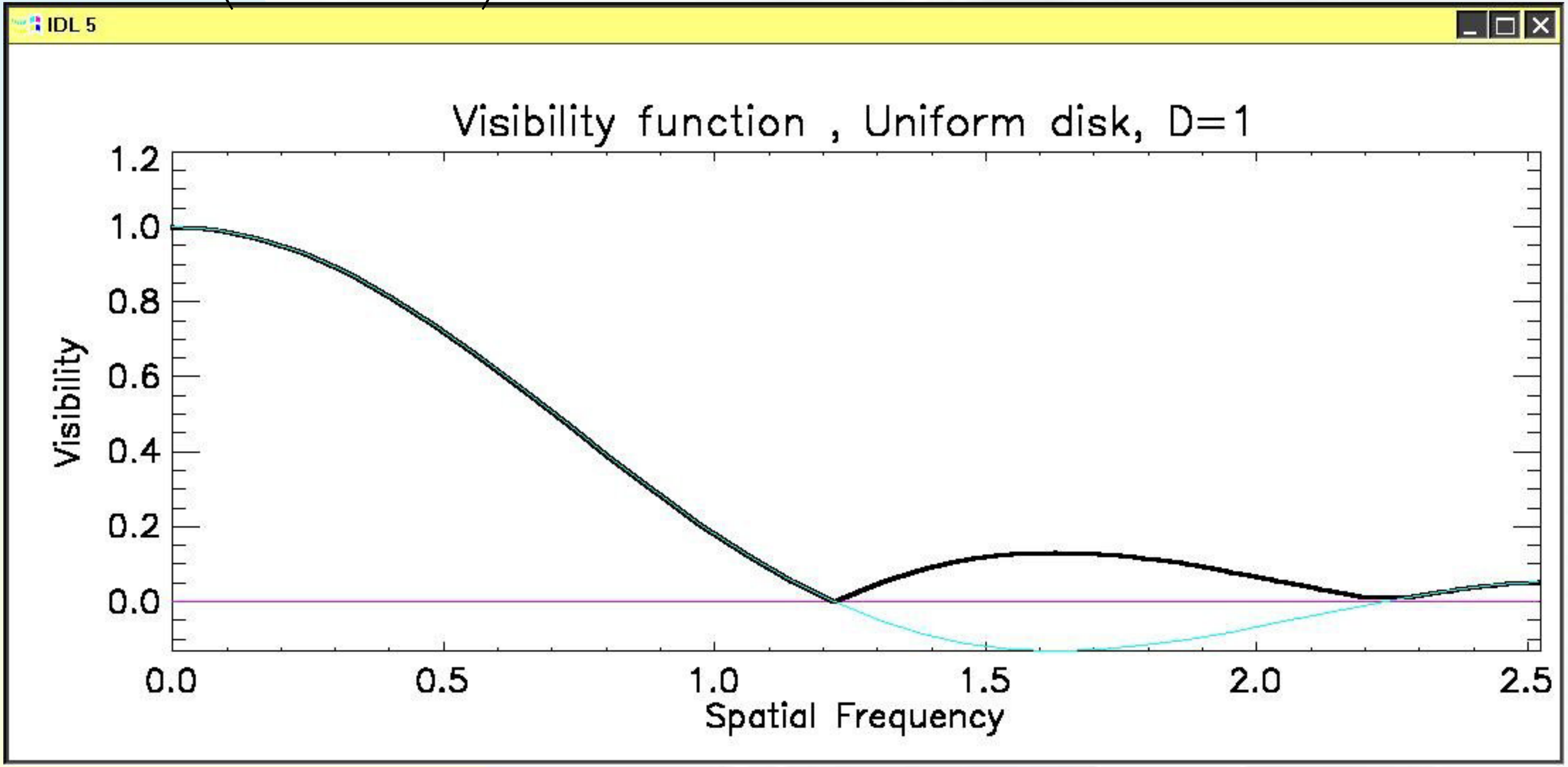


Variation of the fringe contrast as a function of the angular separation between the two stars:

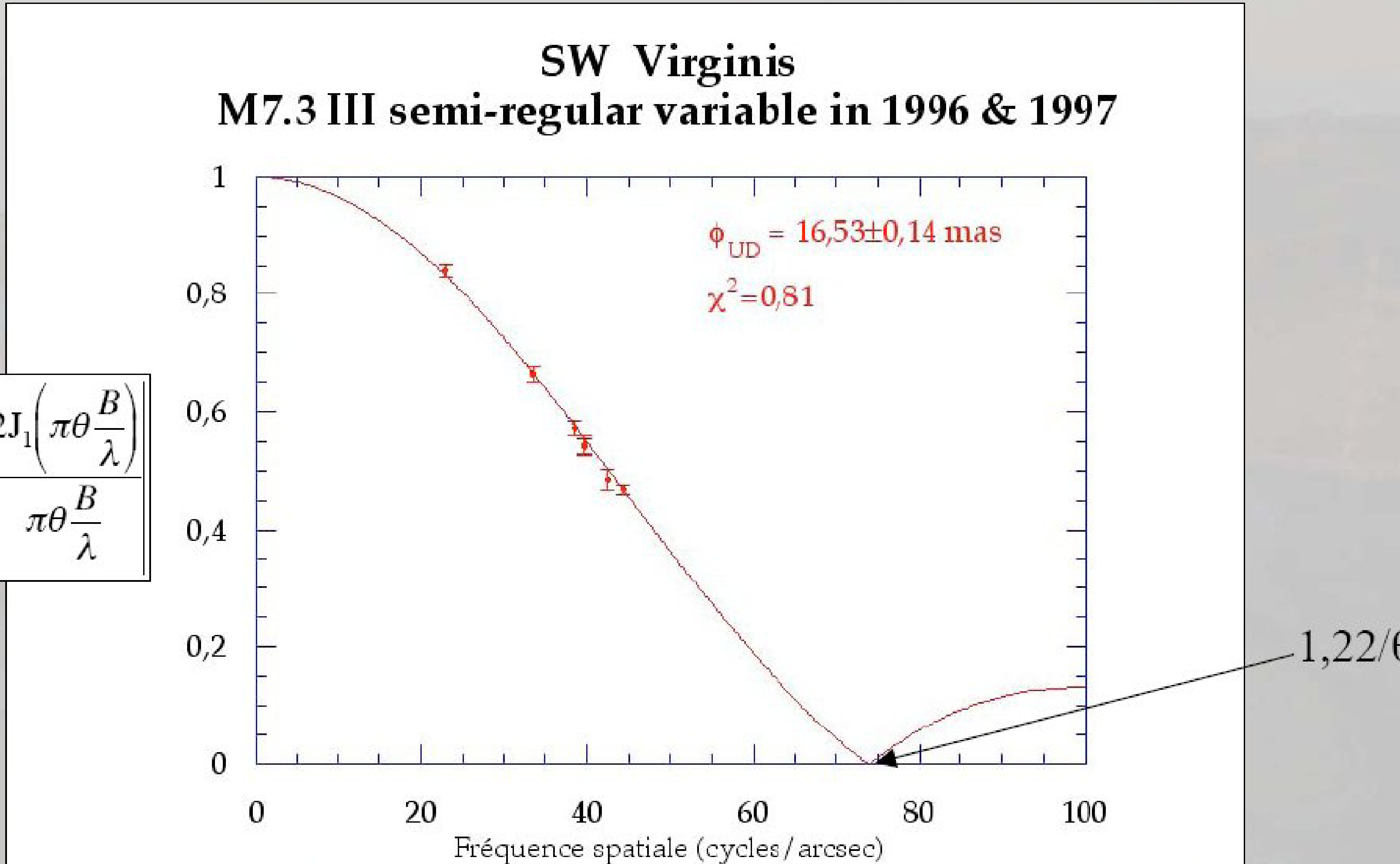


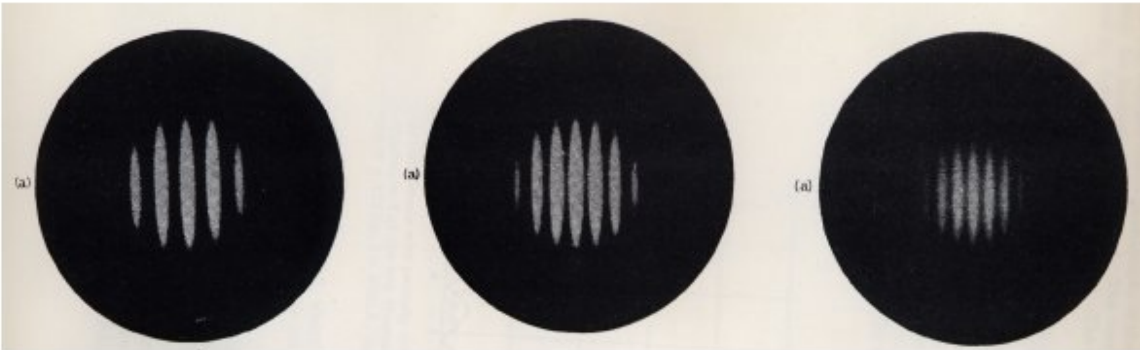
If the source is characterized by a uniform disk light distribution, the corresponding visibility function is given by

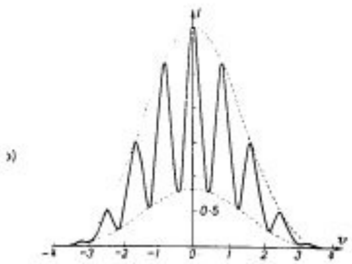
$$\upsilon = \left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \right) = \left| \gamma_{12}(0) \right| = TF(I) = \frac{2J_1(\pi\theta_{UD}B/\lambda)}{\pi\theta_{UD}B/\lambda}$$



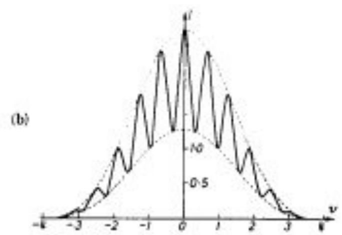
$$V_{DU}(B) = \left| \frac{2J_1\left(\pi\theta\frac{B}{\lambda}\right)}{\pi\theta\frac{B}{\lambda}} \right|$$



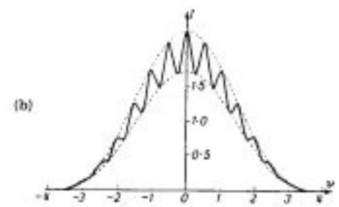




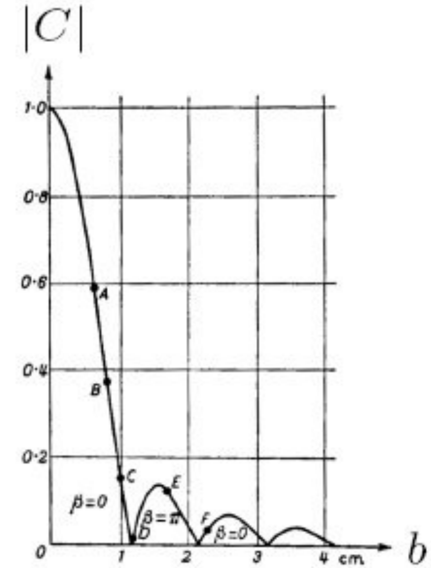
(A): d = 0·6 cm
$|\mu_{12}| = 0{\cdot}593,\ \beta_{12} = 0$



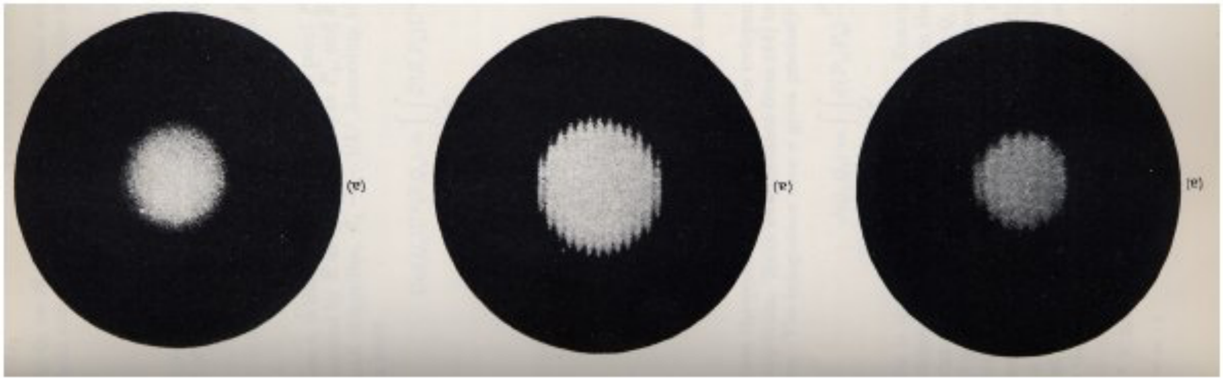
(B): d = 0·8 cm
$|\mu_{12}| = 0{\cdot}361,\ \beta_{12} = 0$

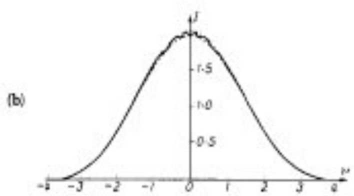


(C): d = 1 cm
$|\mu_{12}| = 0{\cdot}146,\ \beta_{12} = 0$

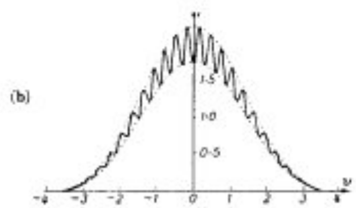


$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

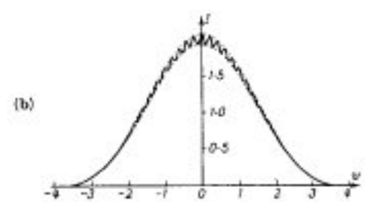




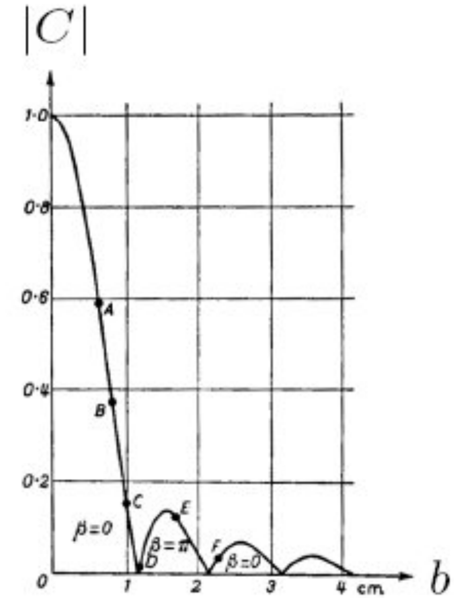
(D): $d = 1{\cdot}2$ cm
$|\mu_{12}| = 0{\cdot}015,\ \beta_{12} = \pi$



(E): $d = 1{\cdot}7$ cm
$|\mu_{12}| = 0{\cdot}123,\ \beta_{12} = \pi$



(F): $d = 2{\cdot}3$ cm
$|\mu_{12}| = 0{\cdot}035,\ \beta_{12} = 0$



$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C|\cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

For the case of the Sun:

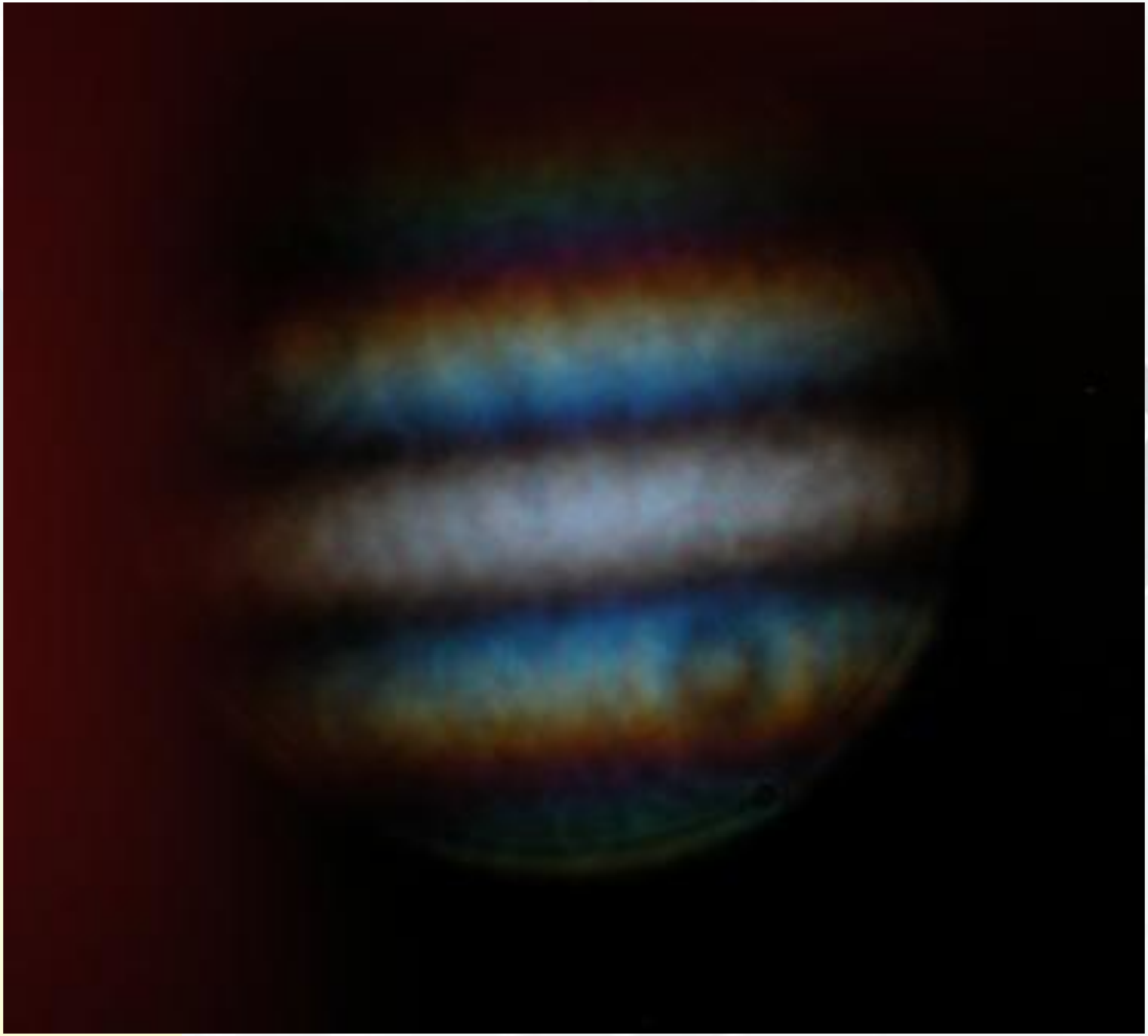
$\vartheta_{UD} = 1.22\lambda / B = 1.22\ 0.55\ / B(\mu) = 30' \times 60'' / 206265$

$B(\mu) = 206265 \times 1.22 \times 0.55 / (30 \times 60) = 76.9\ \mu$

$d(\mu) = 7.2$ or $14.4\ \mu$ ➔ $\sigma = 2.44\ \lambda / d = 7.8°$ or $3.9°$

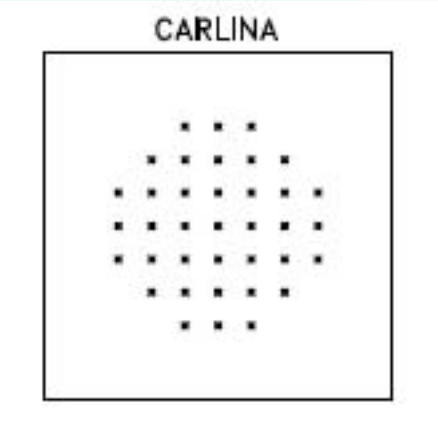
See the masks!

First fringes on the Sun: 9/4/2010



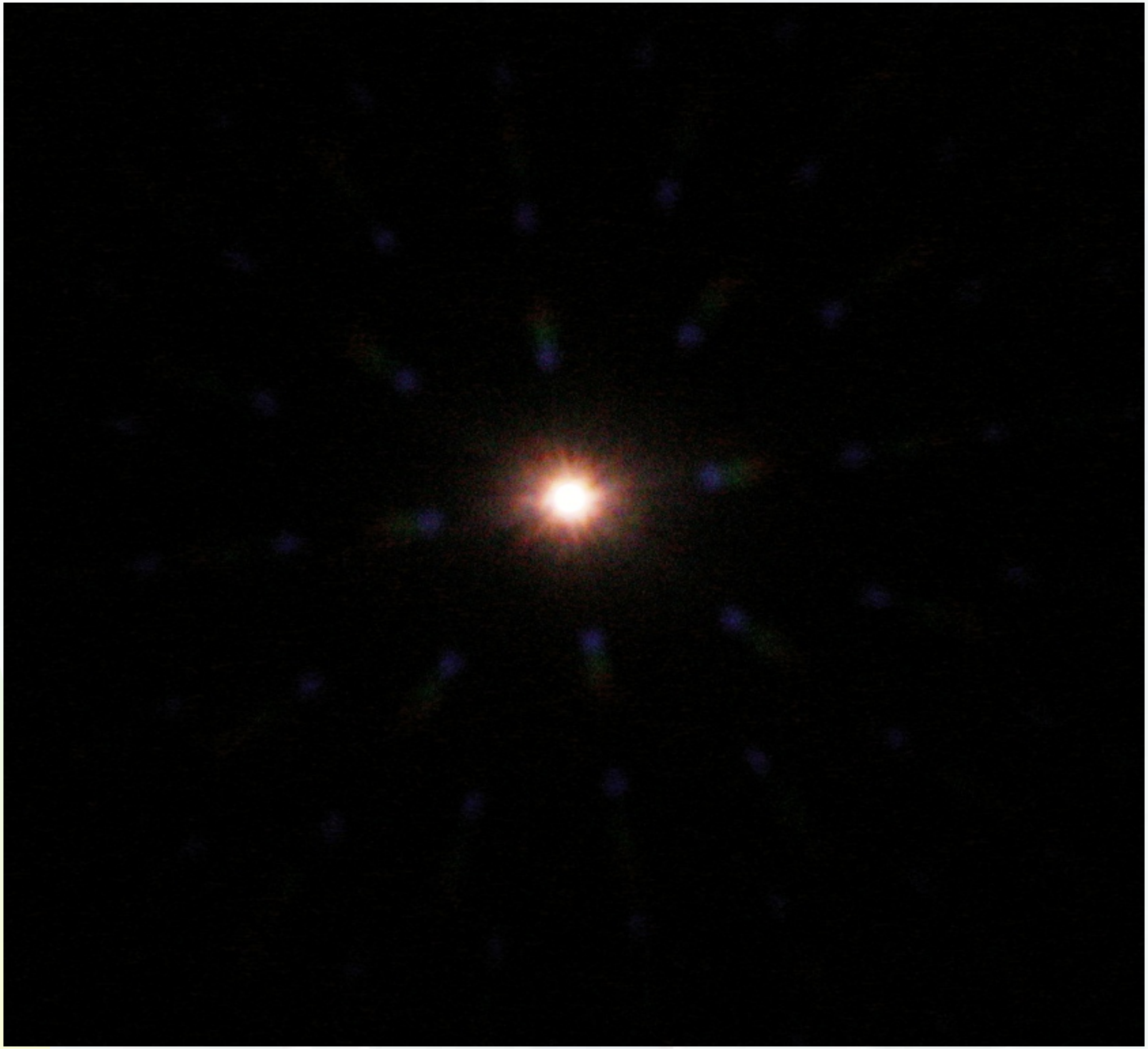
B = 29.4μ
d = 11.8μ

Carlina PSF

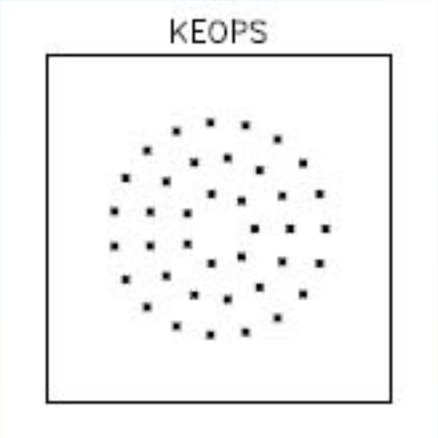


↔ 50μ

. 14μ



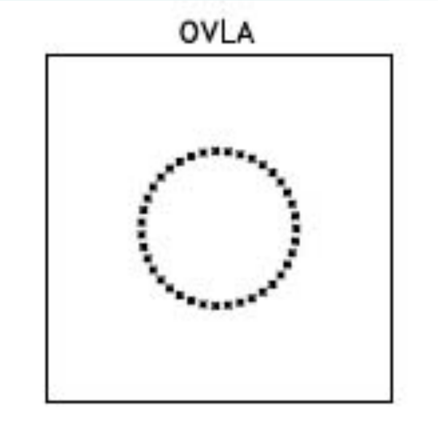
KEOPS PSF



↔ 50μ

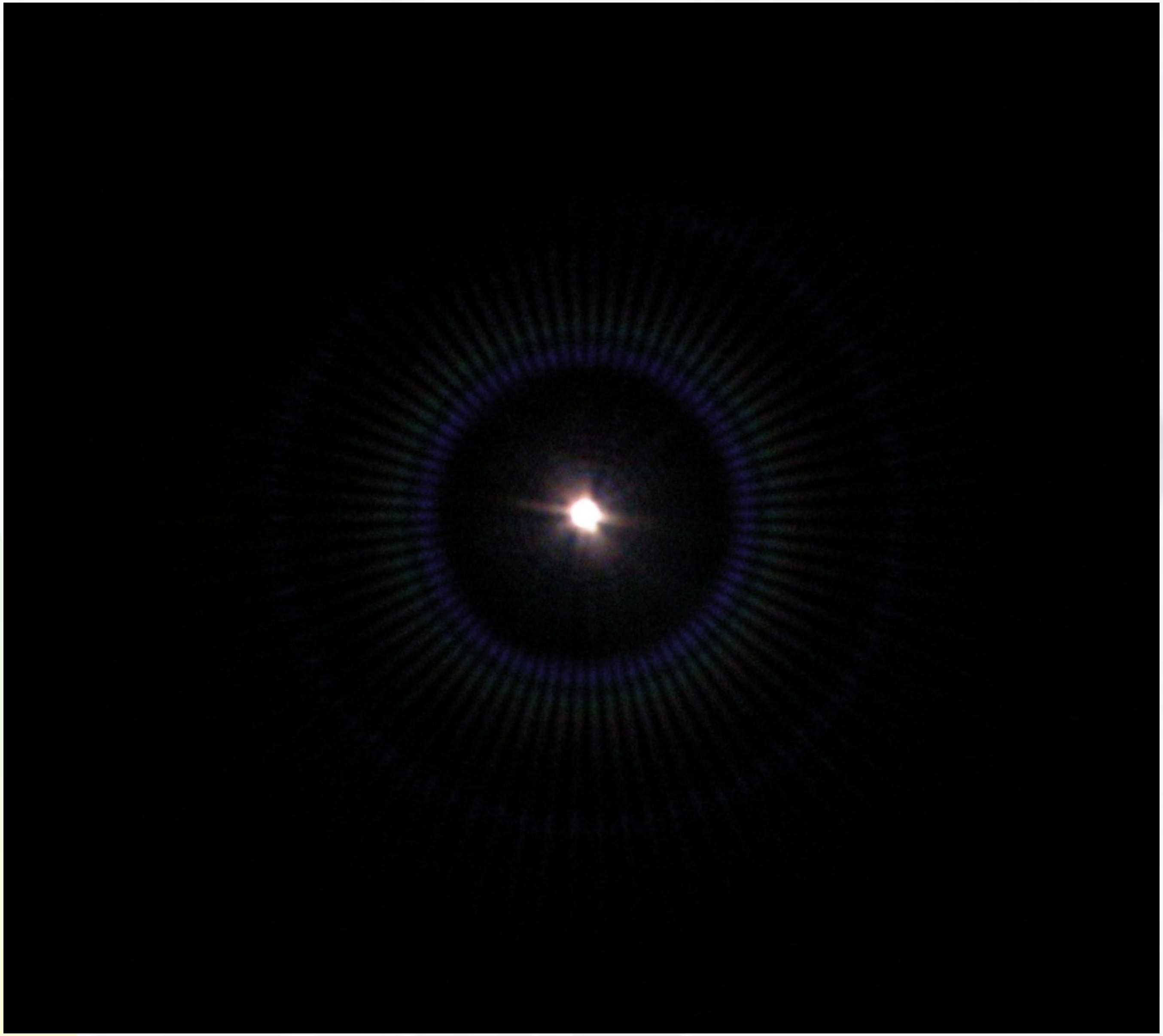
. 14μ

OVLA PSF

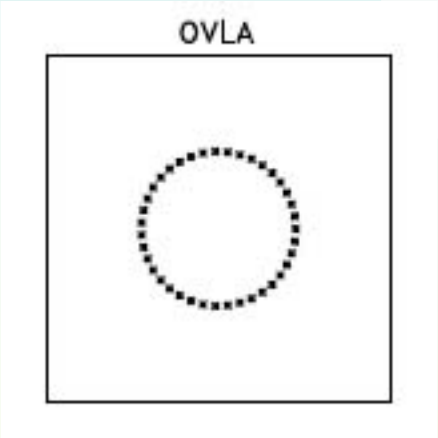


↔ 50μ

. 14μ



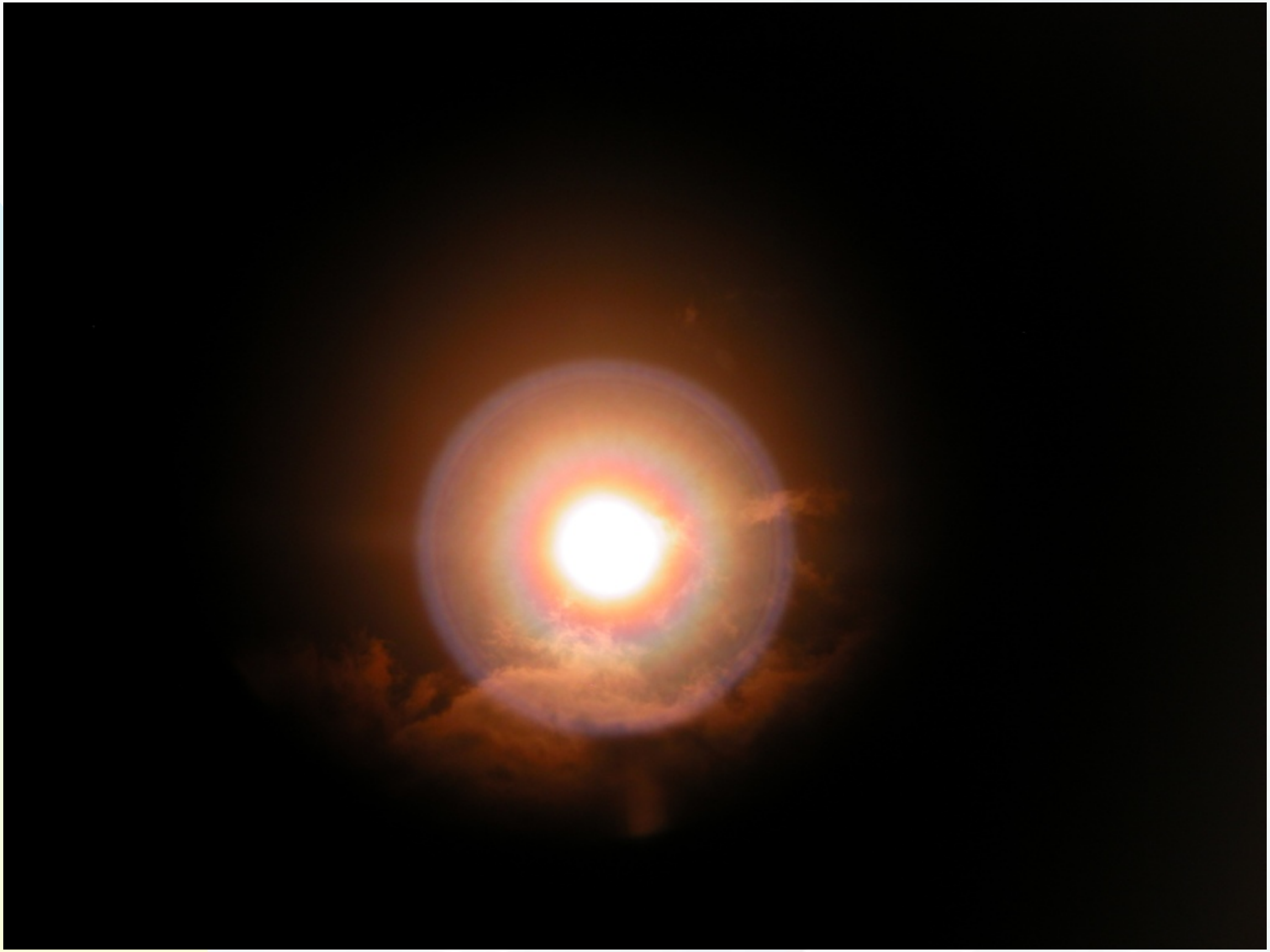
OVLA PSF



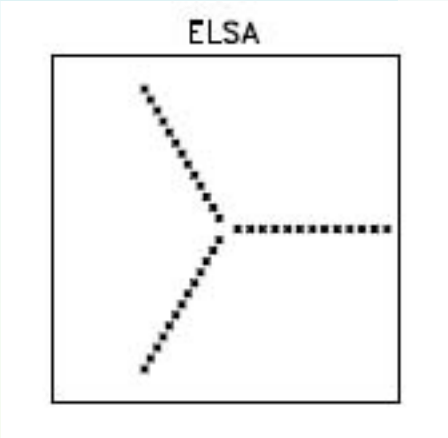
↔ 50μ

. 14μ

OVLA_Sun_2



ELSA PSF

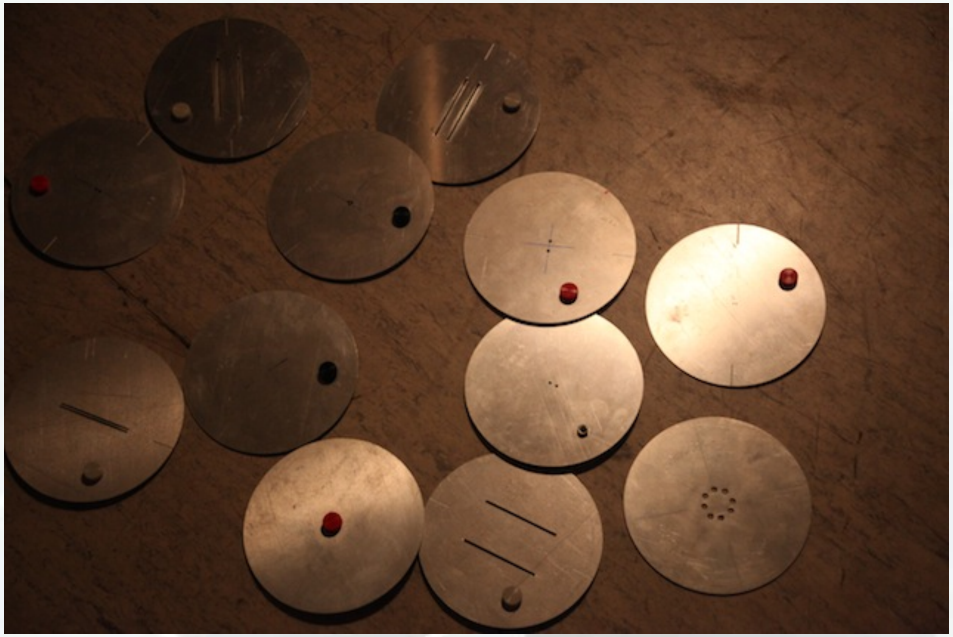


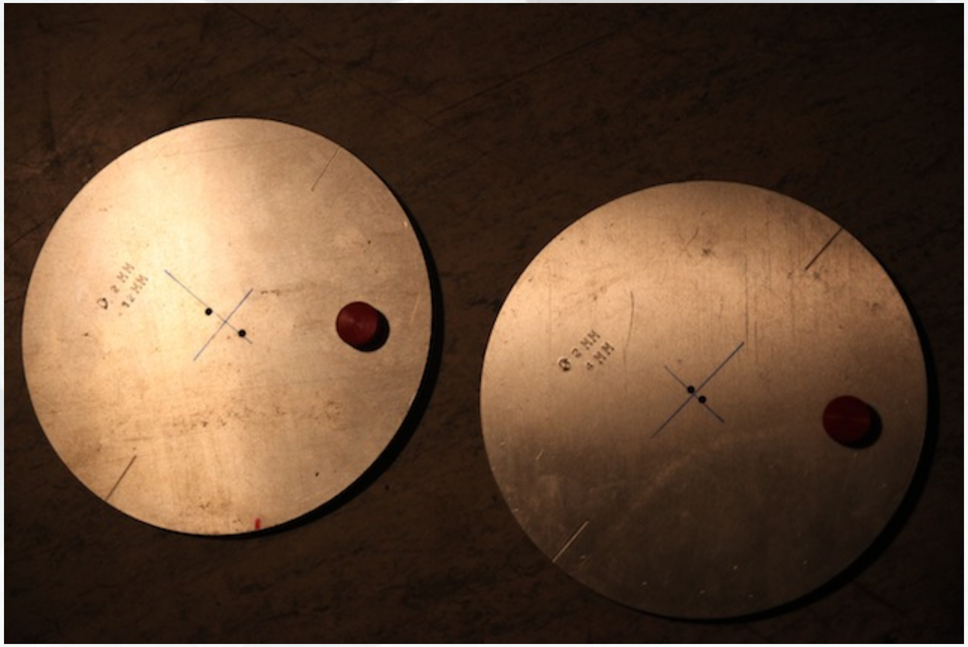
↔ 50μ

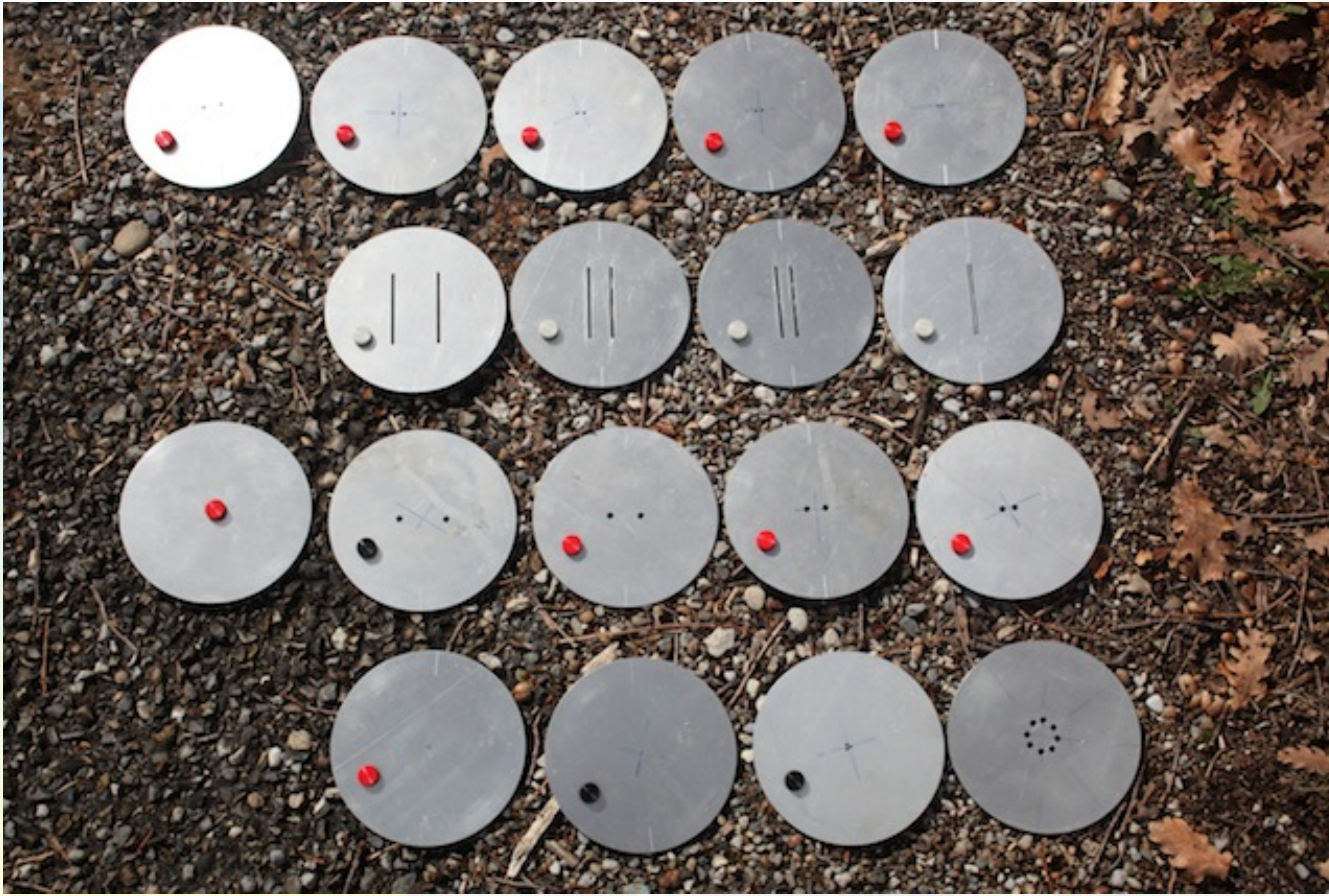
. 14μ

ELSA_Sun_24

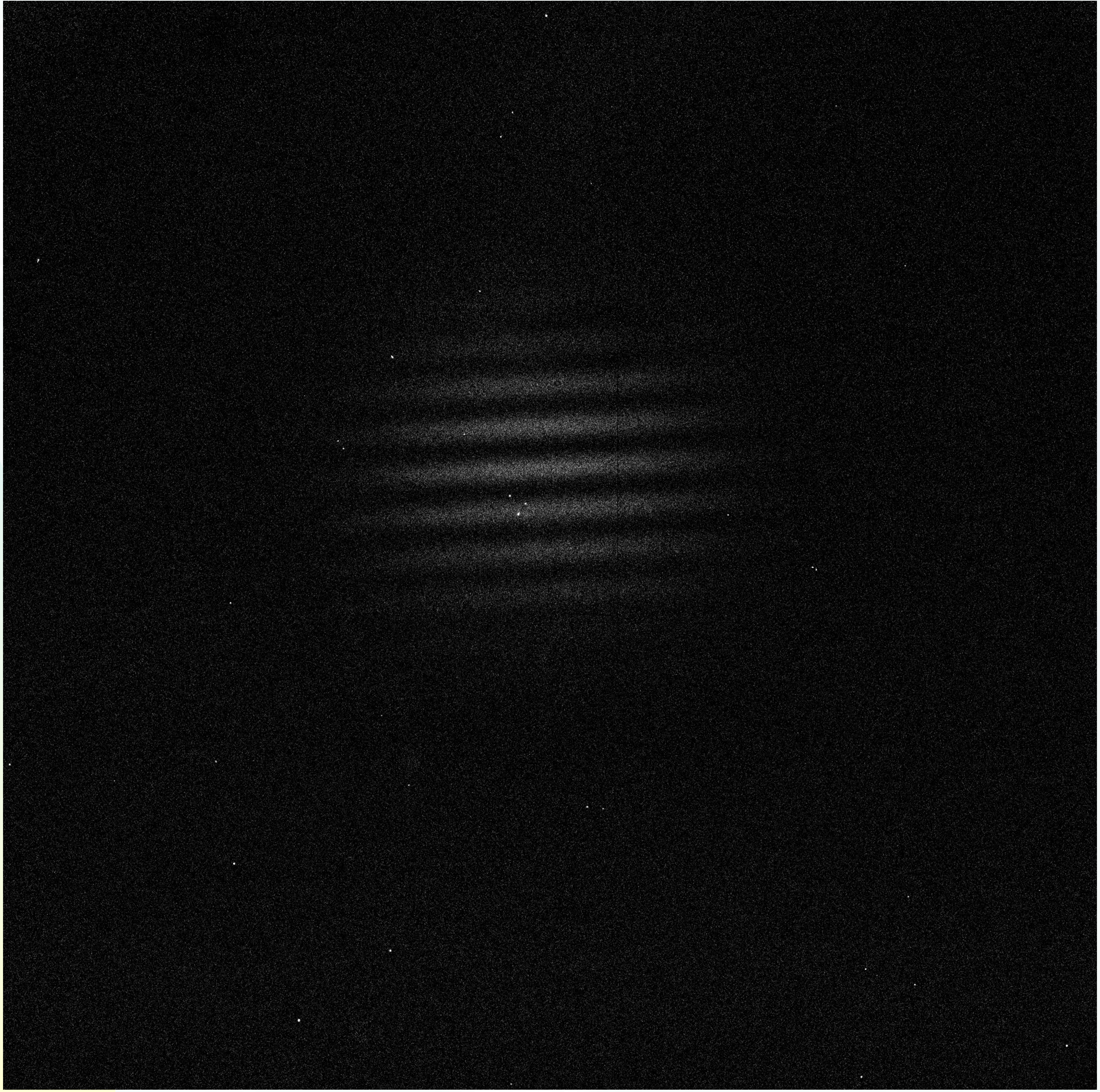
Interferometric observations on 10/4/2010 of Procyon, Mars and Saturn, using the 80cm telescope at Haute-Provence Observatory and adequate masks (coll. with Hervé le Coroller) …



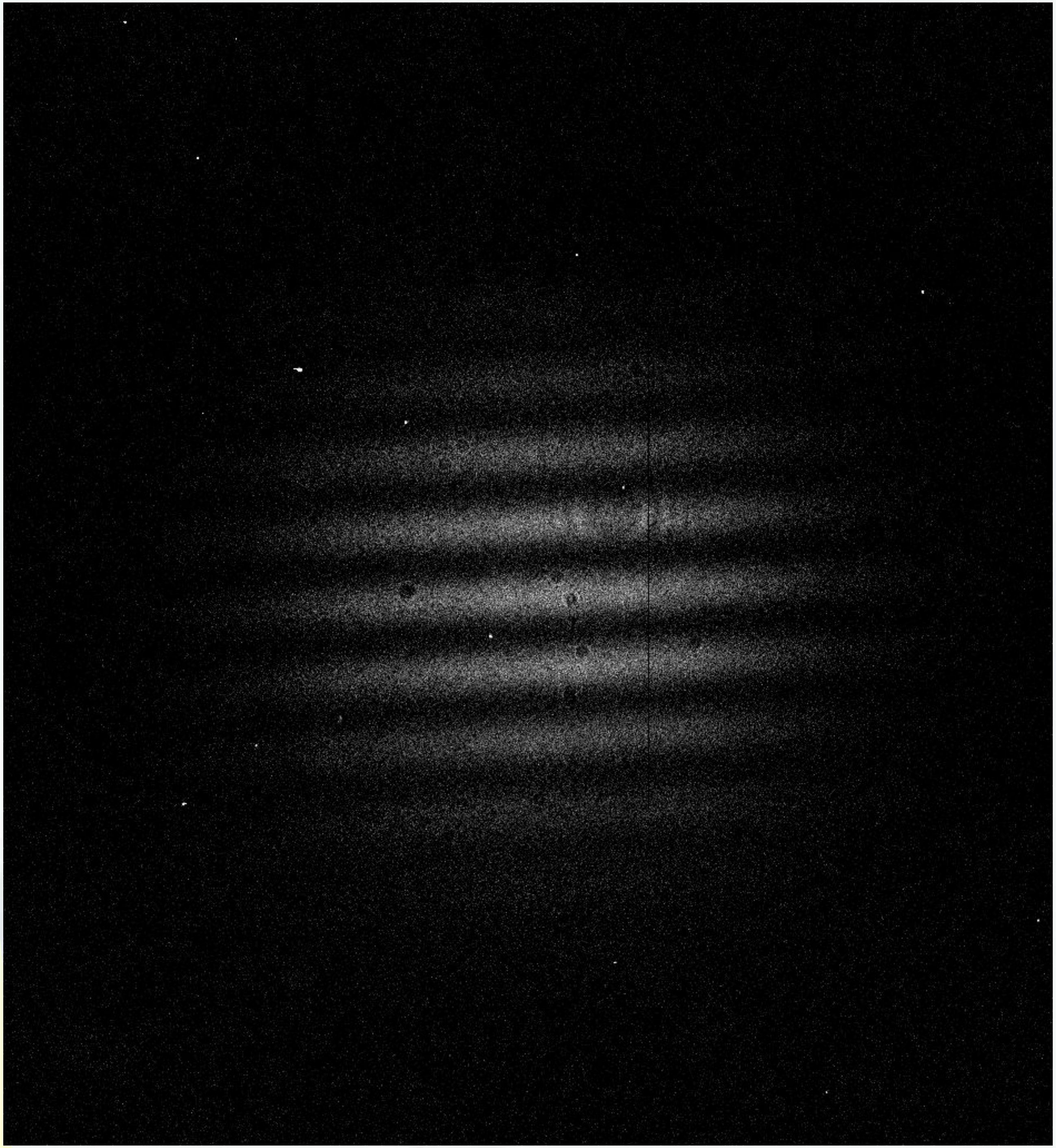




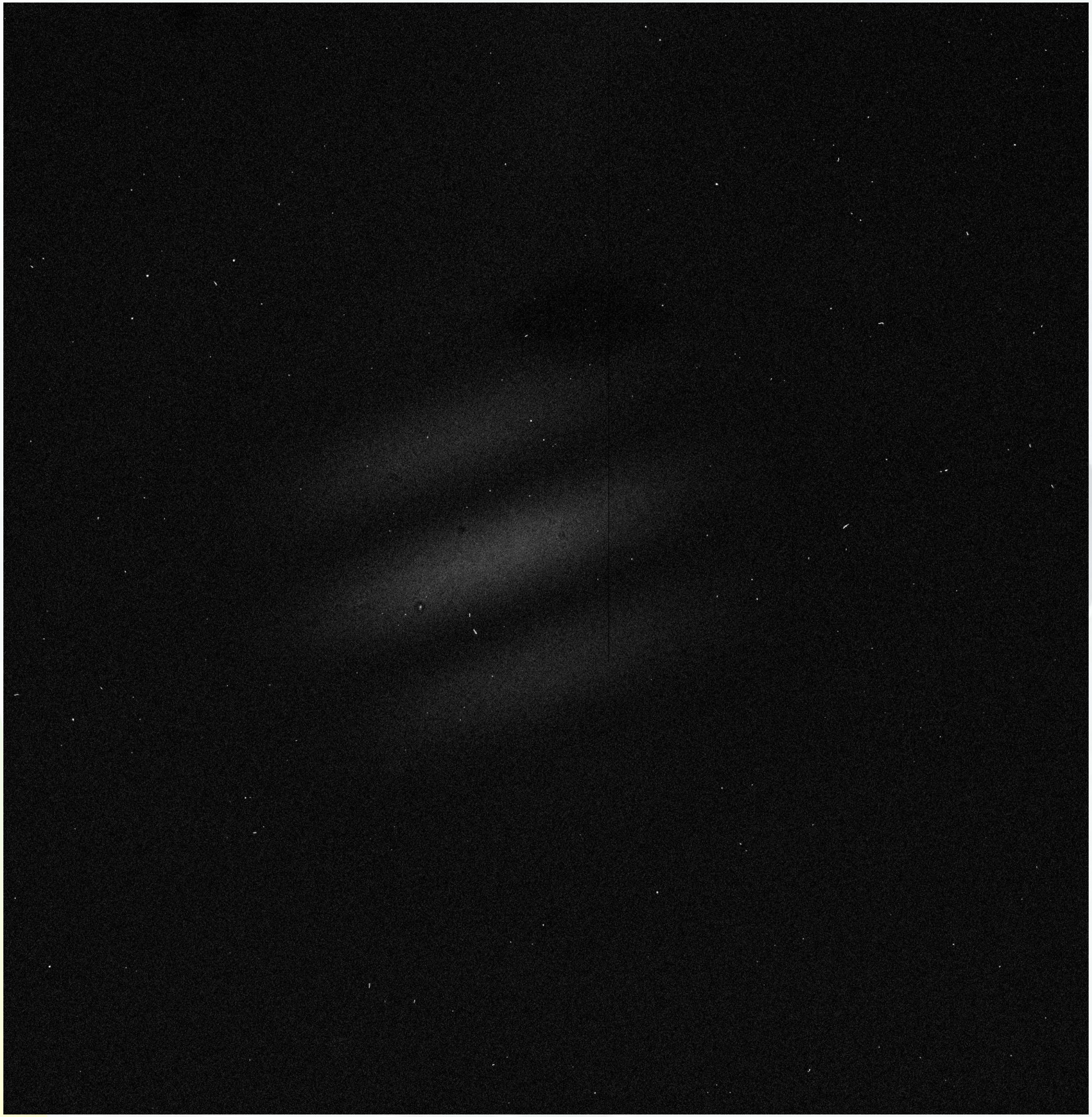
Procyon
B = 12 mm
d = 2 mm



Mars
B = 12 mm
d = 2 mm



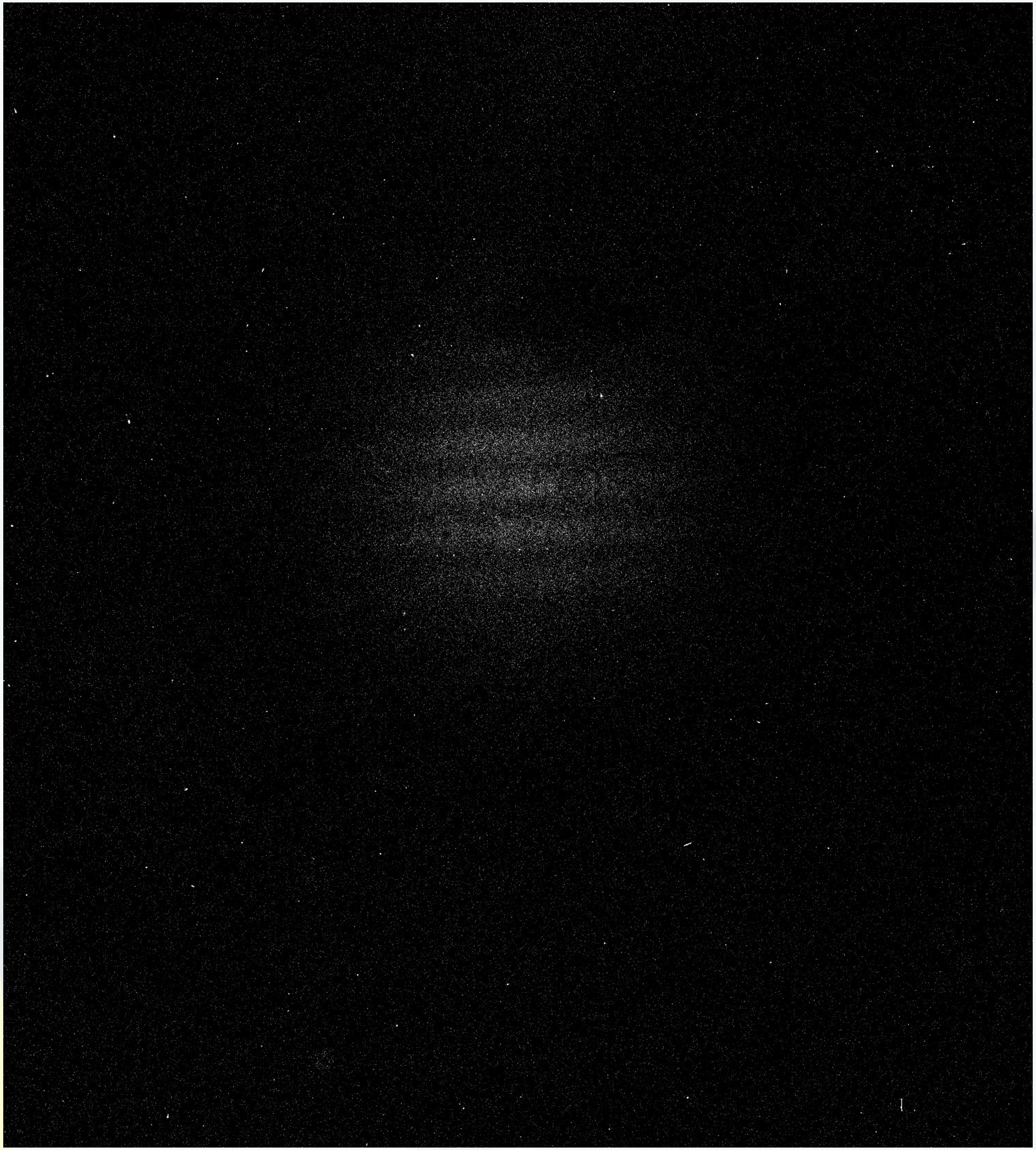
Saturn
B = 4 mm
d = 2 mm



Saturn

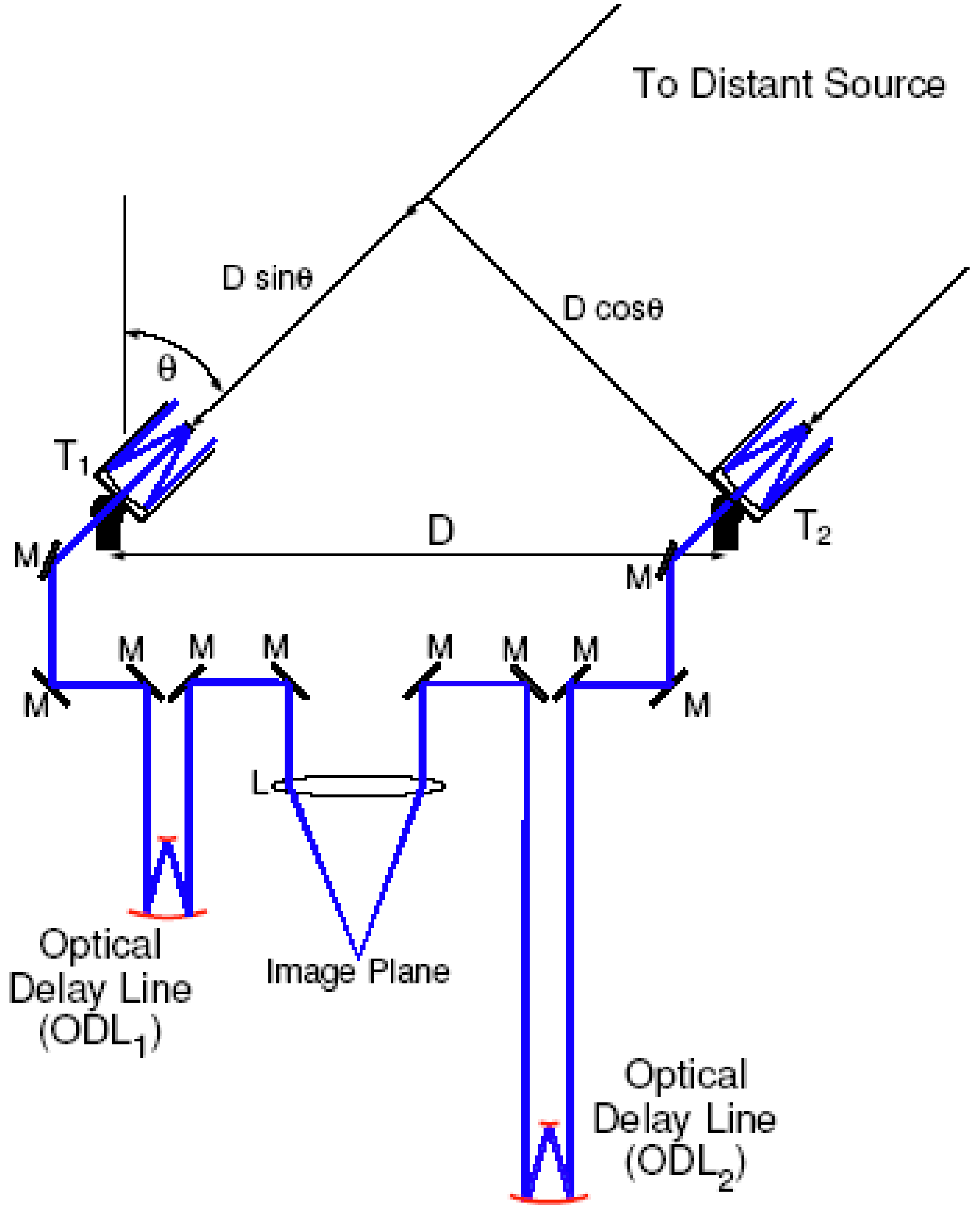
B = 12 mm

d = 2 mm

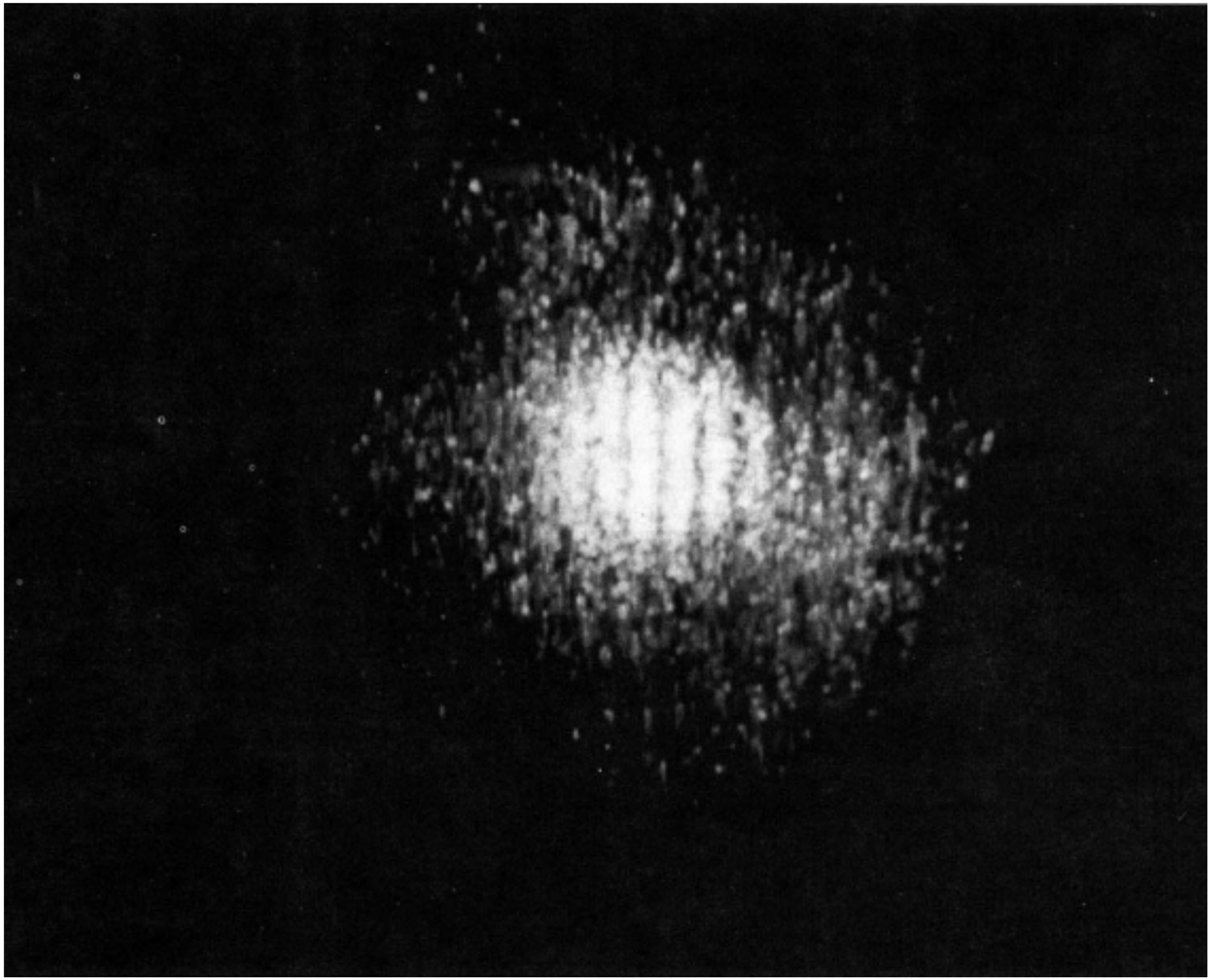


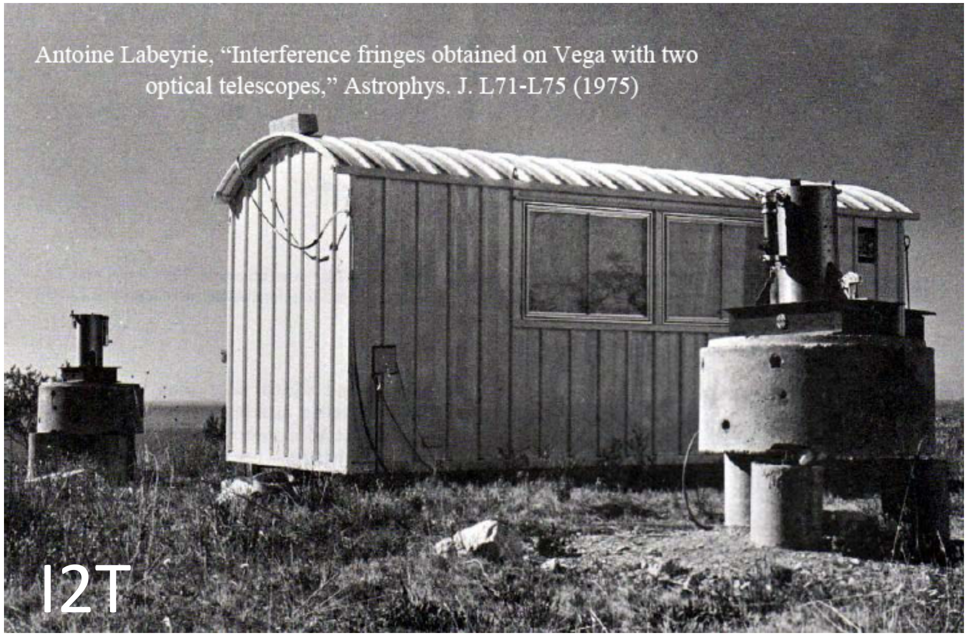
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers


To Distant Source
D sinθ
D cosθ
θ
T1
T2
D
M
M
M
M
M
M
M
M
M
M
M
L
Optical Delay Line (ODL1)
Image Plane
Optical Delay Line (ODL2)

First fringes with I2T

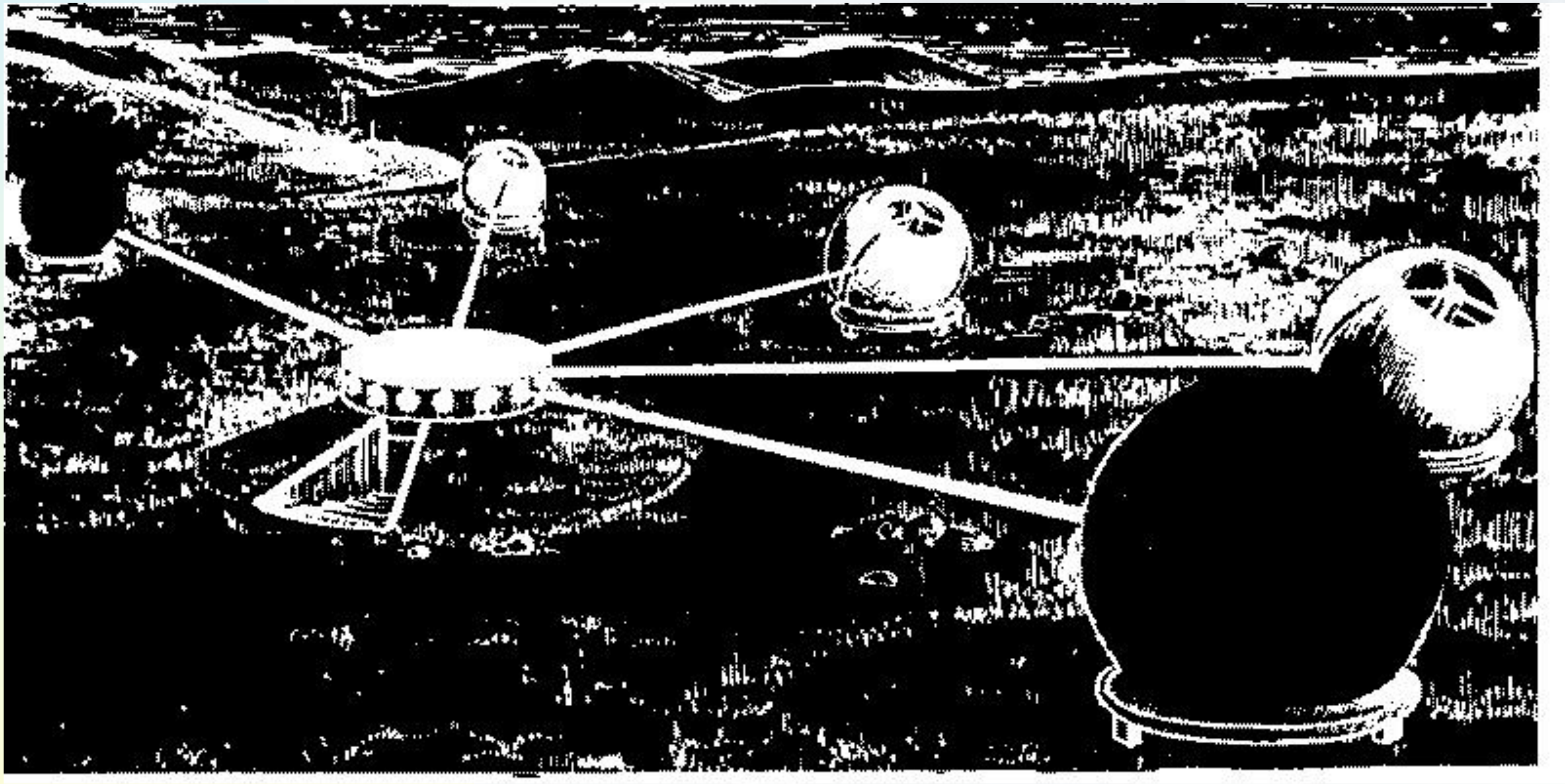



Antoine Labeyrie, "Interference fringes obtained on Vega with two optical telescopes," Astrophys. J. L71-L75 (1975)
I2T


GI2T

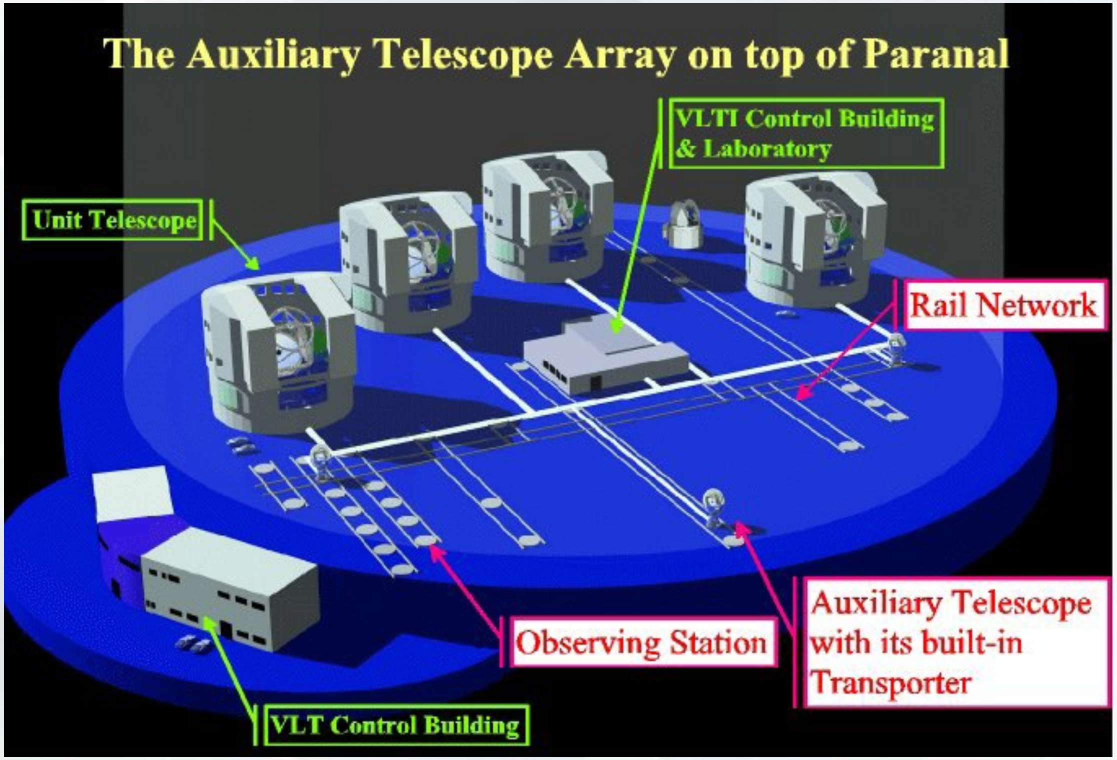
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers



An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers



http://www.aeos.ulg.ac.be/HARI/

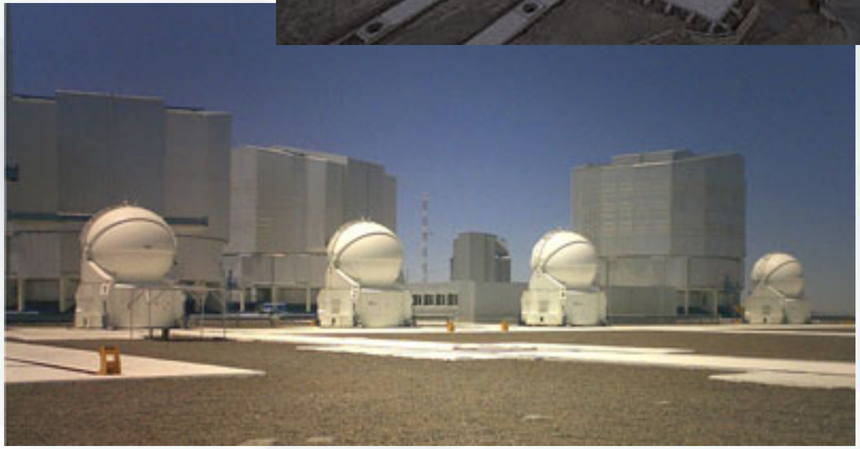
An introduction to optical/IR interferometry

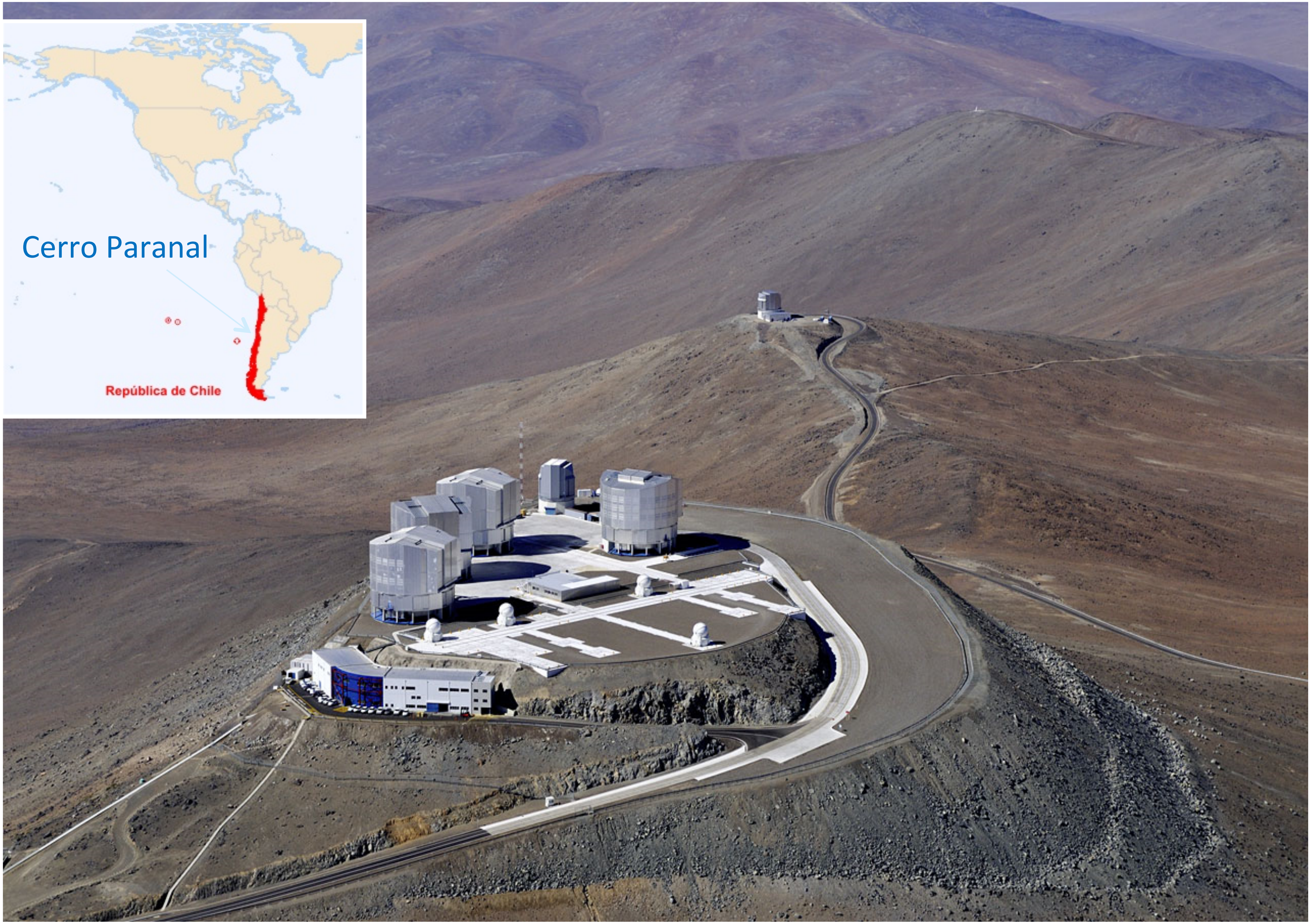
- 6 Some examples of optical interferometers

Interferometry to-day is:

Very Large Telescope Interferometer (VLTI)

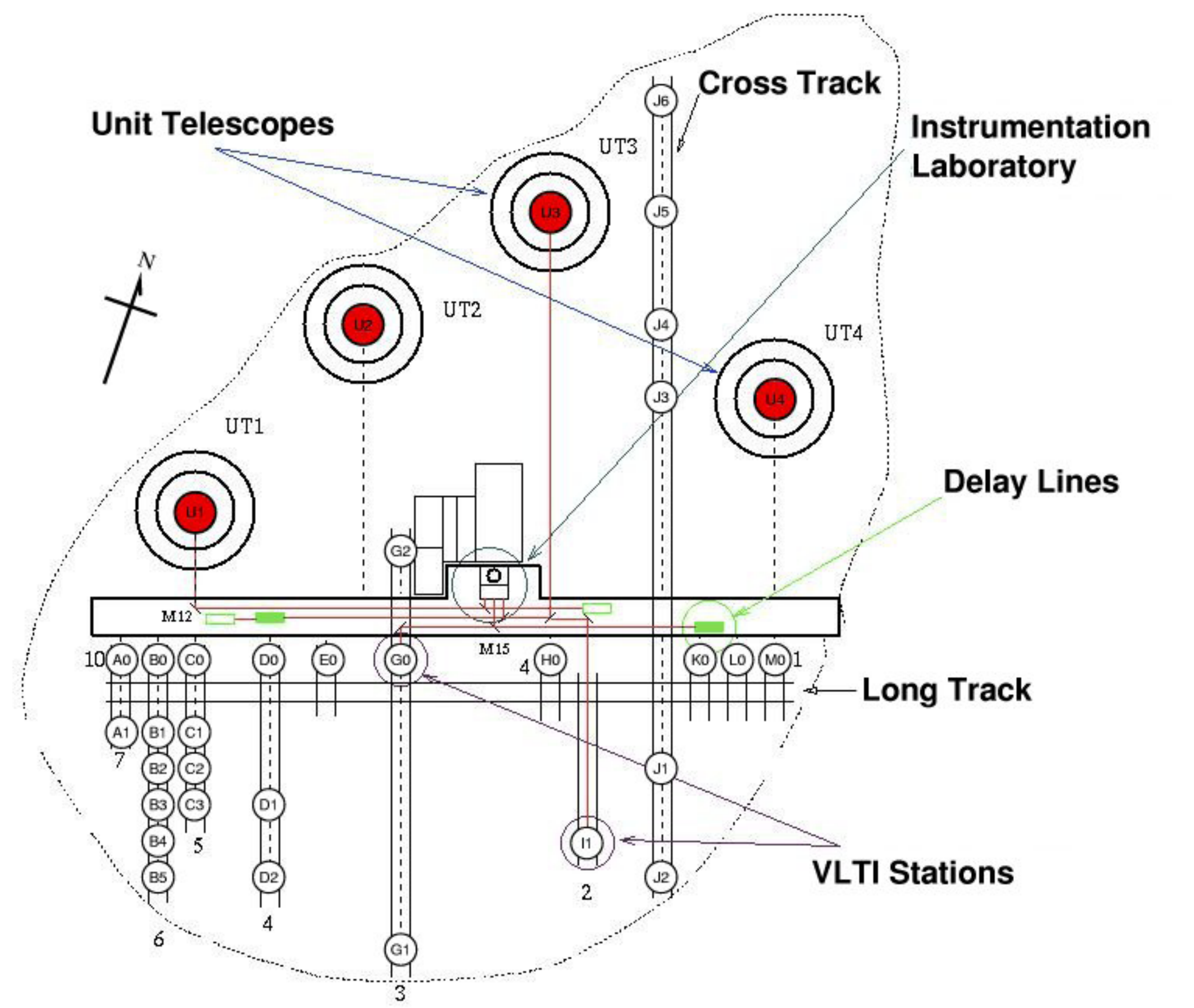
- 4 x 8.2m UTs
- 4 x 1.8m ATs
- Max. Base: 200m




Cerro Paranal
República de Chile

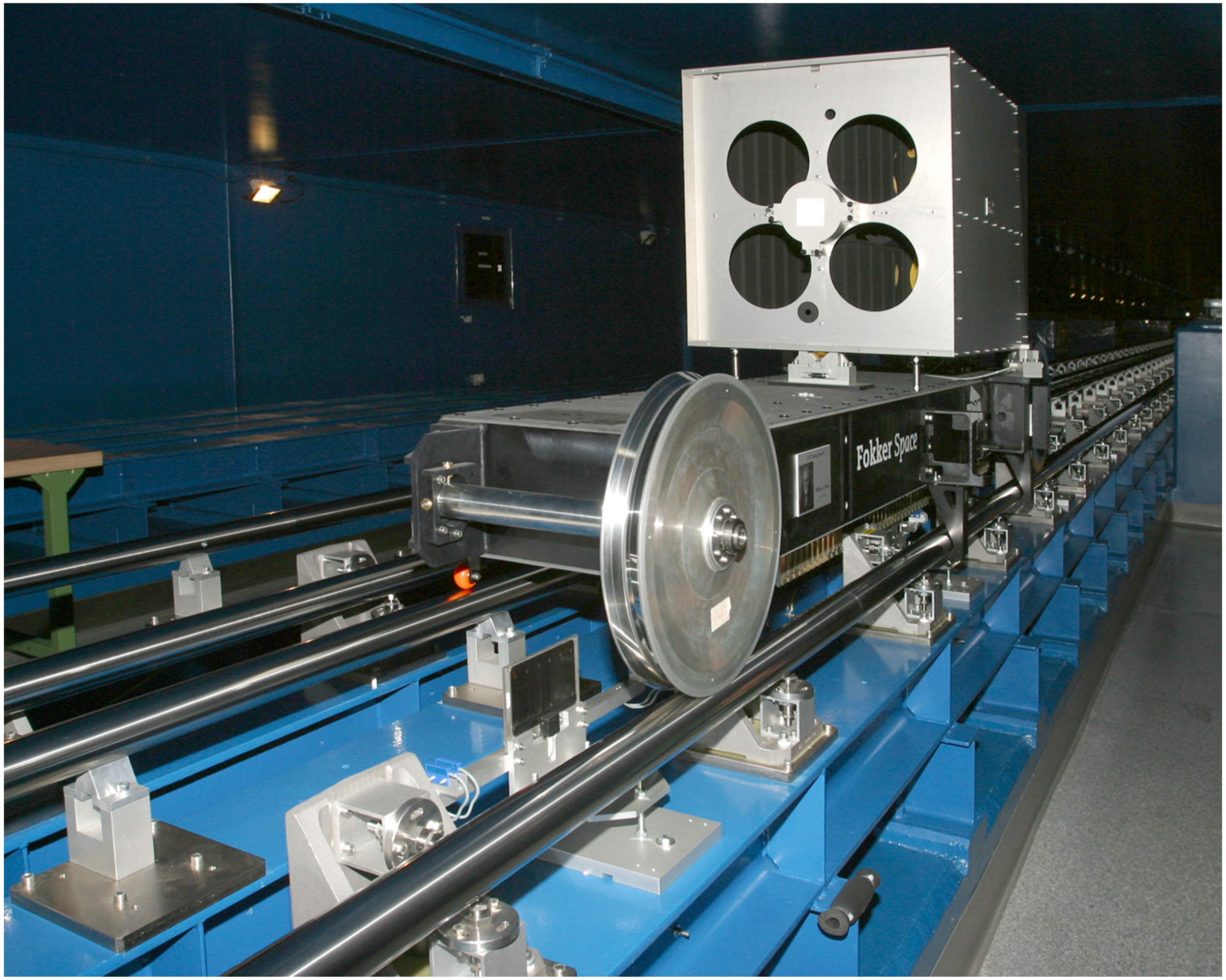
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

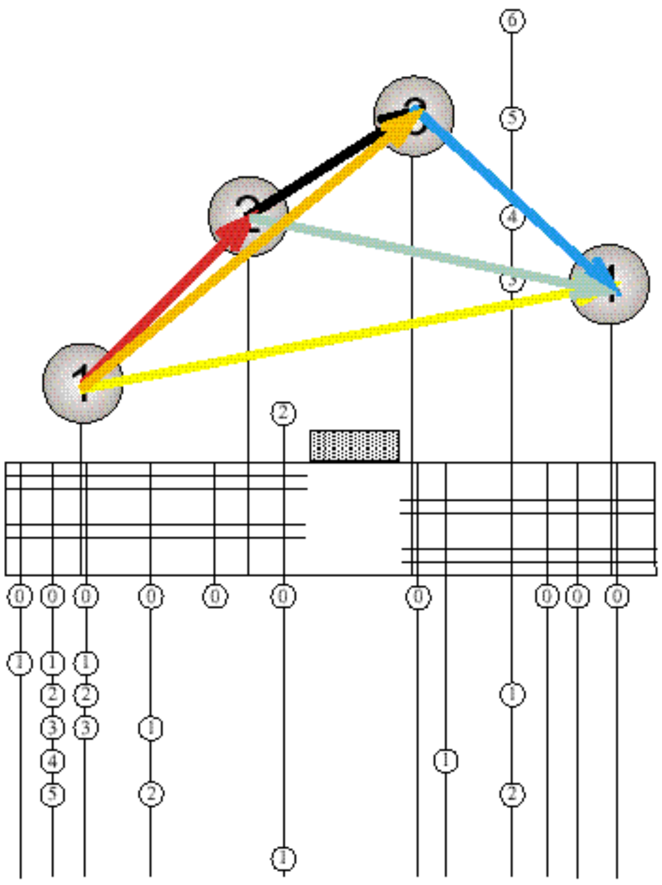


Cross Track
Unit Telescopes
Instrumentation Laboratory
UT3
U3
J6
J5
J4
J3
UT2
U2
UT4
U4
N
UT1
U1
Delay Lines
G2
M12
M15
10
A0
B0
C0
D0
E0
G0
4
H0
K0
L0
M0
1
Long Track
A1
B1
C1
7
B2
C2
J1
B3
C3
D1
B4
5
I1
VLTI Stations
B5
D2
J2
2
4
6
G1
3

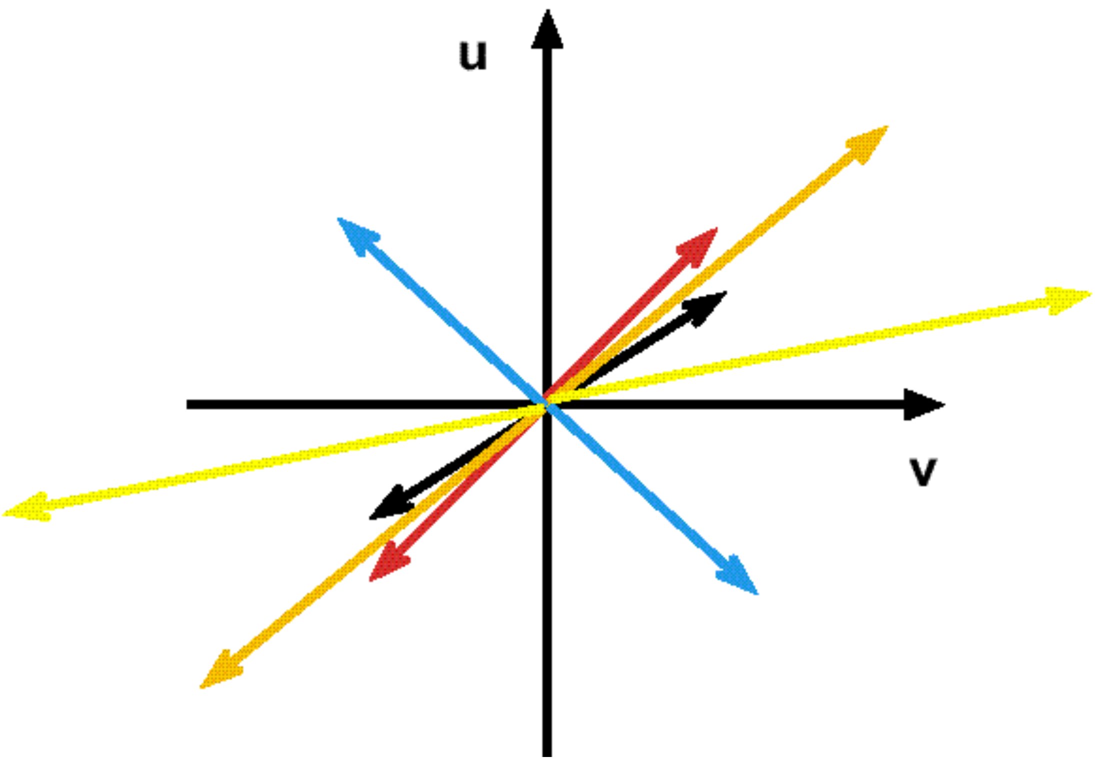
VLTI delay lines



uv plane coverage



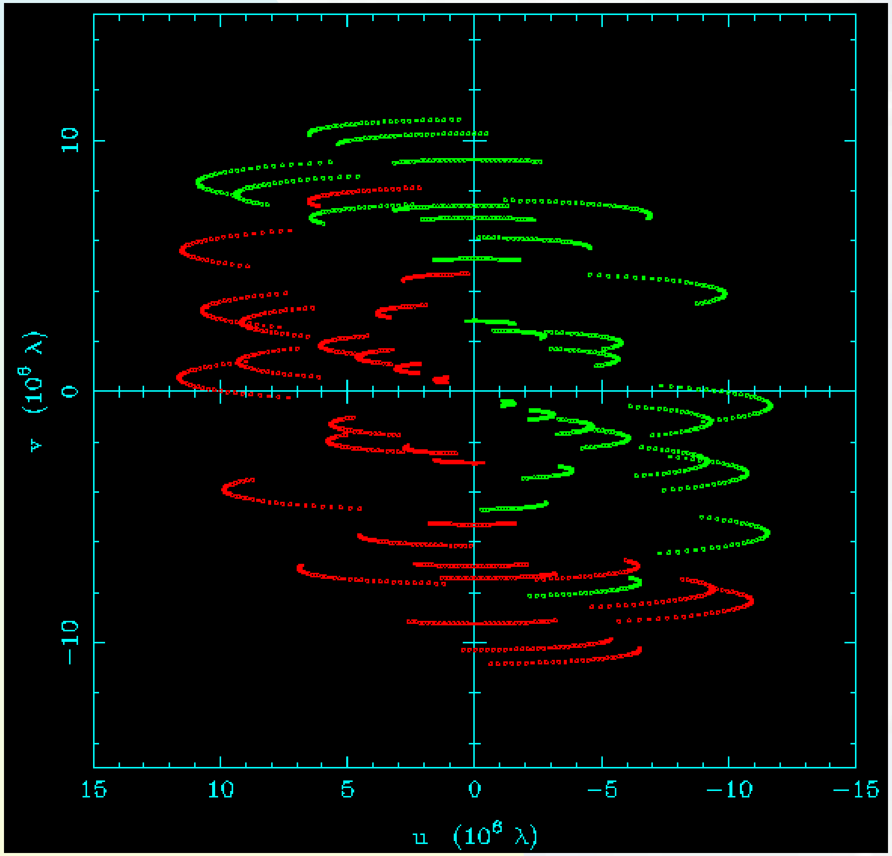
uv plane:



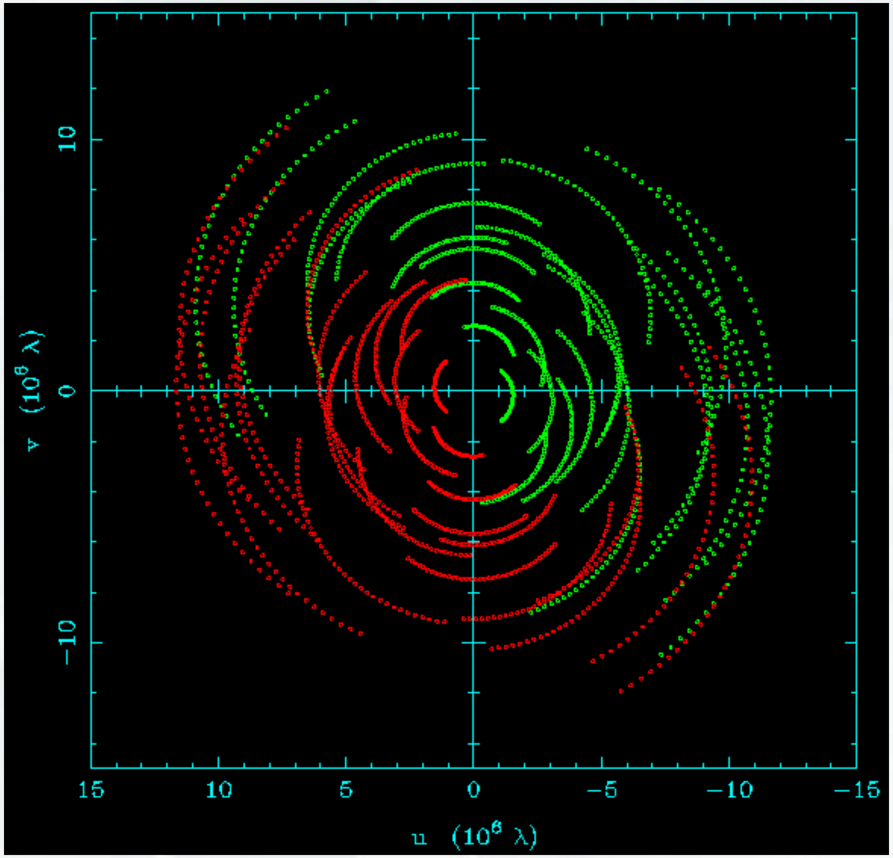
Note: *uv* plane coverage for an object at zenith. More generally, the projected baselines must be used.

Examples of *uv* plane coverage

Dec -15

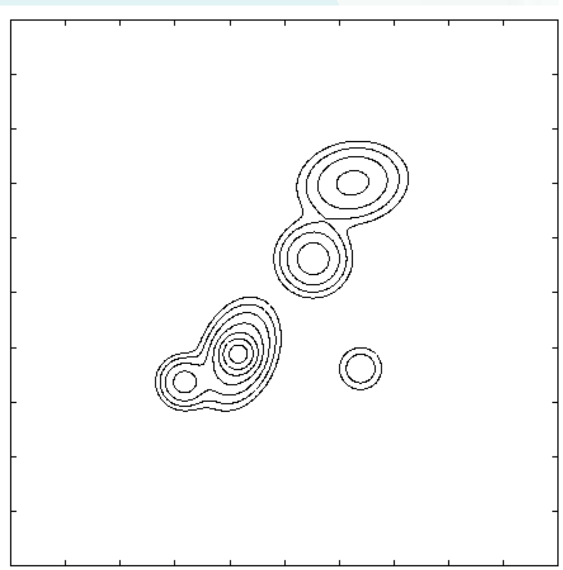


Dec -65

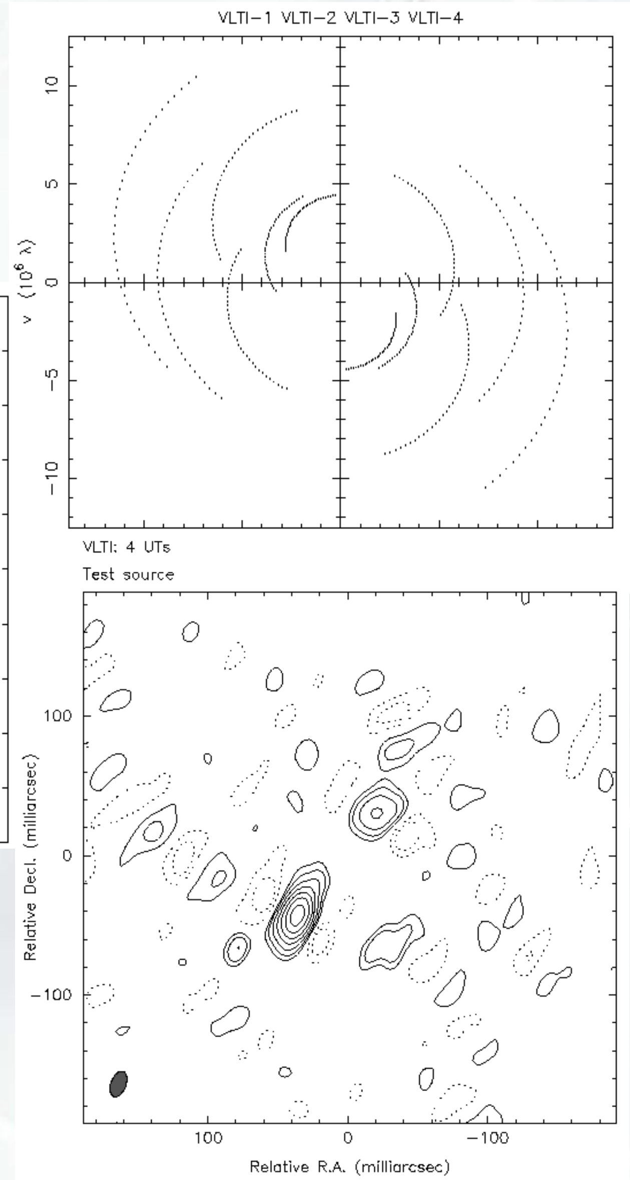


How does the *uv* plane coverage affect imagery?

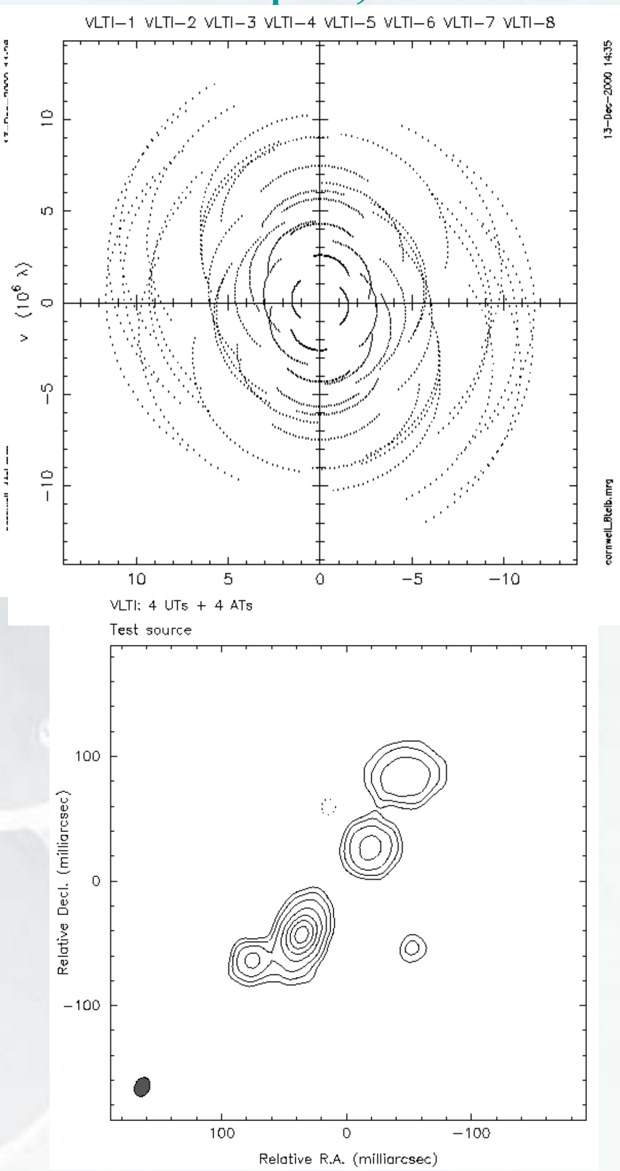
Model



4 telescopes, 6 hrs



8 telescopes, 6 hrs



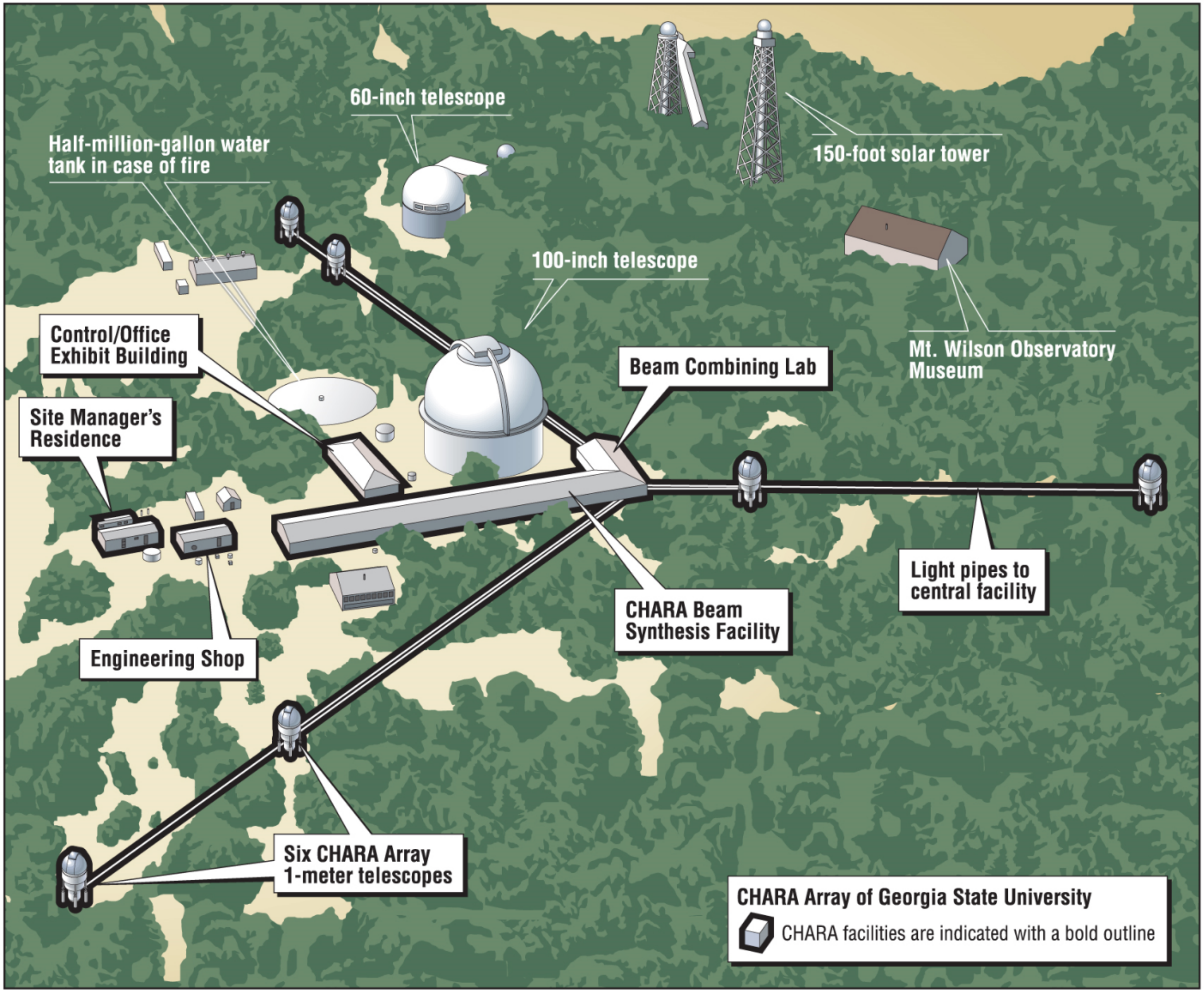
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day is also:

The CHARA interferometer

• 6 x 1m telescopes
• Max. Base: 330m



60-inch telescope
150-foot solar tower
Half-million-gallon water tank in case of fire
100-inch telescope
Control/Office Exhibit Building
Beam Combining Lab
Mt. Wilson Observatory Museum
Site Manager's Residence
Light pipes to central facility
CHARA Beam Synthesis Facility
Engineering Shop
Six CHARA Array 1-meter telescopes
CHARA Array of Georgia State University
CHARA facilities are indicated with a bold outline

An introduction to optical/IR interferometry

6 Some examples of optical interferometers

Interferometry to-day is also:

Palomar Testbed Interferometer (PTI)

- 3 x 40cm telescopes
- Max. Base: 110m

An introduction to optical/IR interferometry

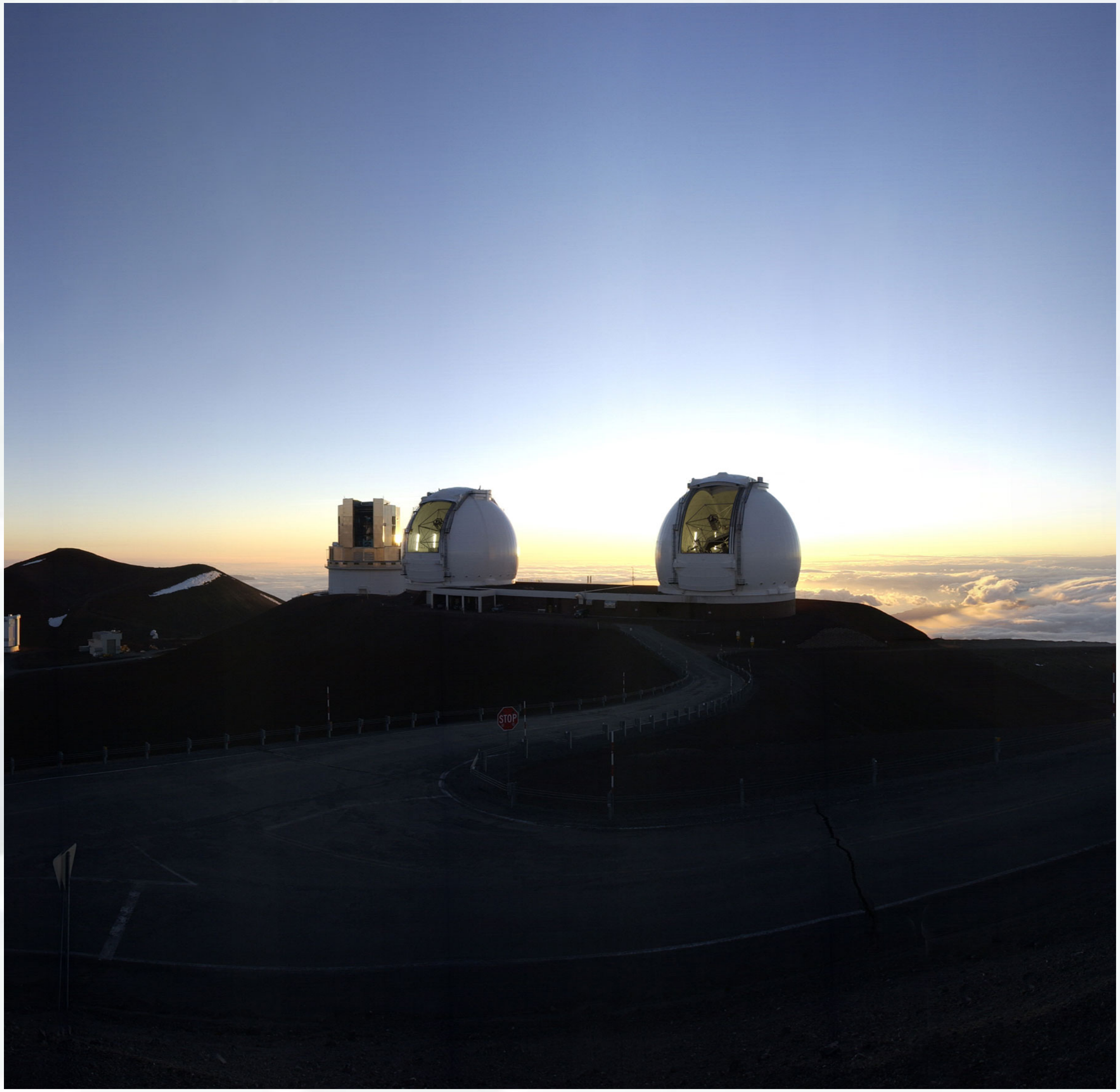
- 6 Some examples of optical interferometers

Interferometry to-day

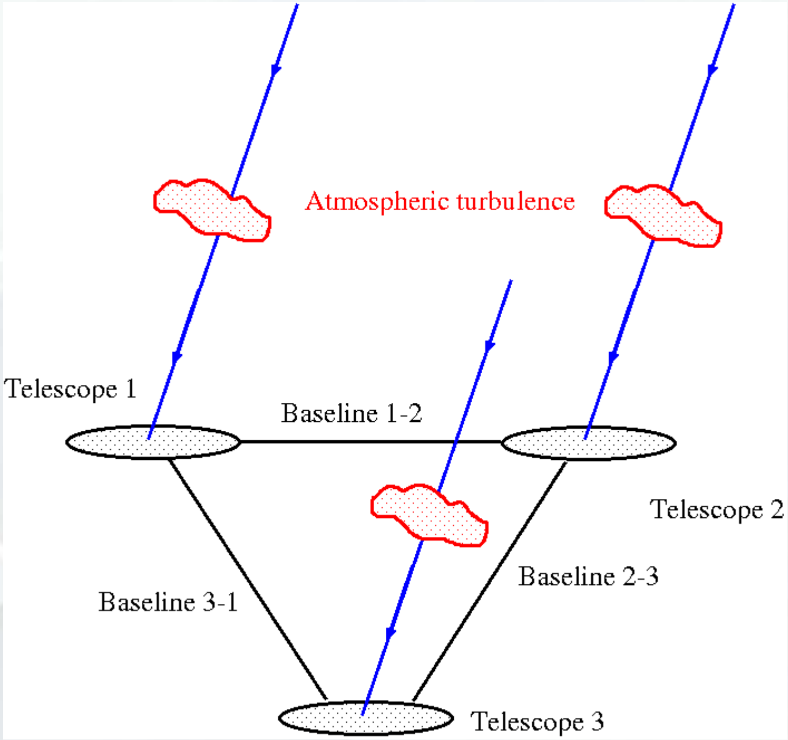
is also:

Keck interferometer

- 2 x 10m telescopes
- Base: 85m



Closure phases – what are these?



- Measure visibility phase (Φ) on three baselines simultaneously.

- Each is perturbed from the true phase (ϕ) in a particular manner:

$\Phi_{12} = \phi_{12} + \varepsilon_1 - \varepsilon_2$
$\Phi_{23} = \phi_{23} + \varepsilon_2 - \varepsilon_3$
$\Phi_{31} = \phi_{31} + \varepsilon_3 - \varepsilon_1$

- Construct the linear combination of these:

$\Phi_{12} + \Phi_{23} + \Phi_{31} = \phi_{12} + \phi_{23} + \phi_{31}$

The error terms are antenna dependent – they vanish in the sum.
The source information is baseline dependent – it remains.
We still have to figure out how to use it!

Closure phase is a peculiar linear combination of the true Fourier phases:

– In fact, it is the argument of the product of the visibilities on the baselines in question, hence the name triple product (or bispectrum):

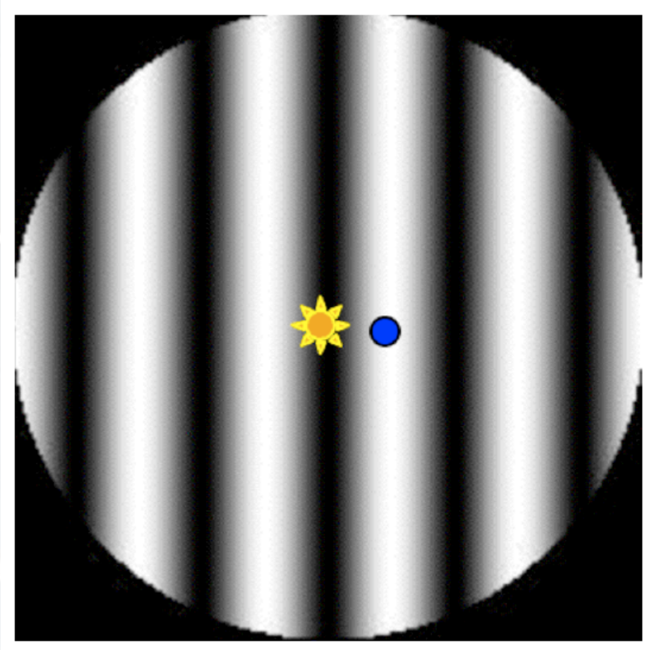
$$V_{12}V_{23}V_{31} = |V_{12}|\,|V_{23}|\,|V_{31}| \exp(i2\pi[\Phi_{12} + \Phi_{23} + \Phi_{31}]) = T_{123}$$

– So we have to use the closure phases as additional constraints In some nonlinear iterative inversion scheme.

An introduction to optical/IR interferometry

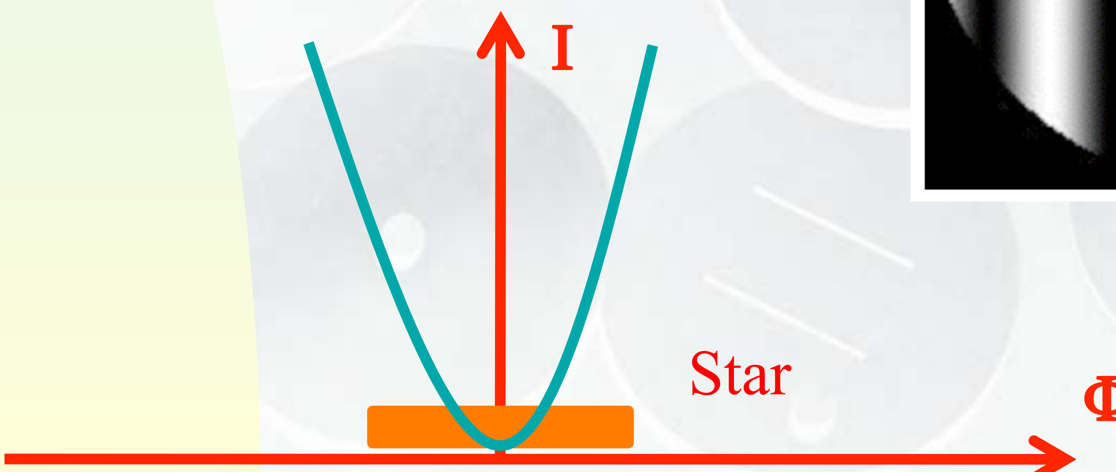
6 Some examples of optical interferometers

Interferometry to-day is also:



Nulling interferometry

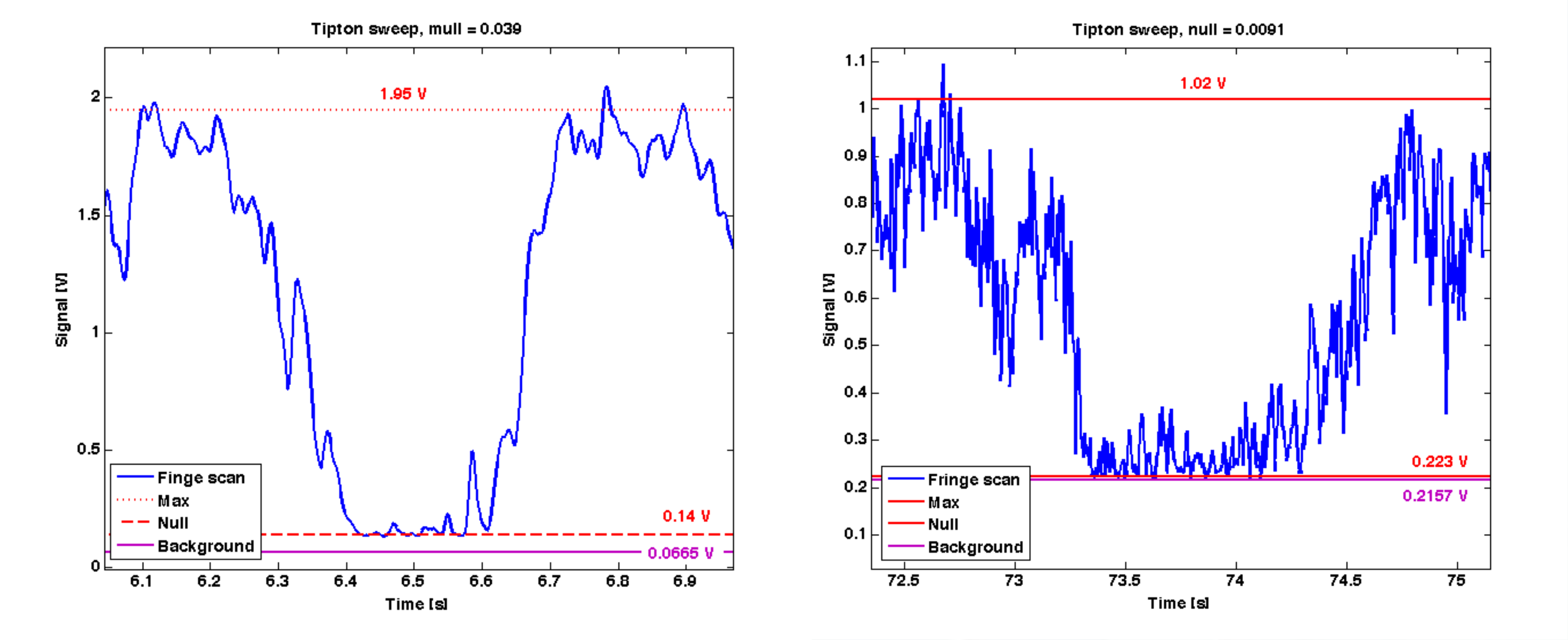
- Measurement of « stellar leakage »
- Allow to resolve stars with a
  a small size interferometer



An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day is also:



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- 6 Other examples of interferometers: ALMA

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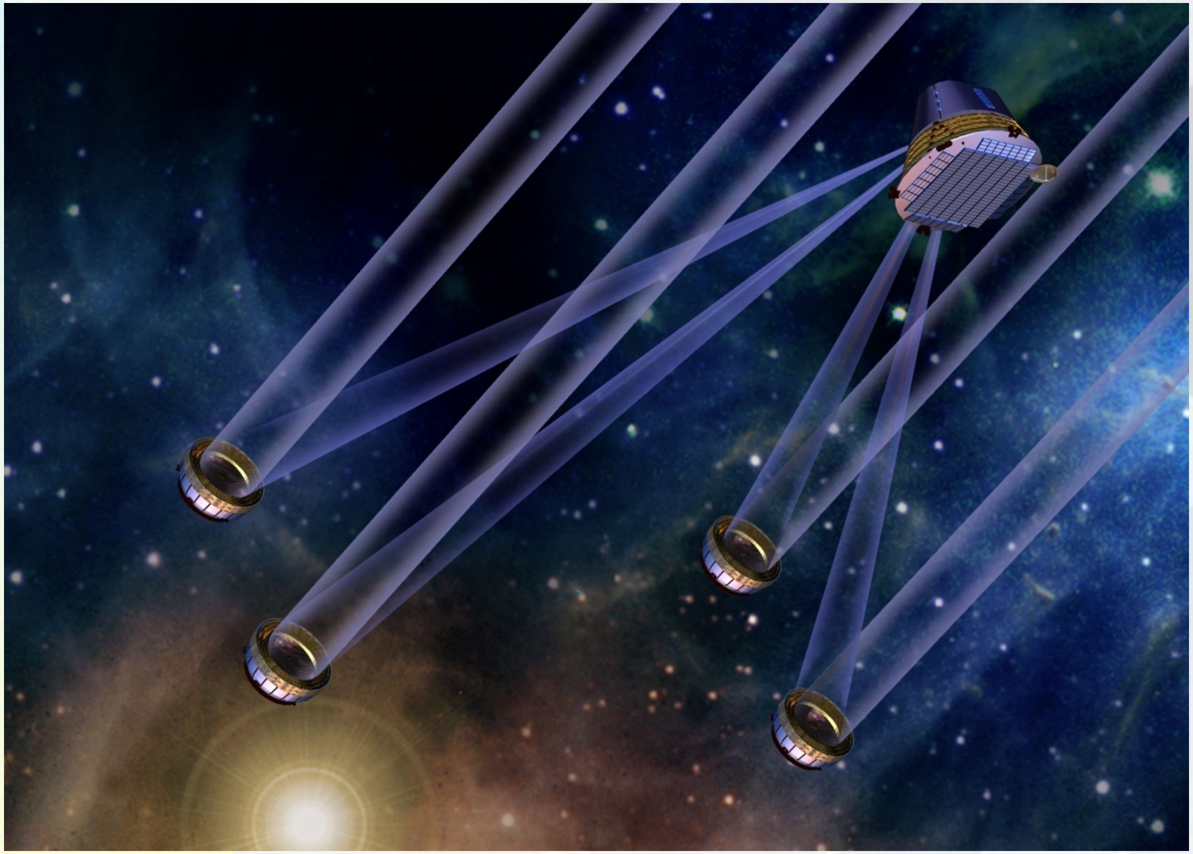
- 6 Other examples of interferometers: DARWIN



An introduction to optical/IR interferometry

- 7 Some results

Star	Spectral type	Luminosity class	Angular diameter $\times 10^{-3}$ seconds of arc
α Boo	K2	Giant	20
α Tau	K5	Giant	20
α Sco	M1–M2	Super-giant	40
β Peg	M2	Giant	21
σ Cet	M6e	Giant	47
α Ori	M1–M2	Super-giant variable	34→47

Table 2.1. Stars measured with Michelson's interferometer. From Pease (1931).

An introduction to optical/IR interferometry

- 7 Some results

Table 2. Diamètres stellaires mesurés à l'I2T

NOM	SPECTRE	DIAMÈTRE λ = 0,55 μm en ms. d'arc	MESURÉ λ = 2,2 μm en ms. d'arc	R/R ⊙	TEMPÉRATURE EFFECTIVE λ = 0,55 μm en degrés Kelvin	TEMPÉRATURE EFFECTIVE λ = 2,2 μm en degrés Kelvin	DISTANCE en parsecs (1 pc = 3,26 al)
α Cas	K0II	5.4 ± 0.6		28 ± 6	4700 ± 300		45 ± 9
β And	M0III	13.2 ± 1.7	14.4 ± 0.5	33 ± 9	3800 ± 250	3711 ± 64	23 ± 3
γ And	K3II	6.8 ± 0.6		60 ± 14	4650 ± 250		75 ± 15
α Per	F5Ib	2.9 ± 0.4		55 ± 9	7000 ± 600		176 ± 6
α Cyg	A2Ia	2.7 ± 0.3		145 ± 45	8200 ± 600		500 ± 100
α Ari	K2III	7.6 ± 1		15 ± 5	4300 ± 350		23 ± 4
β Gem	K0III	7.8 ± 0.6		8 ± 2	4900 ± 220		11 ± 1
β Umi	K4III	8.9 ± 1		30 ± 9	4220 ± 300		31 ± 11
γ Dra	K5III	8.7 ± 0.8	10.2 ± 1.4	45 ± 10	4300 ± 230	3960 ± 270	59 ± 21
δ Dra	G9III	3.8 ± 0.3		16 ± 5	4530 ± 220		36 ± 6
μ Gem	M3III		14.6 ± 0.8	94 ± 30		3900 ± 95	60 ± 15
α Tau	K5III		20.7 ± 0.4	47 ± 7		3904 ± 34	21 ± 3
α Boo	K2III		21.5 ± 1.2	26 ± 6		4240 ± 120	11 ± 2
α Aur$_A$	G5III	8.0 ± 1.2		11.7 ± 2	6400 ± 200		13.7 ± 0.6
α Aur$_B$	G0III	4.8 ± 1.5		7.1 ± 2	5950 ± 200		13.7 ± 0.6
α Lyr	A0V	3.0 ± 0.2		2.6 ± 0.2			8.1 ± 0.3

An introduction to optical/IR interferometry

8 Three important theorems … and some applications

8.1 The fundamental theorem

8.2 The convolution theorem

8.3 The Wiener-Khintchin theorem

Réf.: P. Léna; Astrophysique: méthodes physiques de l'observation (Savoirs Actuels / CNRS Editions)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

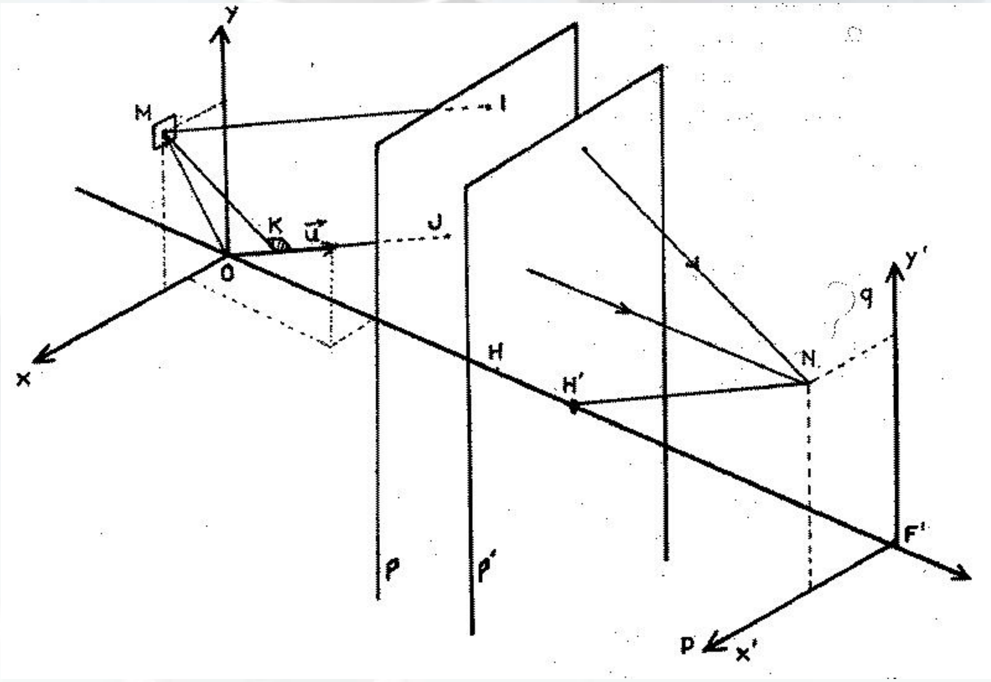
a(p,q) = TF_(A(x,y))(p,q),

$$a(p,q) = \int_{R^2} A(x,y)\exp\left[-i2\pi\left(px+qy\right)\right]dxdy,$$

with

p = x’ /(λ f)

q = y’ /(λ f)



8.1 The fundamental theorem

The distribution of the complex amplitude a(p,q) in the focal plane is given by the Fourier transform of the distribution of the complex amplitude A(x,y) in the entrance pupil plane.

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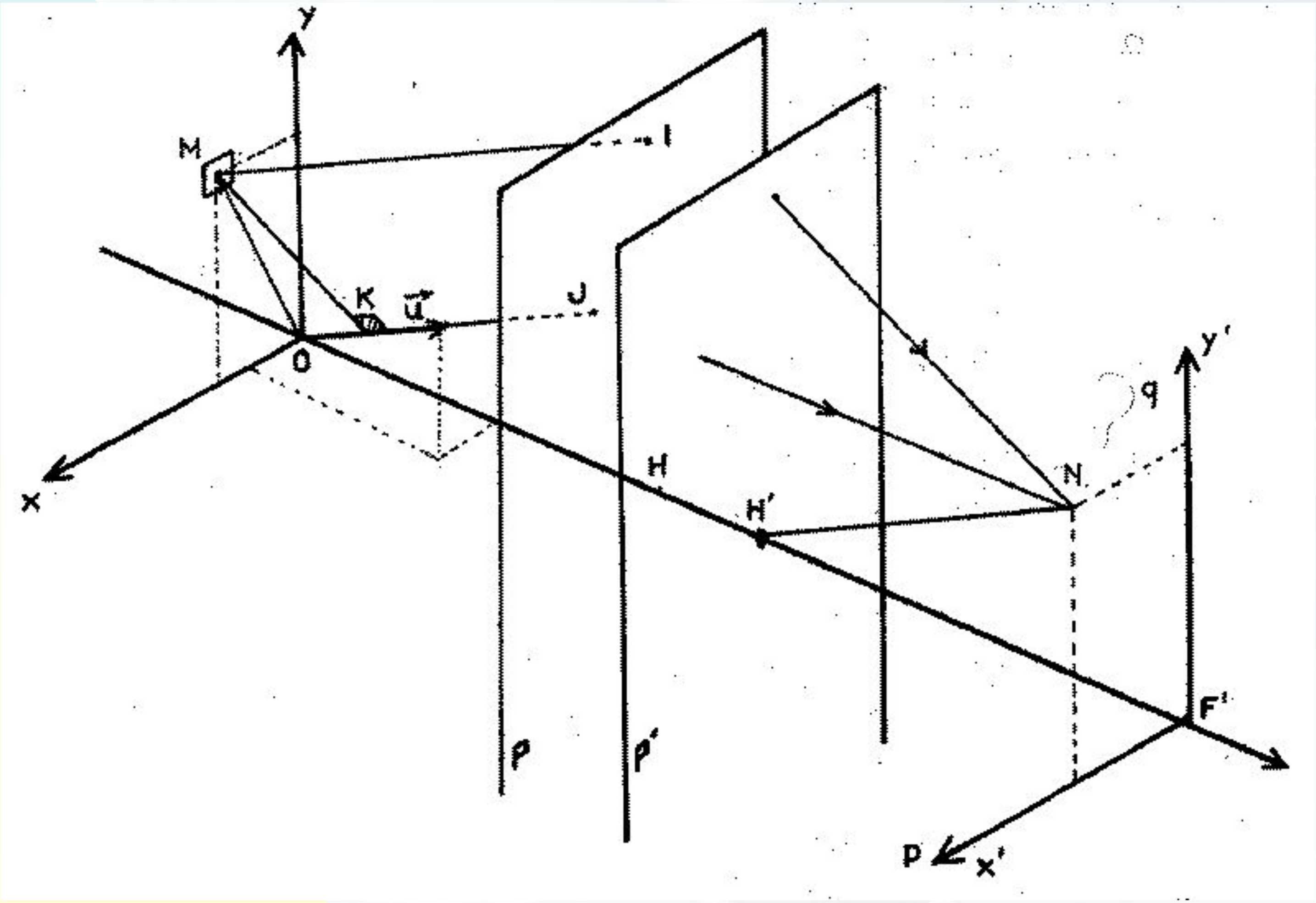
8.1 The fundamental theorem



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8.1 The fundamental theorem

- **Démonstration**

$A(x,y)\ \exp(i2\pi\nu t),$ (8.1.3.1)

$A(x,y) = \mathcal{A}(x,y)\ \exp(i\phi(x,y))\ P_0(x,y).$ (8.1.3.2)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

- **Démonstration**

$A(x,y)\ \exp(i2\pi\nu t + i\psi)$, (8.1.3.3)

$\delta = d(M\ I\ N) - d(O\ J\ N)$, (8.1.3.4)

$\psi = 2\pi\ \delta\ /\ \lambda$. (8.1.3.5)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

- **Démonstration**

$\delta = -d(O, K) = -|(\mathbf{OM}\ \mathbf{u})|,$ (8.1.3.6)

$A(x,y) \exp(i2\pi(\nu t - xx'/\lambda f - yy'/\lambda f)).$ (8.1.3.7)

$p = x'/\lambda f,\ q = y'/\lambda f,$ (8.1.3.8)

$\exp(i2\pi\nu t)\ A(x,y) \exp(-i2\pi(xp + yq)).$ (8.1.3.9)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

- **Démonstration**

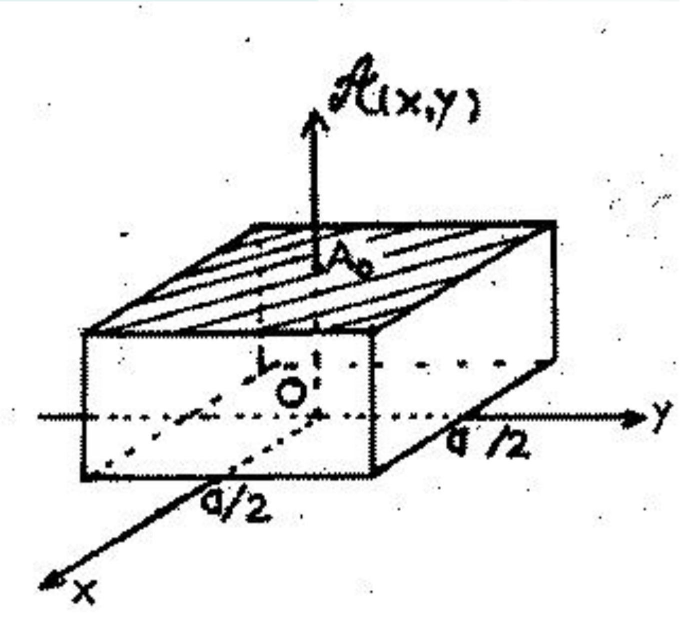
$$a(p,q) = \int_{R^2} A(x,y) \exp\left[-i2\pi\left(px+qy\right)\right] dxdy, \qquad (8.1.3.10)$$

$$a\left(p,q\right) = TF\_\left[A\left(x,y\right)\right]\left(p,q\right) \qquad (8.1.3.11)$$

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination



$$A(x,y) = A_0\, P_0(x,y), \qquad (8.1.1)$$

$$P_0(x,y) = \Pi(x/a)\, \Pi(y/a). \qquad (8.1.2)$$

An introduction to optical/IR interferometry

8.1 The fundamental theorem

$$a(p,q) = TF\_[A(x,y)](p,q) = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2} A_0 \exp[-i2\pi(px+qy)]dxdy \qquad (8.1.3)$$

$$a(p,q) = A_0 \int_{-a/2}^{a/2} \exp[-i2\pi px]dx \int_{-a/2}^{a/2} \exp[-i2\pi qy]dy \qquad (8.1.4)$$

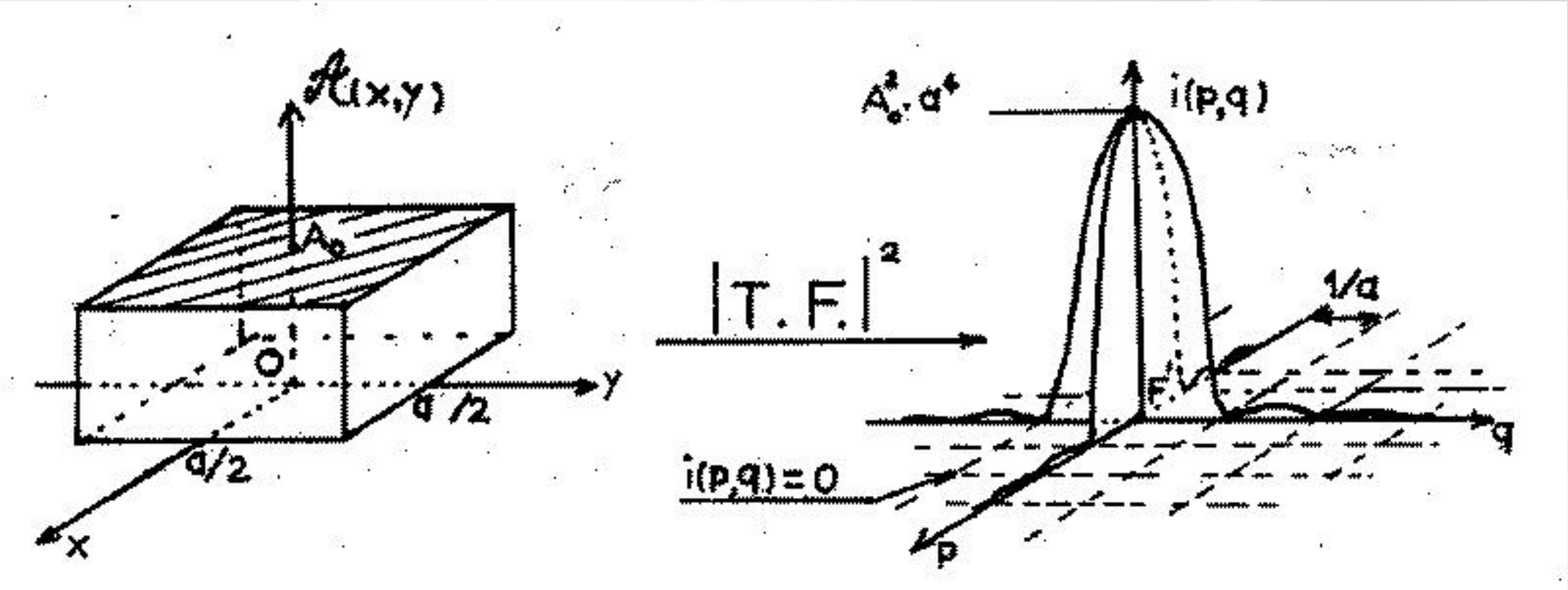
$$a(p,q) = A_0\, a^2\, [\sin(\pi pa) / (\pi pa)]\, [\sin(\pi qa) / (\pi qa)]. \qquad (8.1.5)$$

$$i(p,q) = a(p,q)\, a^*(p,q) = |a(p,q)|^2 = |h(p,q)|^2 =$$
$$= i_0\, a^4\, [\sin(\pi pa) / (\pi pa)]^2\, [\sin(\pi qa) / (\pi qa)]^2. \qquad (8.1.6)$$

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination



$\Delta p = \Delta x' /(\lambda f)$; $\Delta q = \Delta y' /(\lambda f) = 2/a$ ➔ $\Delta\phi_{x'} = \Delta\phi_{y'} = 2\lambda/a$ (8.1.7)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination when observing a star along another direction

$\psi = 2\pi\, \delta / \lambda = 2\pi(xb/f + yc/f) / \lambda,$ (8.1.5.7)

$A(x,y) = P_0(x,y)\, A_0 \exp[2i\pi(xb/f + yc/f) / \lambda].$ (8.1.5.8)

$$a(p,q) = A_0 \int_{-a/2}^{a/2} \exp\left[-2i\pi(p - b/f\lambda)x\right] dx \int_{-a/2}^{a/2} \exp\left[-2i\pi(q - c/f\lambda)y\right] dy \quad (8.1.5.9)$$

$$a(p,q) = A_0 a^2 \left(\frac{\sin\left(\pi\left(p - b/f\lambda\right)a\right)}{\pi\left(p - b/f\lambda\right)a}\right)\left(\frac{\sin\left(\pi\left(q - c/f\lambda\right)a\right)}{\pi\left(q - c/f\lambda\right)a}\right) \quad (8.1.5.10)$$

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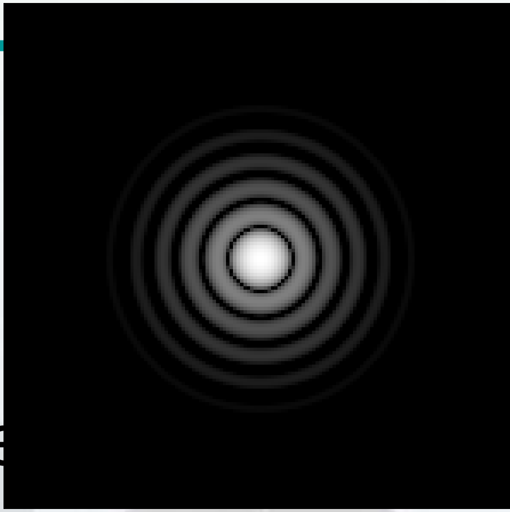
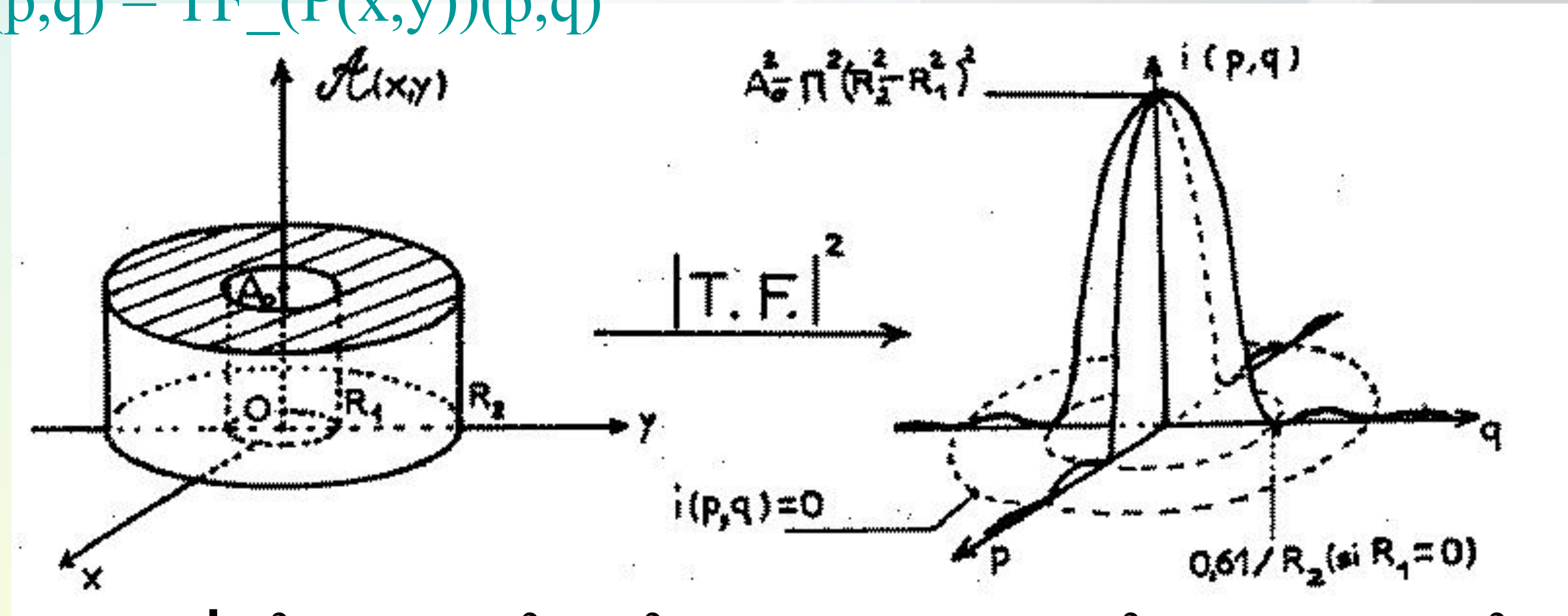


8.1 The fundamental theorem

Application: Point Spread Function dete

h(p,q) = TF_(P(x,y))(p,q)



$$i(\rho') = |a(\rho')|^2 = (A_0\,\pi)^2\,[R_2^2\,2\,J_1(Z_2)/Z_2\; - R_1^2\,2\,J_1(Z_1)/Z_1]^2, \quad (8.1.8)$$

with $Z_2 = 2\pi\,R_2\,\rho' / (\lambda f)$ and $Z_1 = 2\pi\,R_1\,\rho' / (\lambda f)$. (8.1.9)

An introduction to optical/IR interferometry

BESSEL FUNCTIONS (REMINDER)

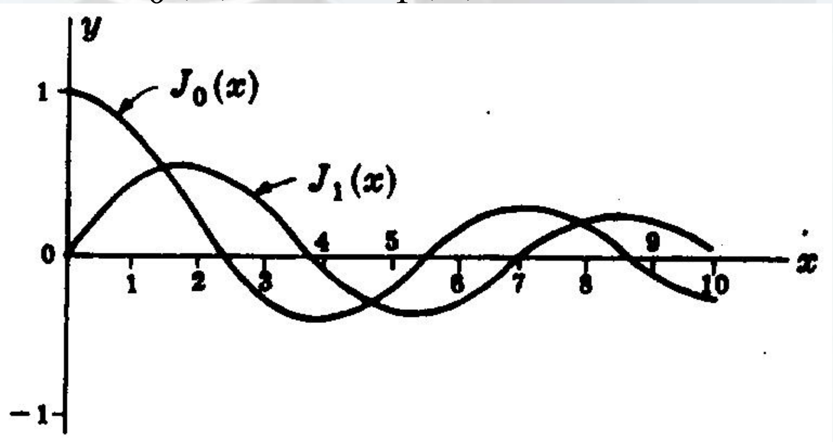
Integral representation of the Bessel functions

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos[x\sin(\vartheta)]d\vartheta \qquad J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos[n\vartheta - x\sin(\vartheta)]d\vartheta$$

Undefined integral

$$\int x' J_0(x')dx' = x J_1(x)$$

Graphs of the $J_0(x)$ and $J_1(x)$ functions



Series development (x ~ 0):

$J_0(x) = 1 - x^2/2^2 + x^4/(2^2 4^2) - x^6/(2^2 4^2 6^2) + \ldots$

$J_1(x) = x/2 - x^3/(2^2 4) + x^5/(2^2 4^2 6) - x^7/(2^2 4^2 6^2 8) + \ldots$

$J_n(x) = (2 / (\pi x))^{1/2} \cos(x - n\pi/2 - \pi/4)$ … and when x is large!

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination

x = ρ cos(θ), y = ρ sin(θ), p = ρ' cos(θ') / (λf), q = ρ' sin(θ') / (λf).

(8.1.5.13)

$$a(\rho',\theta') = A_0\int_{R_1}^{R_2}\int_0^{2\pi} \exp\left[-2i\pi\rho\rho'\cos(\theta-\theta')/(\lambda f)\right] d(\theta-\theta')\rho d\rho$$

(8.1.5.14)

$$a(\rho',\theta') = a(\rho') = A_0\pi\left[\frac{2R_2^2}{Z_2}J_1(Z_2) - \frac{2R_1^2}{Z_1}J_1(Z_1)\right] \quad (8.1.5.15)$$

$$Z_2 = 2\pi R_2\frac{\rho'}{\lambda f} \quad \text{et} \quad Z_1 = 2\pi R_1\frac{\rho'}{\lambda f} \quad (8.1.5.16)$$

Pour le cas $R_1 = 0$ $\quad i(\rho') = |a(\rho')|^2 = 4(A_0\pi)^2 R_2^4\left(\frac{J_1(Z_2)}{Z_2}\right)^2$ (8.1.5.17)

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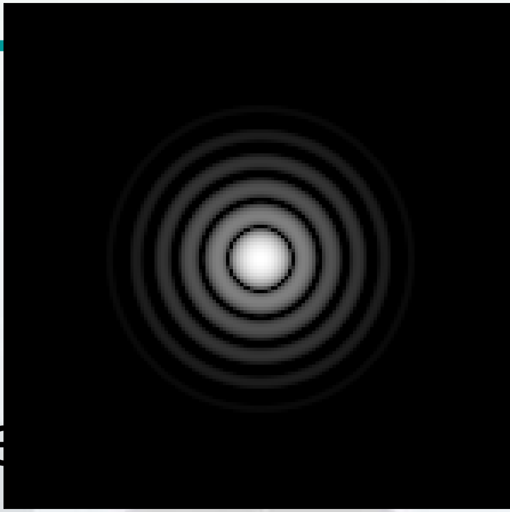

8.1 The fundamental theorem

Application: Point Spread Function dete

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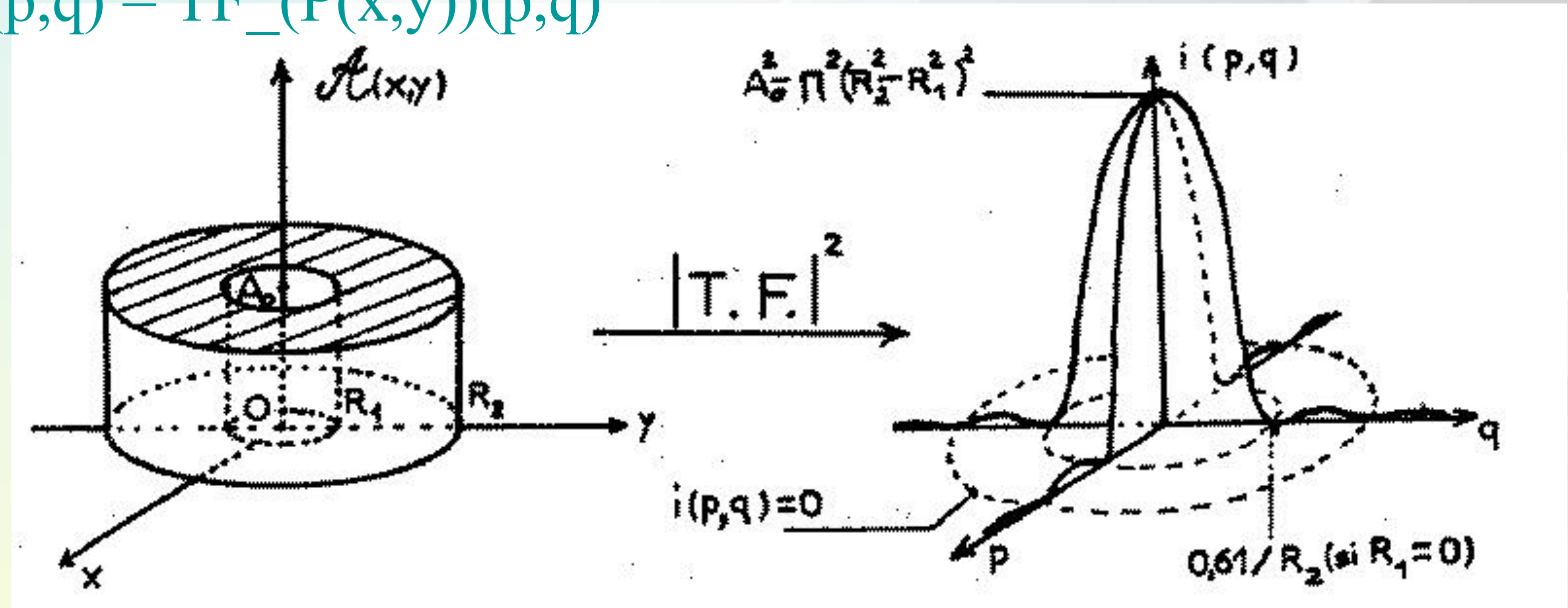

$i(\rho') = |a(\rho')|^2 = (A_0\,\pi)^2\,[R_2^2\,2\,J_1(Z_2)/Z_2 - R_1^2\,2\,J_1(Z_1)/Z_1]^2$, (8.1.8)

with $Z_2 = 2\pi\,R_2\,\rho' / (\lambda f)$ and $Z_1 = 2\pi\,R_1\,\rho' / (\lambda f)$. (8.1.9)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination

ρ' (=r) = 1,22 λ f / D (D = 2 R_2, R_1 = 0). (8.1.5.18)

$$\frac{2\pi\int_0^r i(\rho')\rho' d\rho'}{2\pi\int_0^\infty i(\rho')\rho' d\rho'} = 0{,}84 \qquad (8.1.5.19)$$

h(p,q) = TF_(P(x,y))(p,q). (8.1.5.20)