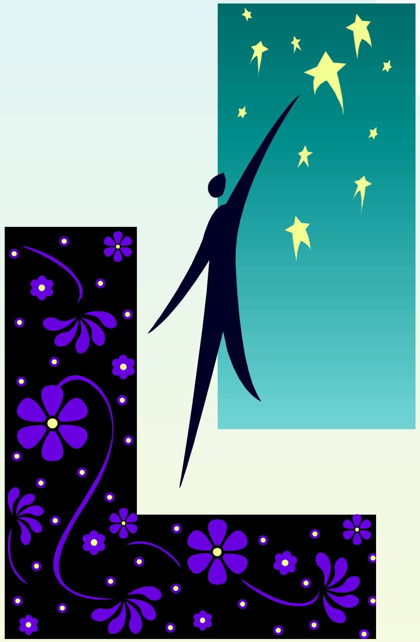
“La vraie faute est celle qu’on ne corrige pas …”

Confucius

An introduction to optical/IR interferometry

Brief summary of main results obtained during the last lecture:



$$V = \left|\gamma_{12}(0,u,v)\right| = \left|\iint_S I'(\zeta,\eta)\exp\left\{-i2\Pi\left(u\zeta + v\eta\right)\right\} d\zeta\, d\eta\right|$$

$$I'(\zeta,\eta) = \iint \gamma_{12}(0,u,v)\exp\left\{i2\Pi\left(\zeta u + \eta v\right)\right\} d(u)d(v)$$

- For the case of a 1D uniformly brightening star whose angular diameter is ϕ = b/z’, we found that the visibility of the fringes is zero when λ/B = b/z’ = ϕ where B is the baseline of the interferometer

- For the case of a double star with an angular separation ϕ = b/z’, we found that the visibility of the fringes is zero when λ/2B = b/z’ = ϕ

An introduction to optical/IR interferometry

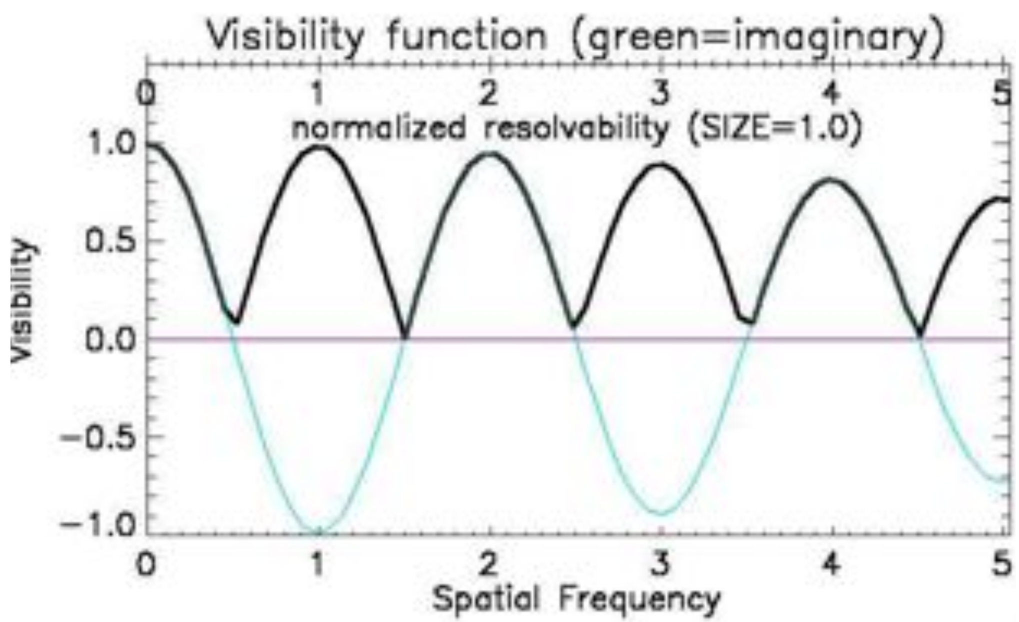
- 5 Light coherence
- **5.5 Aperture synthesis**

Exercises:

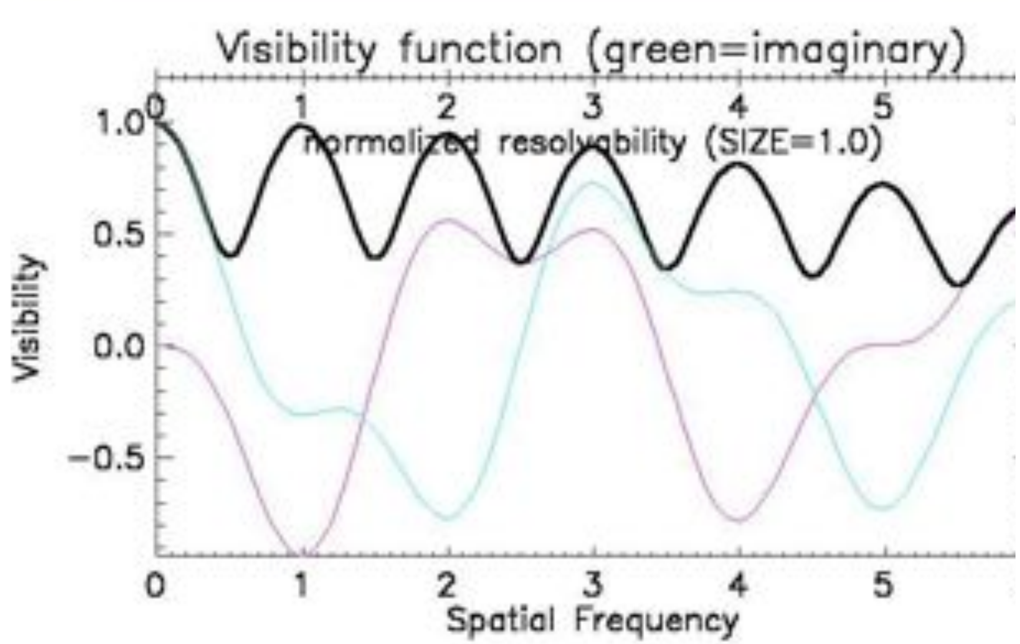
- the case of a gaussian-like source?

- let us assume that the observed visibility $|\Upsilon_{12}(0,u)|$ of a celestial object is $|\cos(\pi u\theta)|$, please retrieve the intensity distribution I' of the source

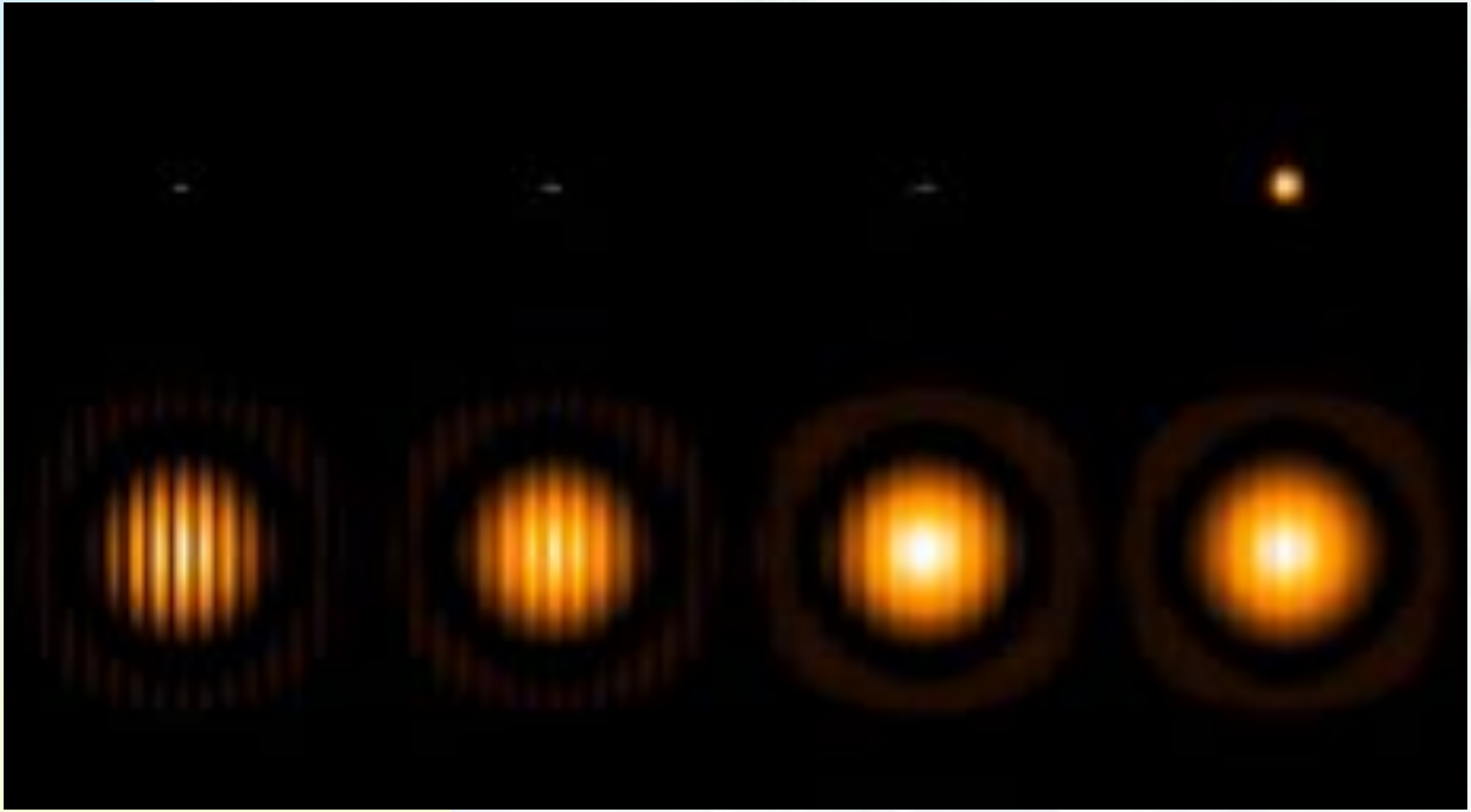
Case of a double point-like source with a flux ratio = 1



Case of a double point-like source with a flux ratio 0.7/0.3

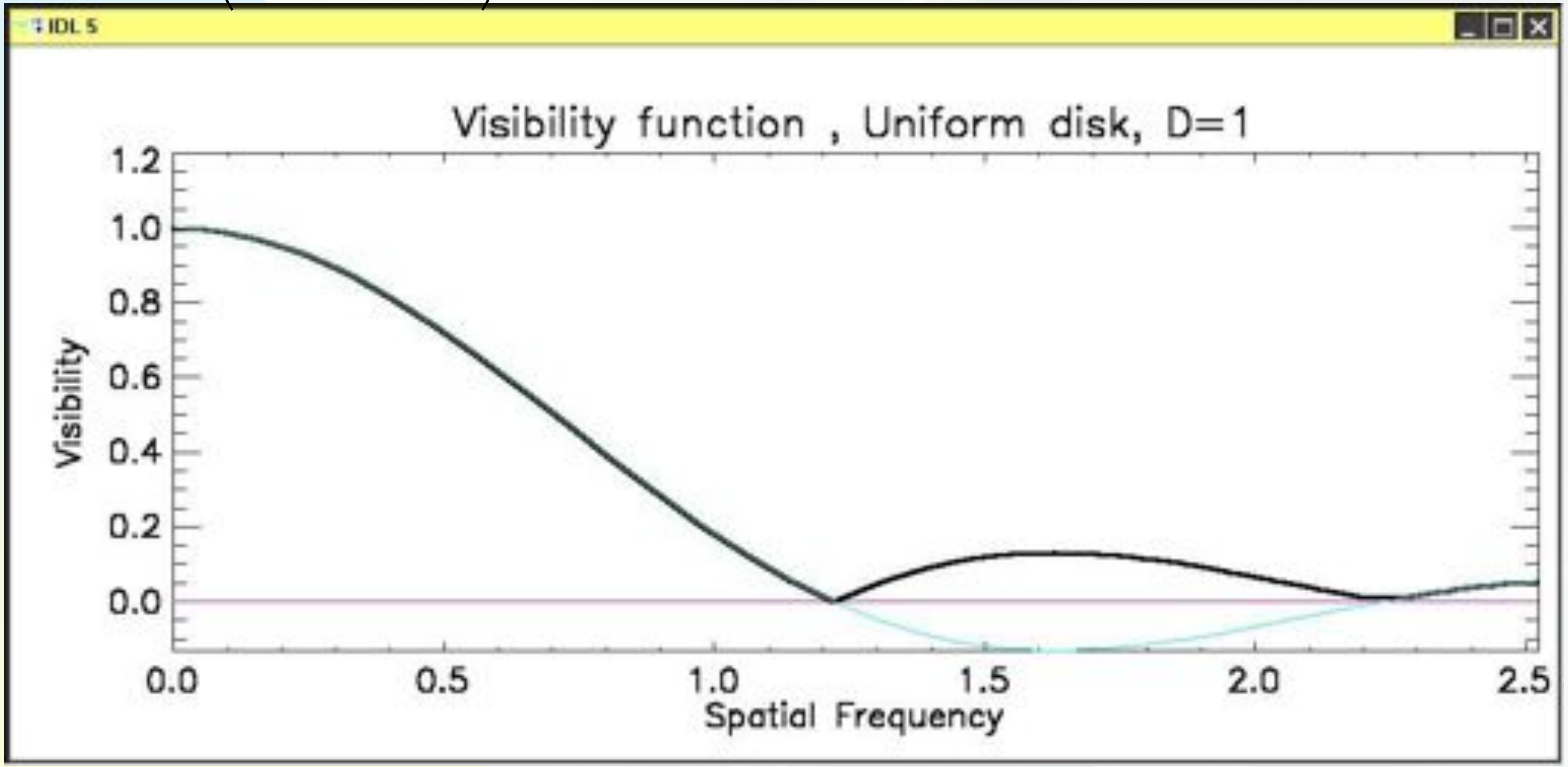


Variation of the fringe contrast as a function of the angular separation between the two stars:

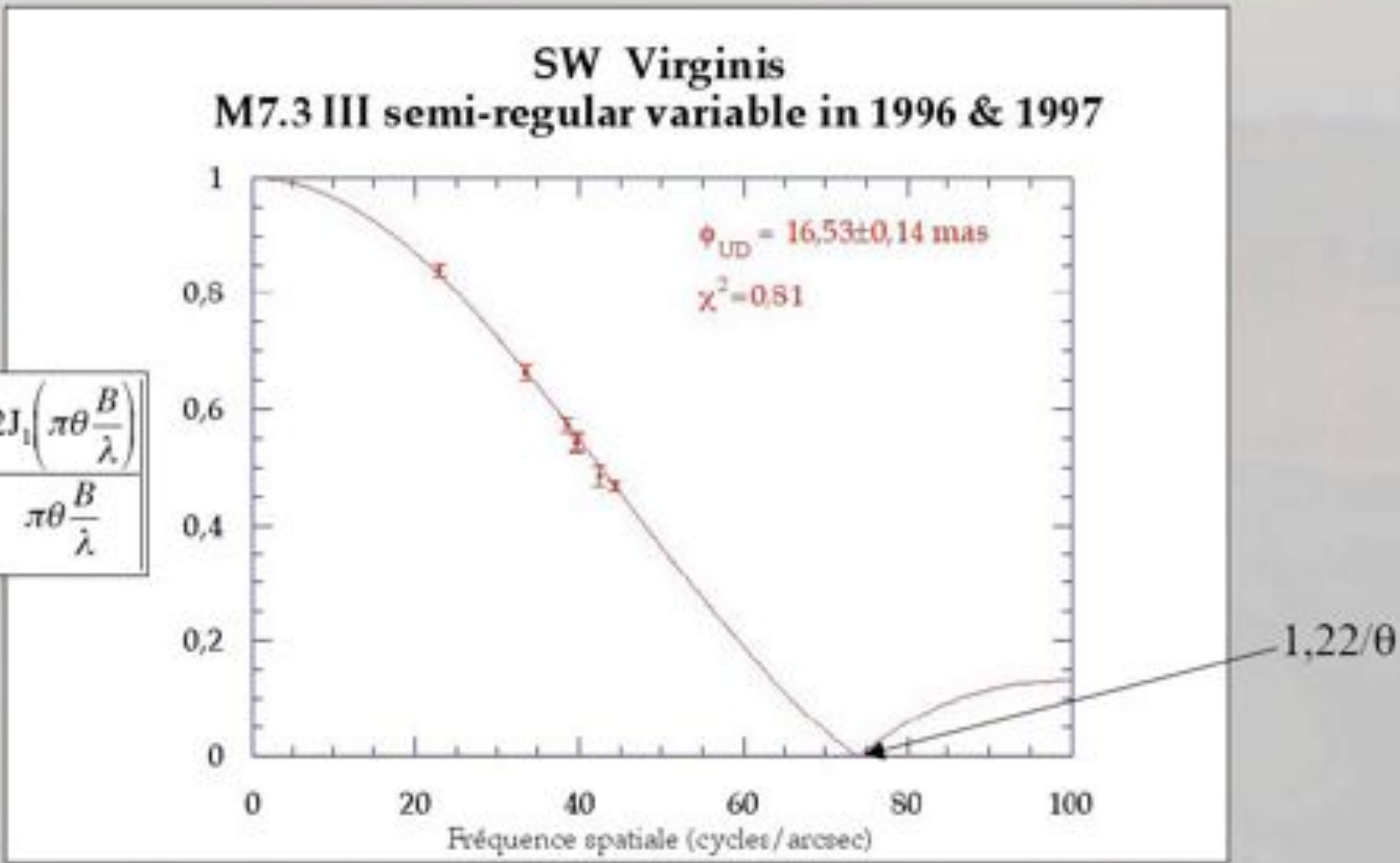


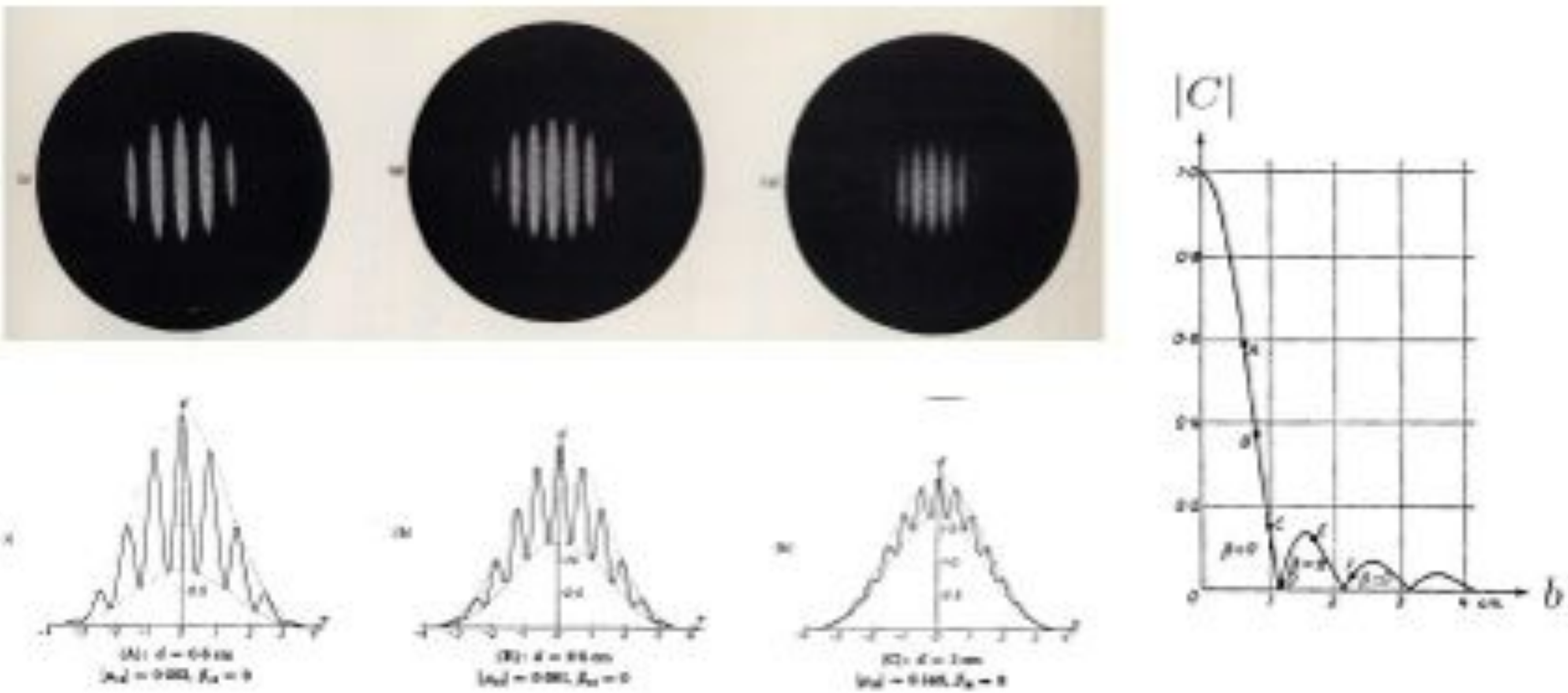
If the source is characterized by a uniform disk light distribution, the corresponding visibility function is given by

$$\upsilon = \left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \right) = \left| \gamma_{12}(0) \right| = TF(I) = \frac{2J_1(\pi\theta_{UD}B/\lambda)}{\pi\theta_{UD}B/\lambda}$$

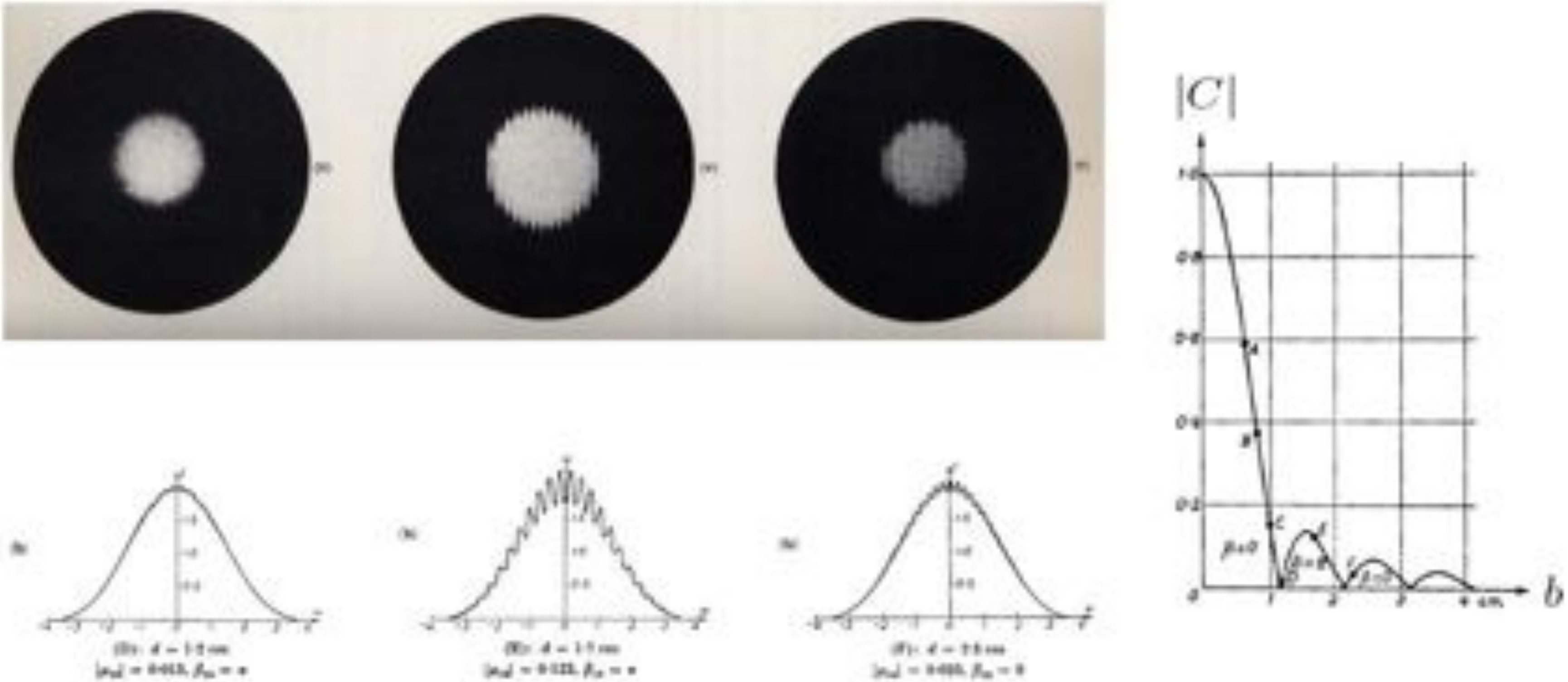


$$V_{DU}(B) = \left| \frac{2J_1\left(\pi\theta \frac{B}{\lambda}\right)}{\pi\theta \frac{B}{\lambda}} \right|$$





$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C|\cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$



$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C|\cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

For the case of the Sun:

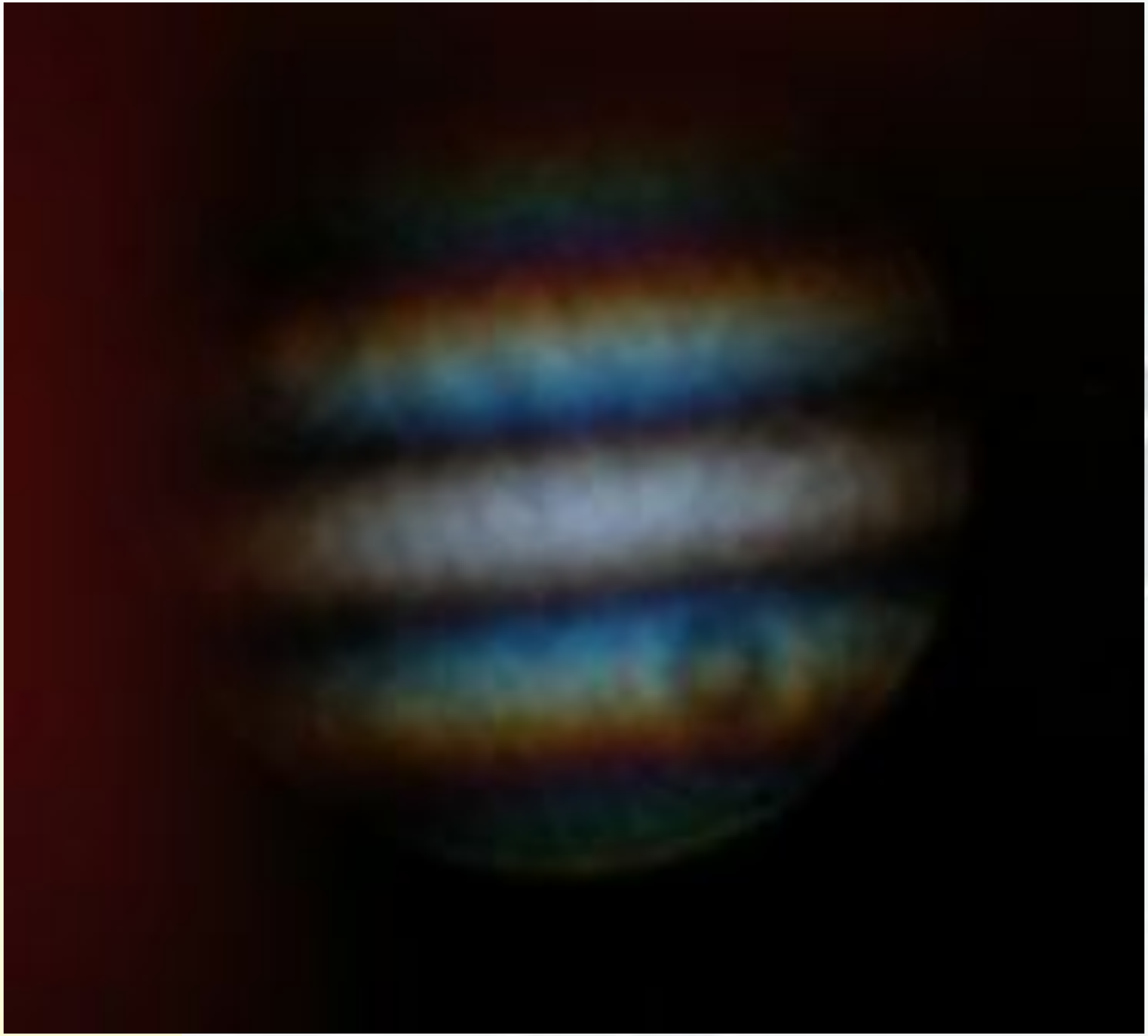
$\vartheta_{UD} = 1.22\lambda / B = 1.22\ 0.55 / B(\mu) = 30' \times 60'' / 206265$

$B(\mu) = 206265 \times 1.22 \times 0.55 / (30 \times 60) = 76.9\ \mu$

$d(\mu) = 7.2$ or $14.4\ \mu$ ➔ $\sigma = 2.44\ \lambda / d = 7.8°$ or $3.9°$

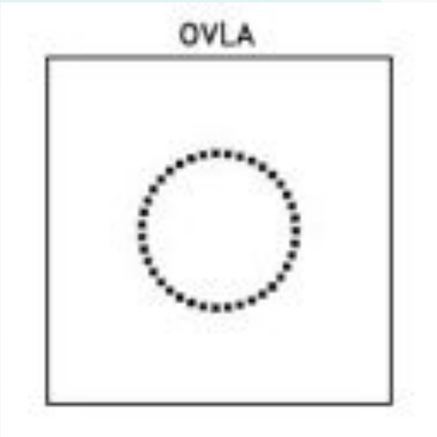
See the masks!

First fringes on the Sun: 9/4/2010



B = 29.4μ
d = 11.8μ

OVLA PSF

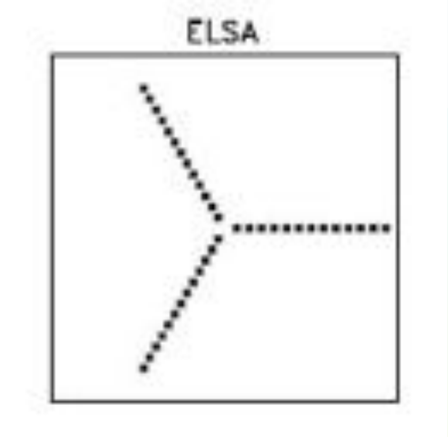


↔ 50μ

• 14μ

OVLA_Sun_2

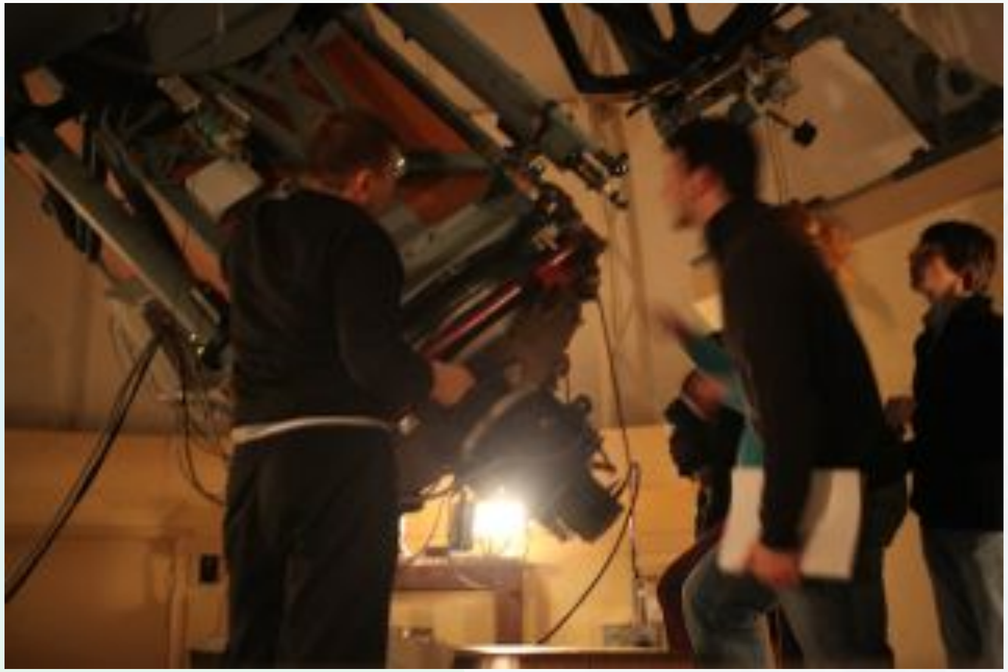
ELSA PSF



↔ 50μ

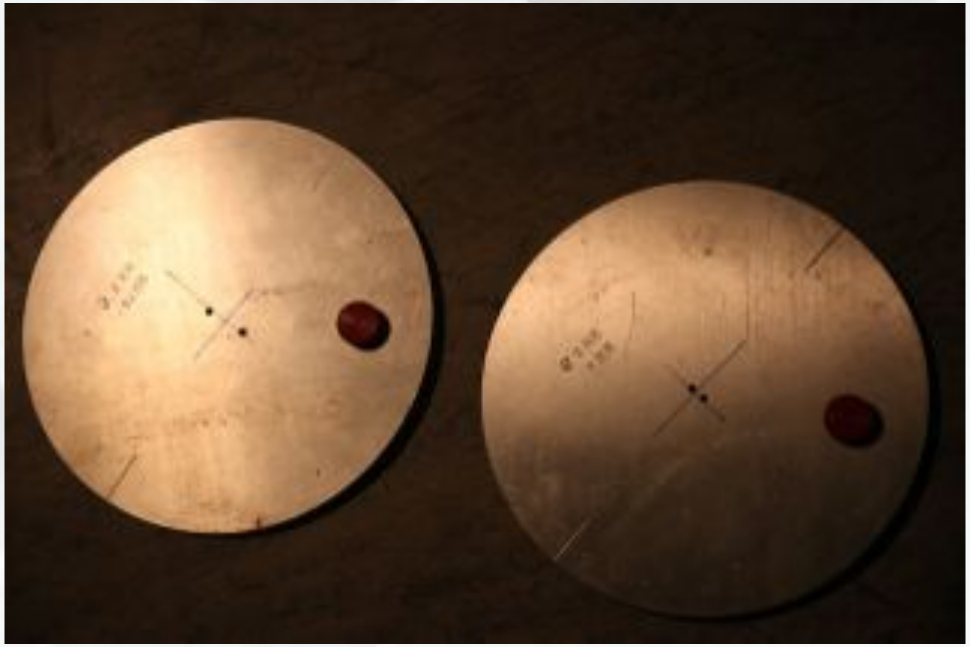
. 14μ

ELSA_Sun_24

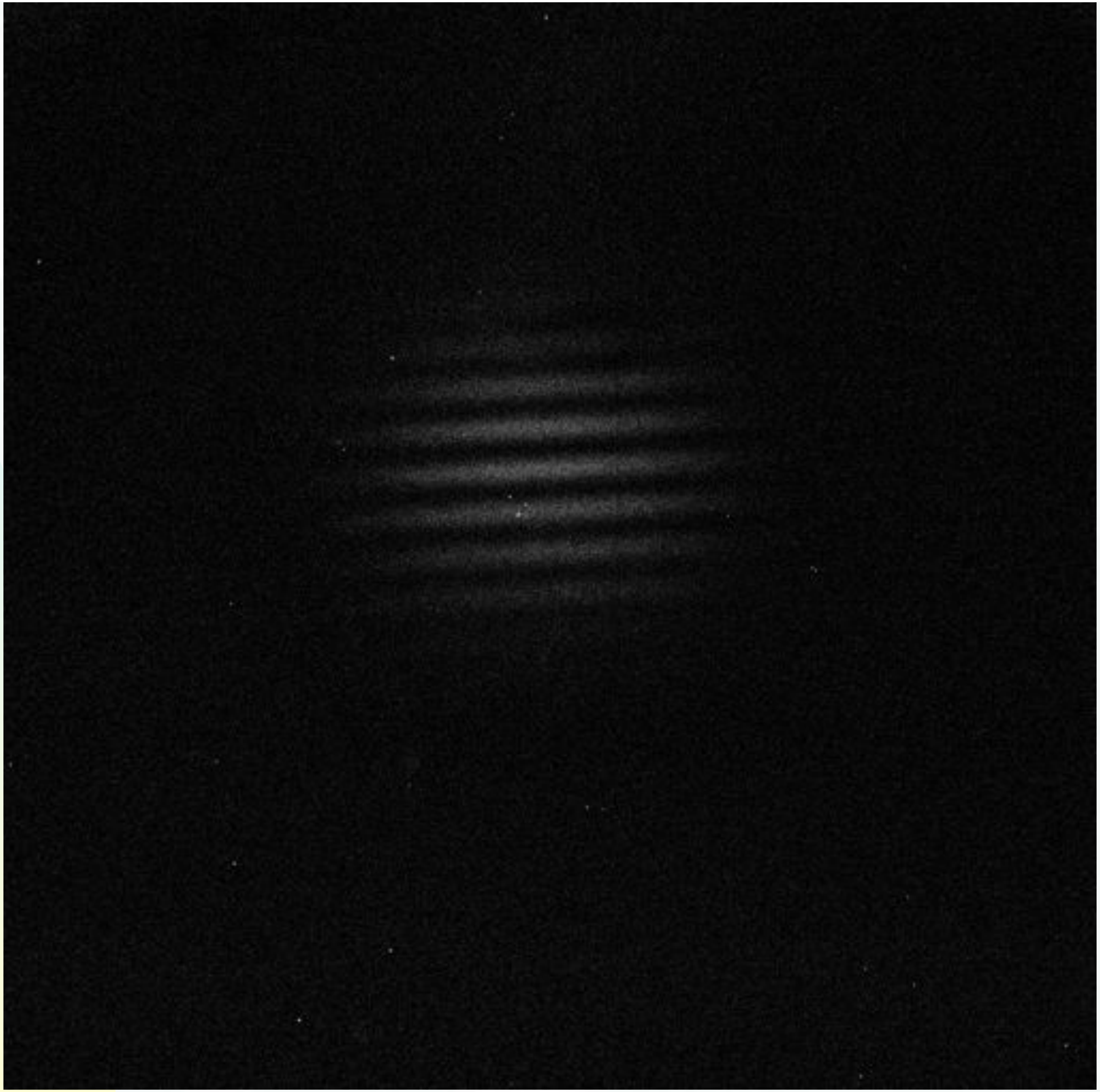


Interferometric observations on 10/4/2010 of Procyon, Mars and Saturn, using the 80cm telescope at Haute-Provence Observatory and adequate masks (coll. with Hervé le Coroller) …

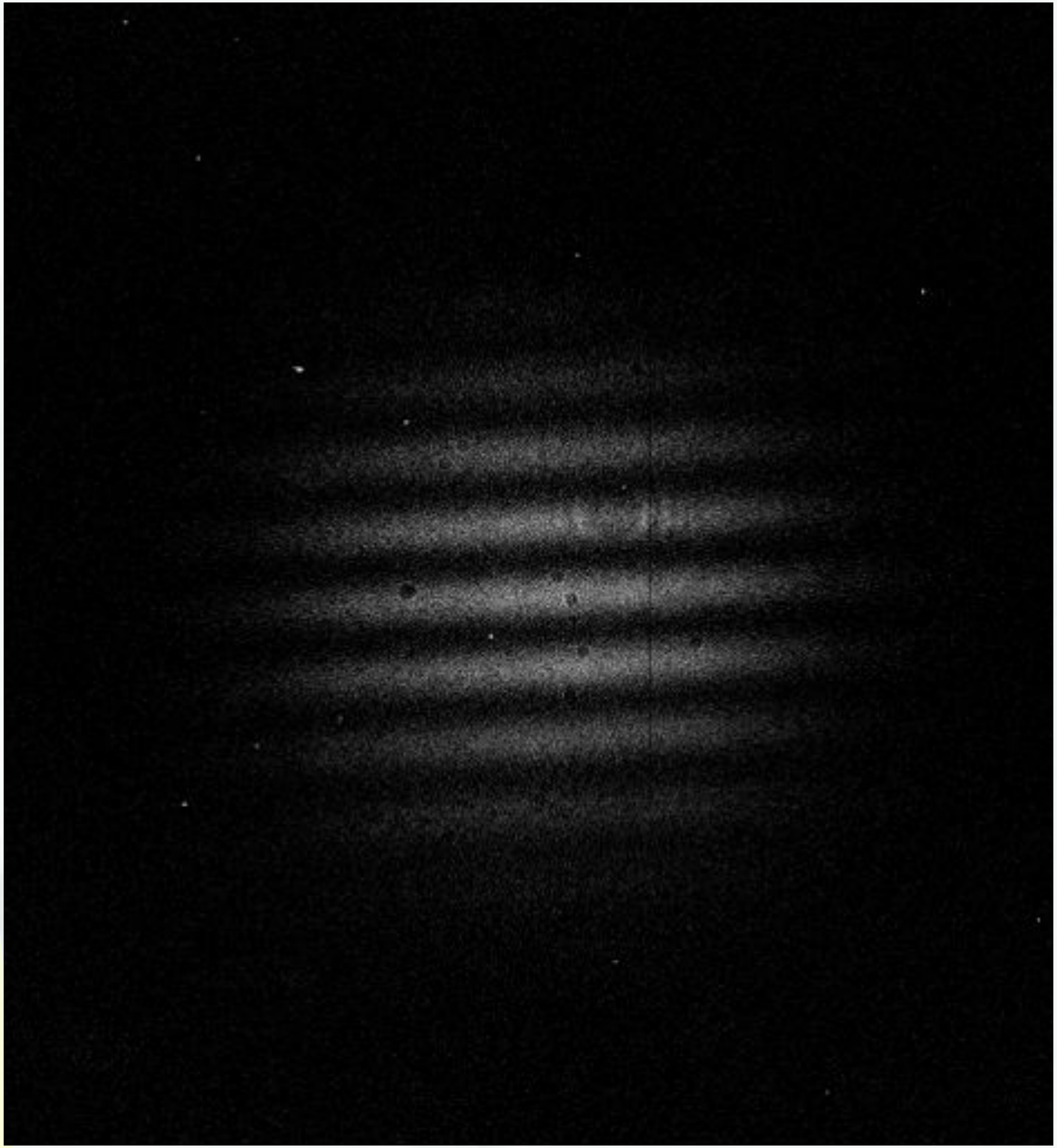




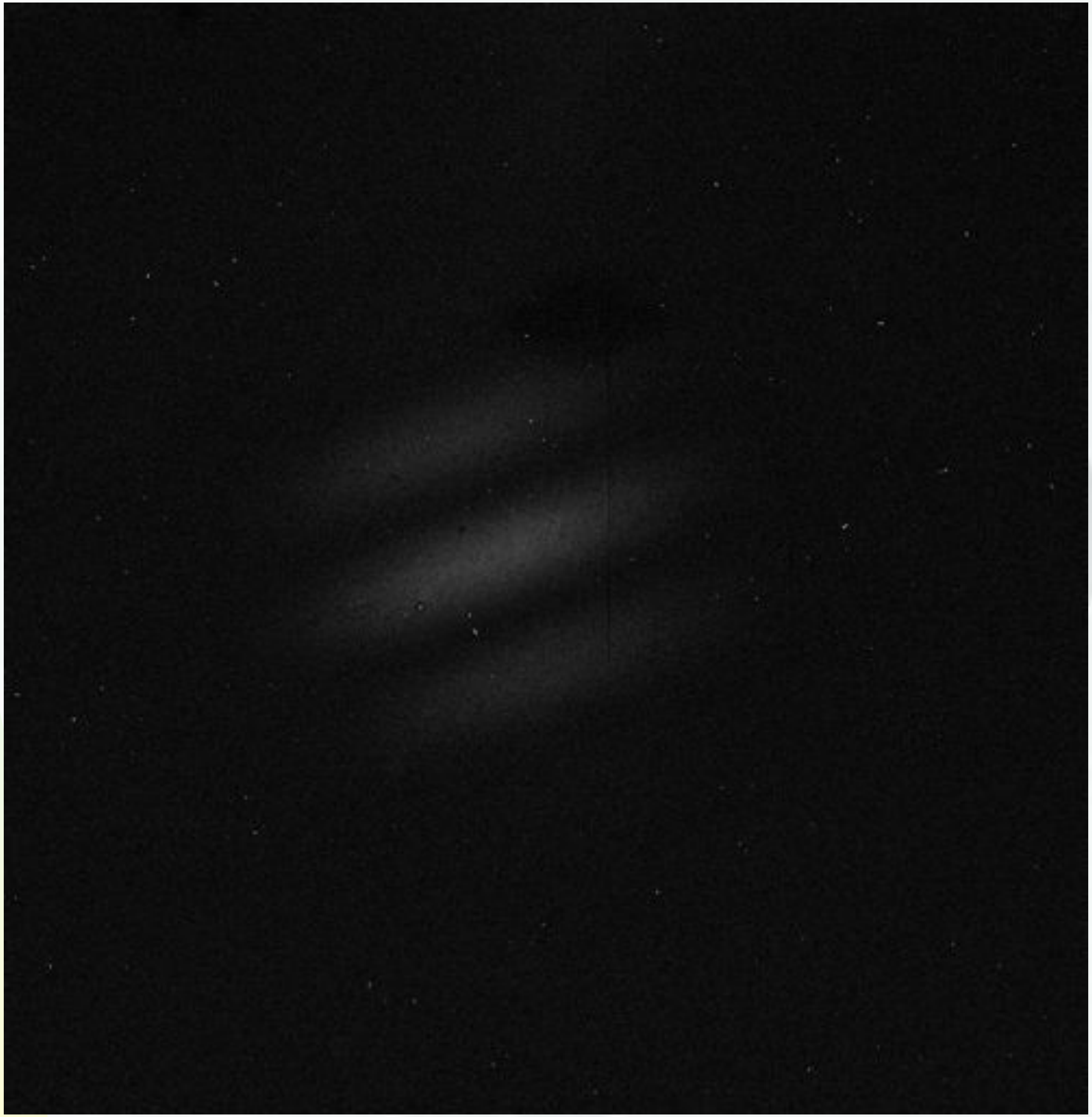
Procyon
B = 12 mm
d = 2 mm



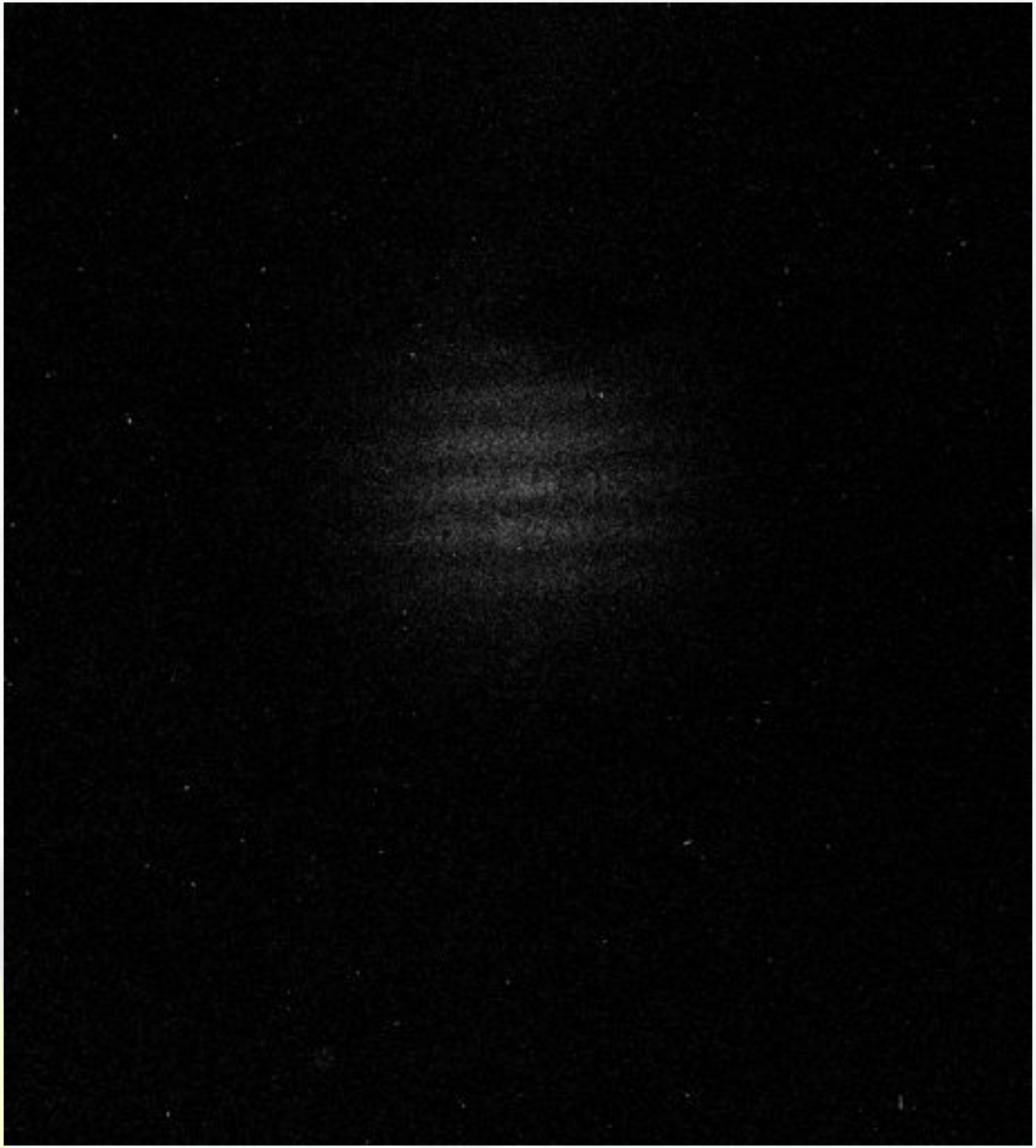
Mars
B = 12 mm
d = 2 mm



Saturn
B = 4 mm
d = 2 mm

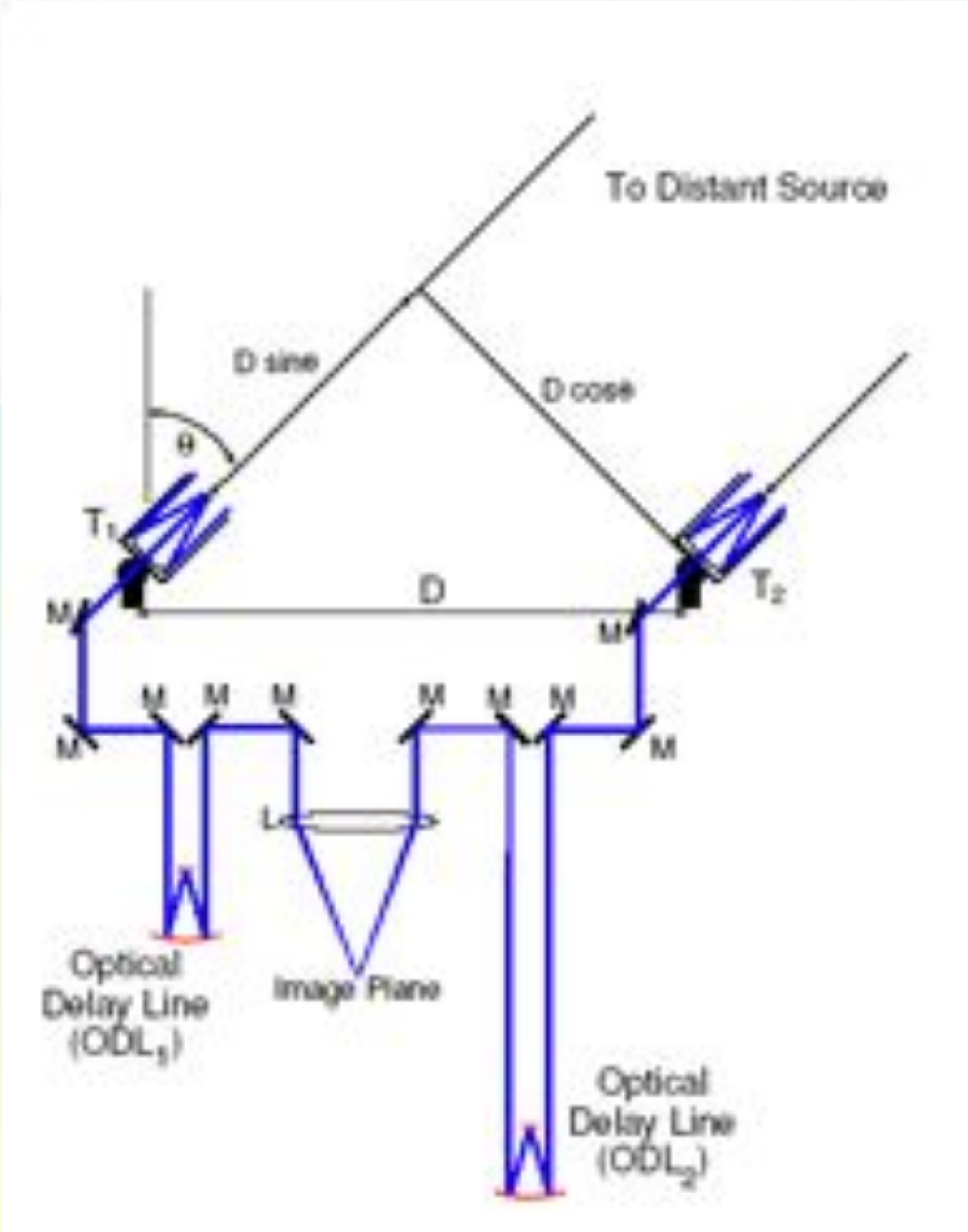


Saturn
B = 12 mm
d = 2 mm



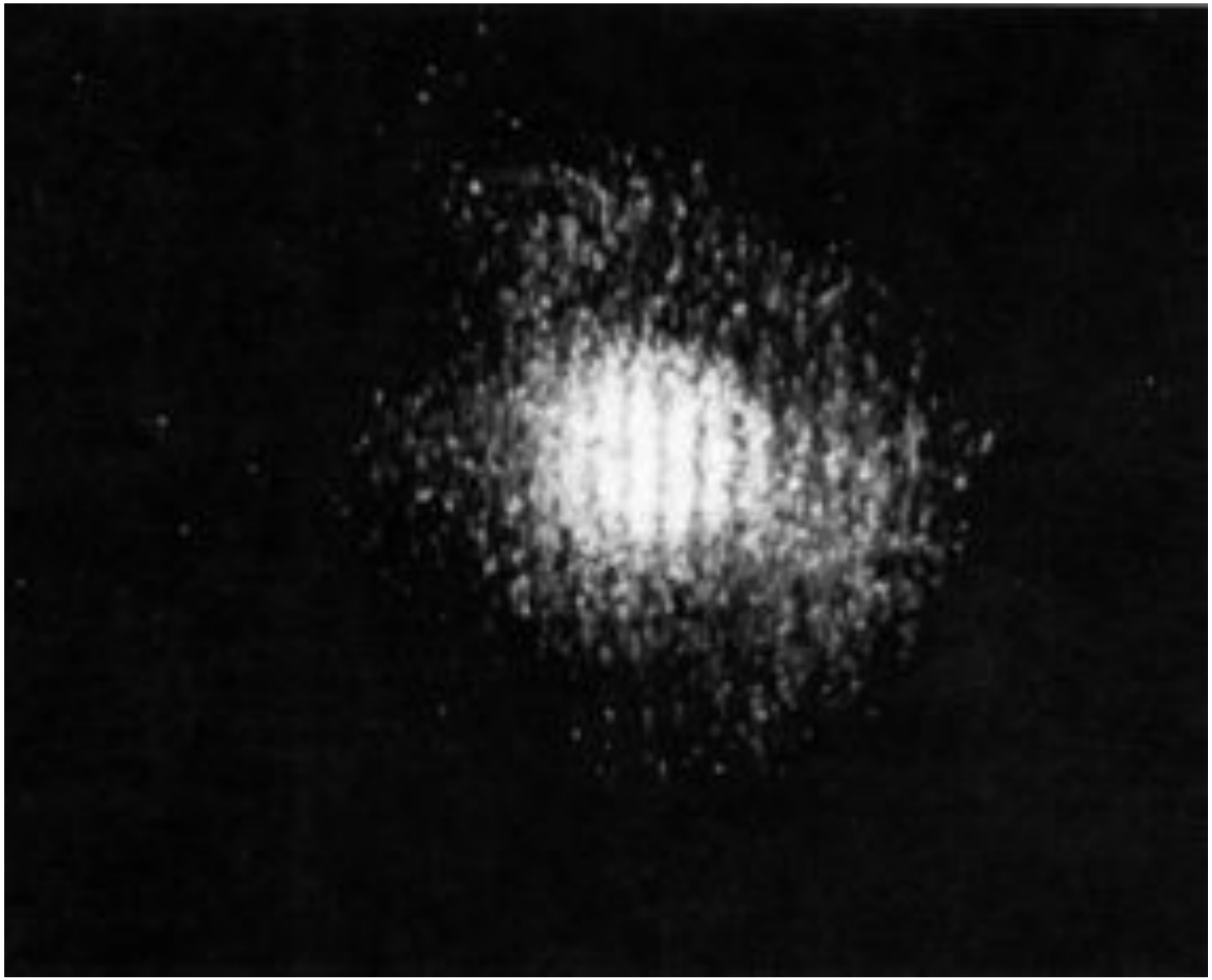
An introduction to optical/IR interferometry

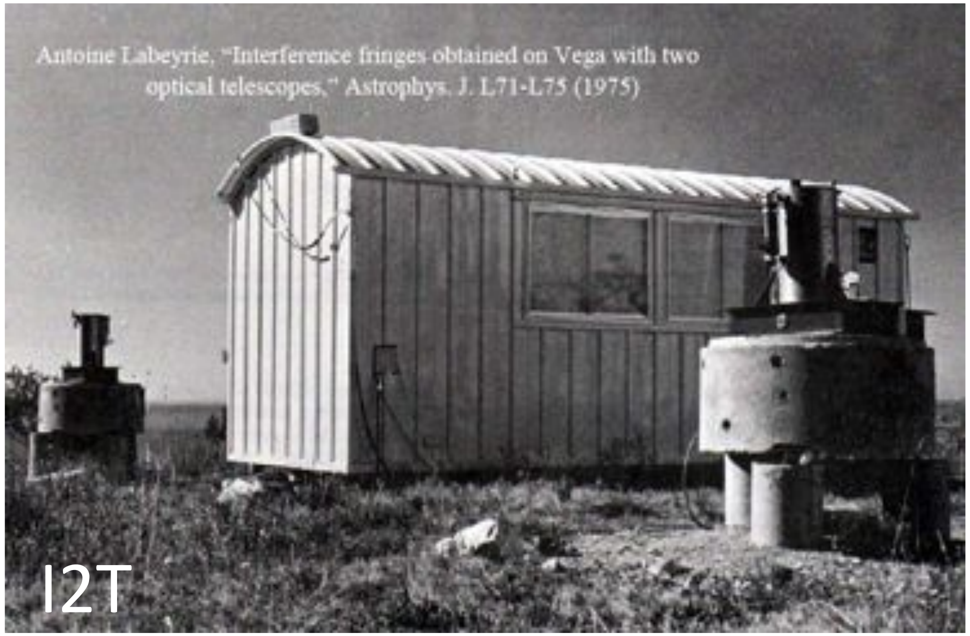
- 6 Some examples of optical interferometers



To Distant Source
D sinθ
D cosθ
θ
T1
T2
D
M
M
M M M M M M
M
M
L
Optical
Delay Line
(ODL1)
Image Plane
Optical
Delay Line
(ODL2)

First fringes with I2T

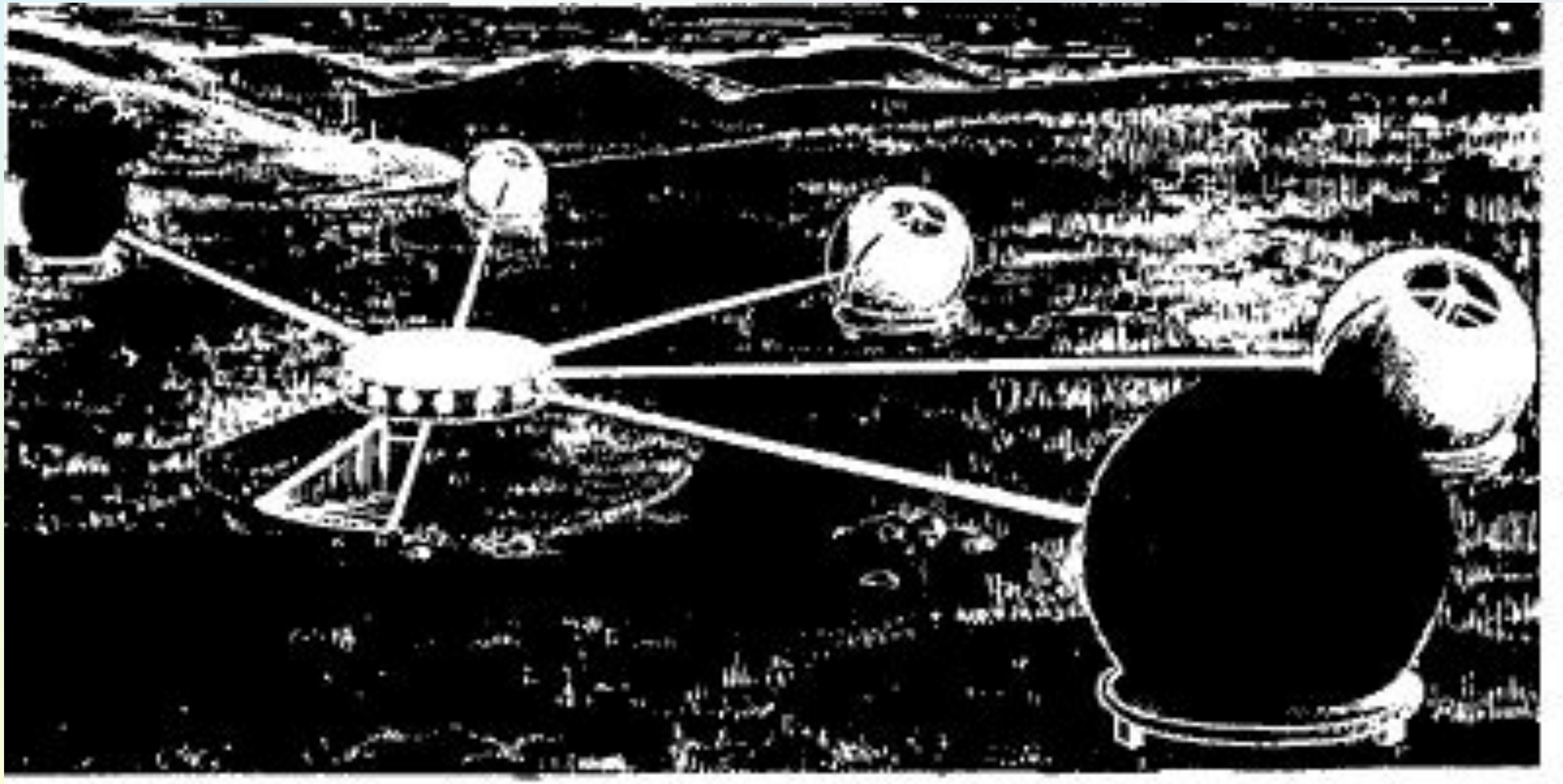



Antoine Labeyrie, "Interference fringes obtained on Vega with two optical telescopes," Astrophys. J. L71-L75 (1975)
I2T


GI2T

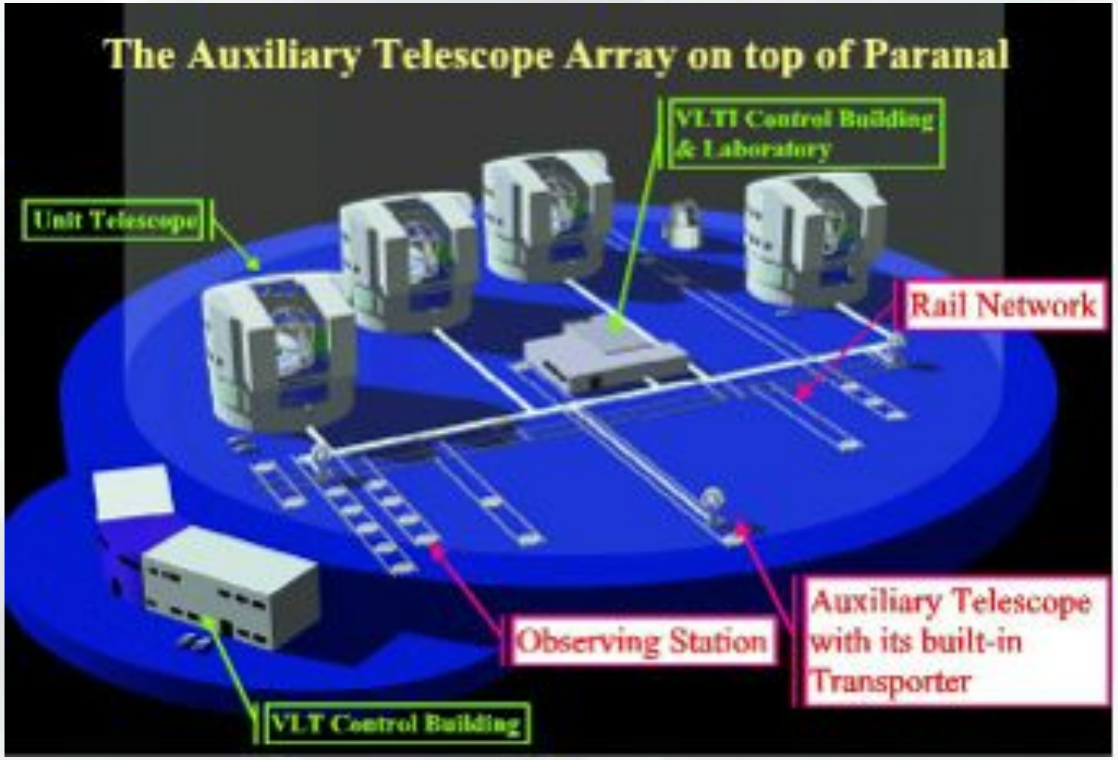
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers



An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers



http://www.aeos.ulg.ac.be/HARI/

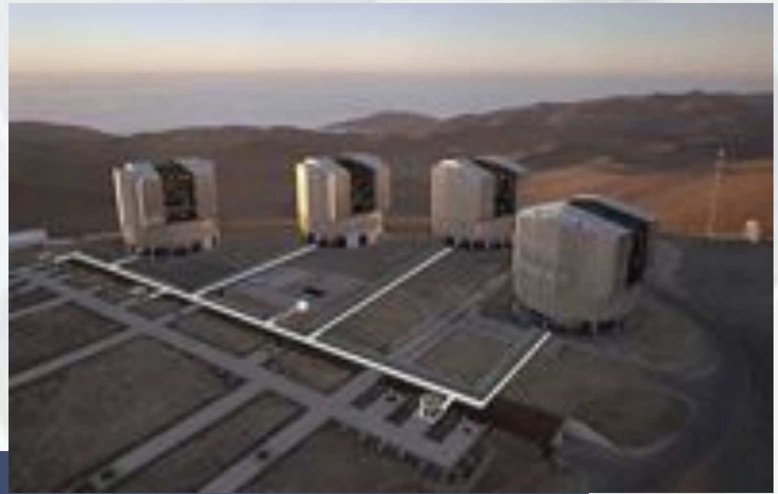
An introduction to optical/IR interferometry

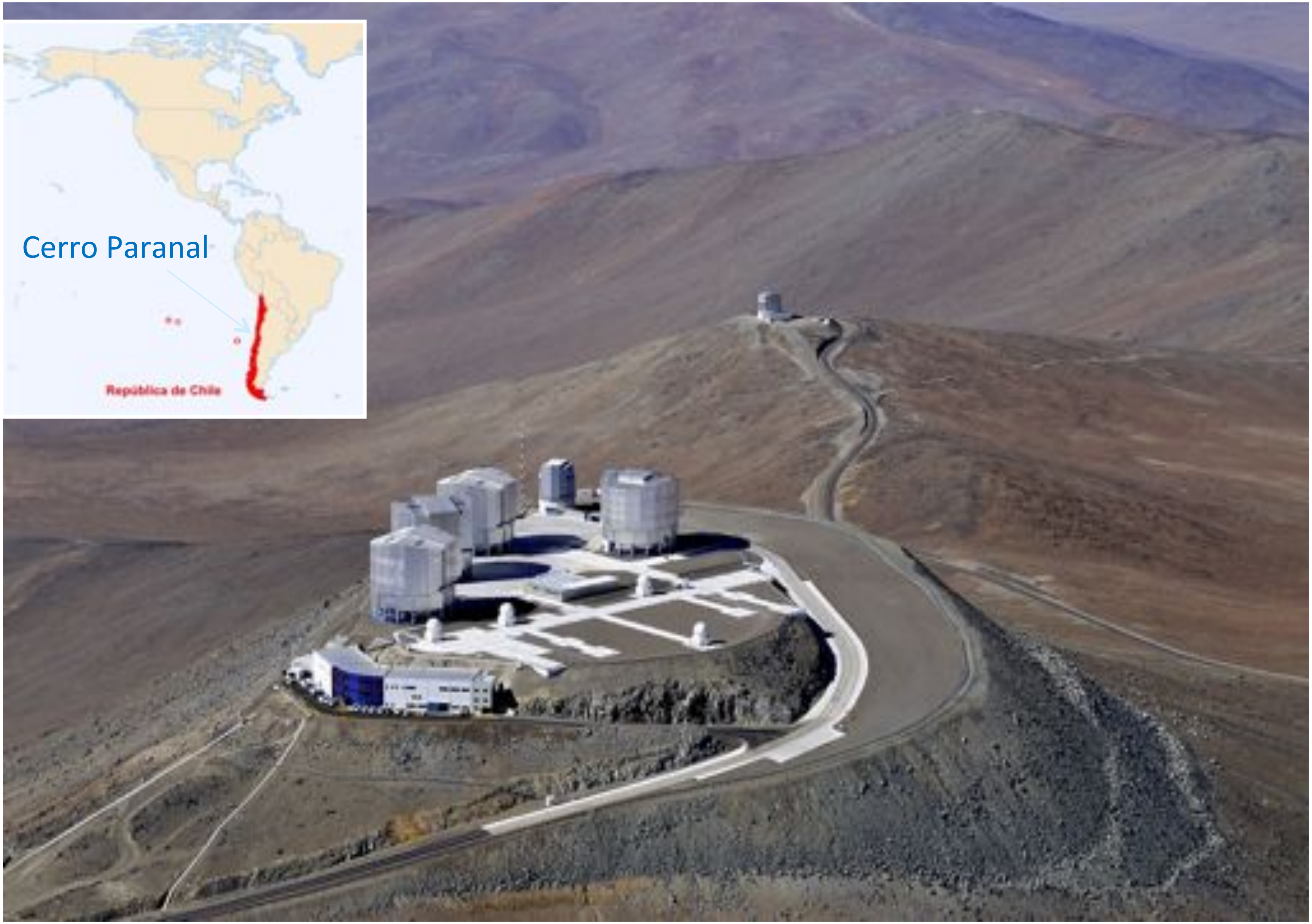
6 Some examples of optical interferometers

Interferometry to-day is:

Very Large Telescope Interferometer (VLTI)

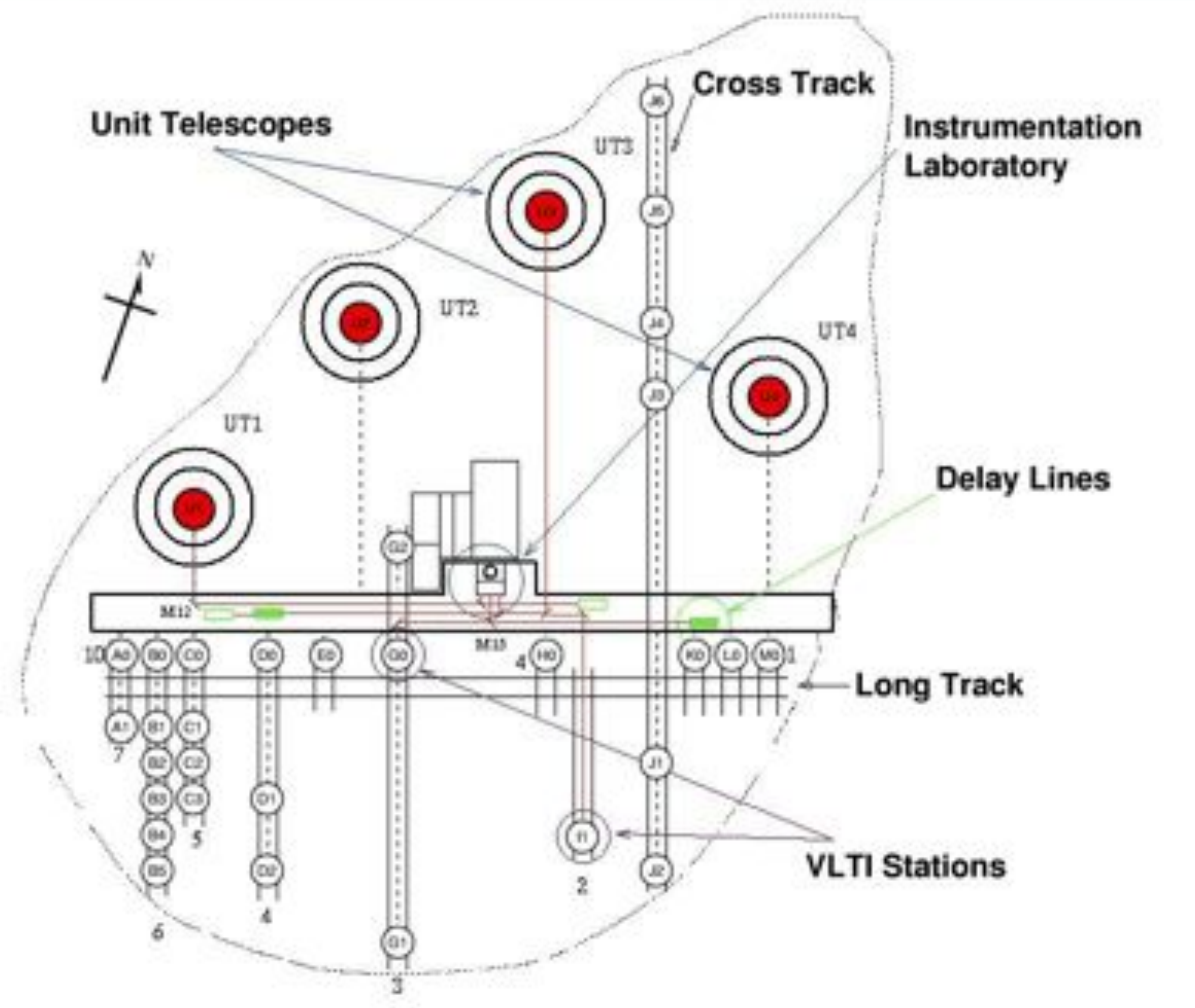
- 4 x 8.2m UTs
- 4 x 1.8m ATs
- Max. Base: 200m




Cerro Paranal
República de Chile

An introduction to optical/IR interferometry

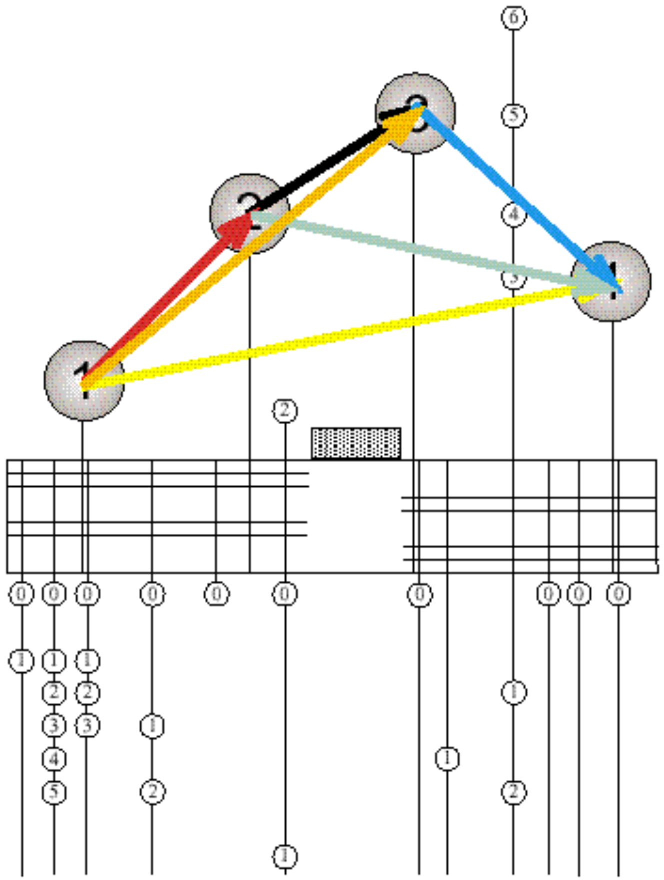
- 6 Some examples of optical interferometers



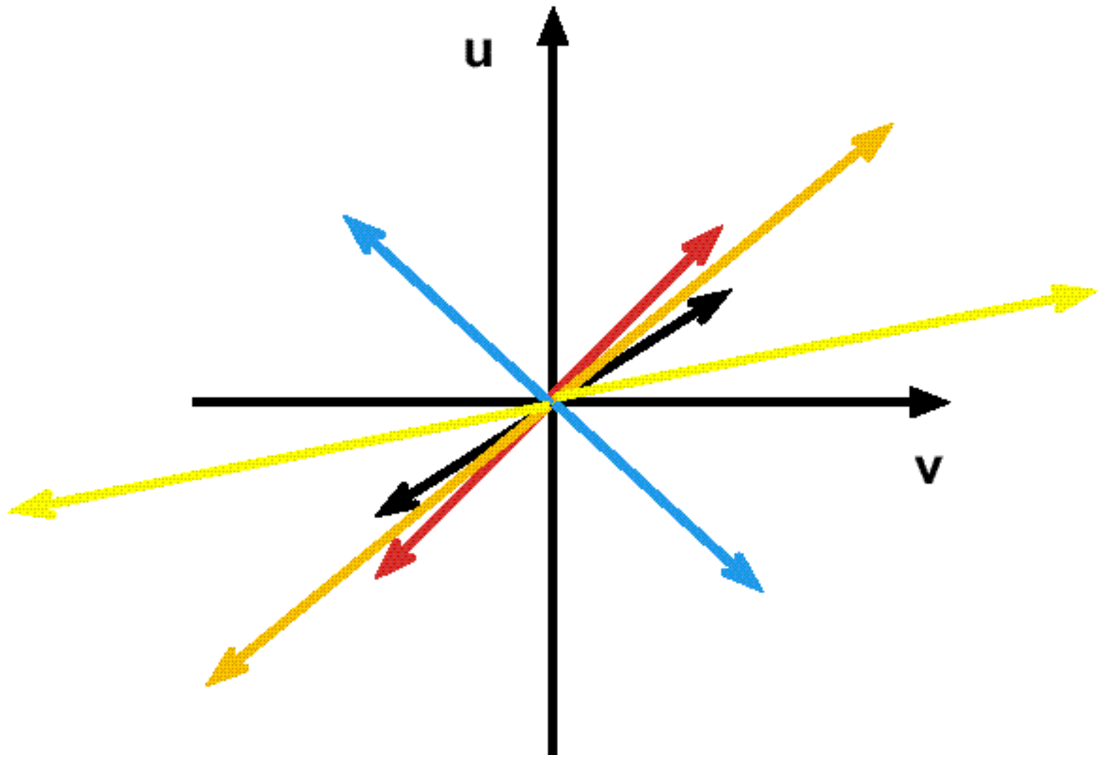
Cross Track
Unit Telescopes
Instrumentation
Laboratory
UT3
UT2
UT4
UT1
N
Delay Lines
M12
M16
Long Track
VLTI Stations

VLTI delay lines

uv plane coverage



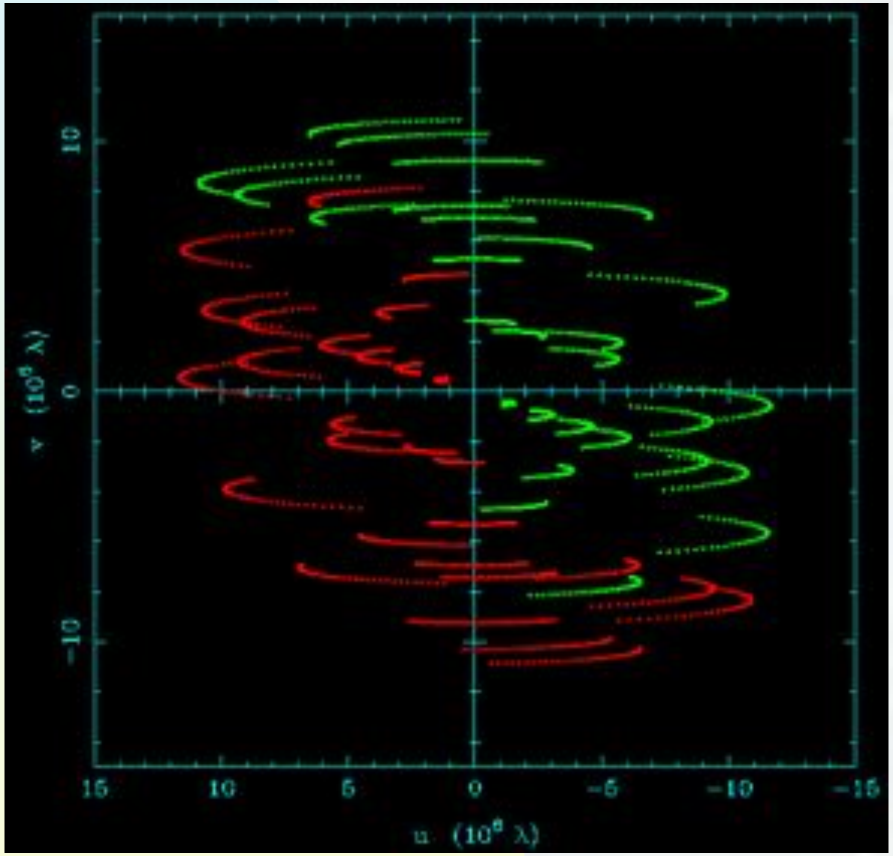
uv plane:



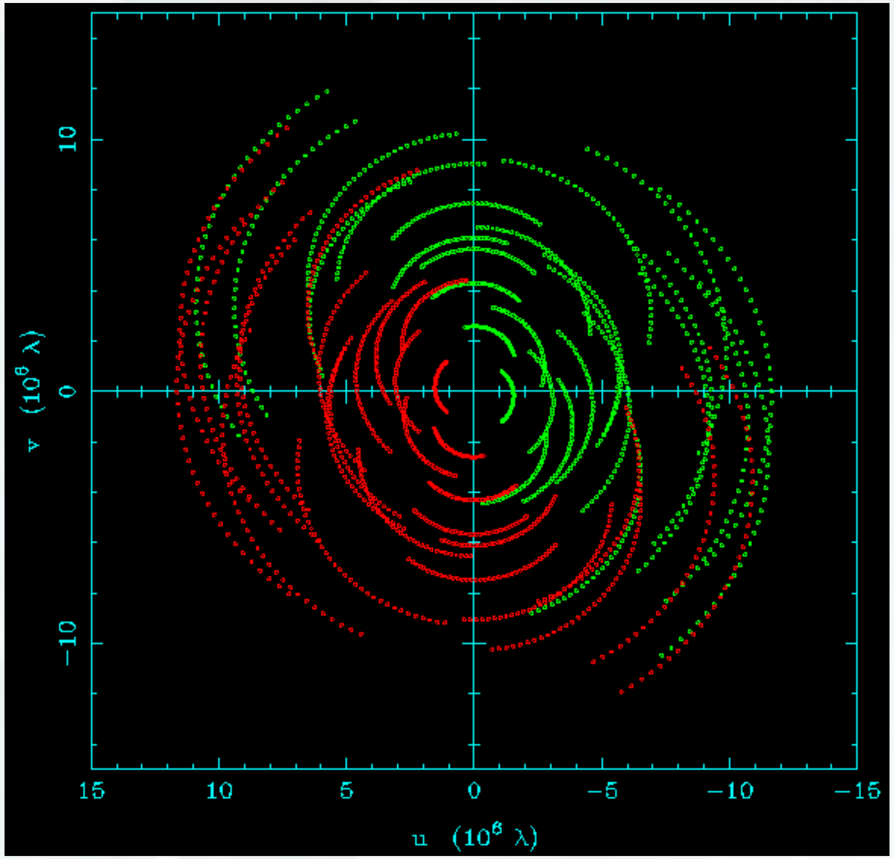
Note: *uv* plane coverage for an object at zenith. More generally, the projected baselines must be used.

Examples of *uv* plane coverage

Dec -15

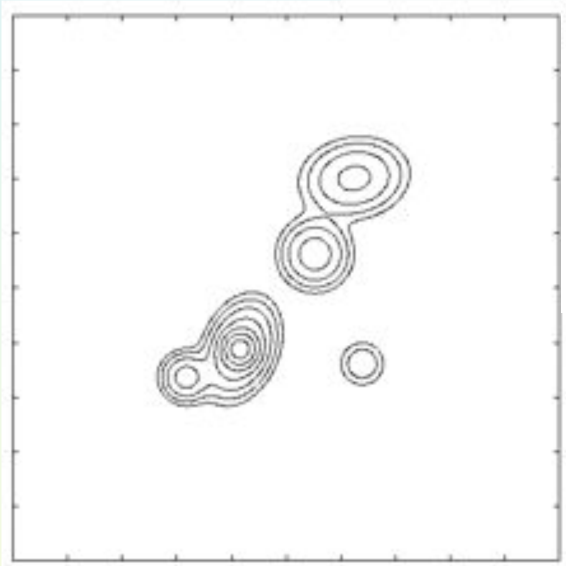


Dec -65

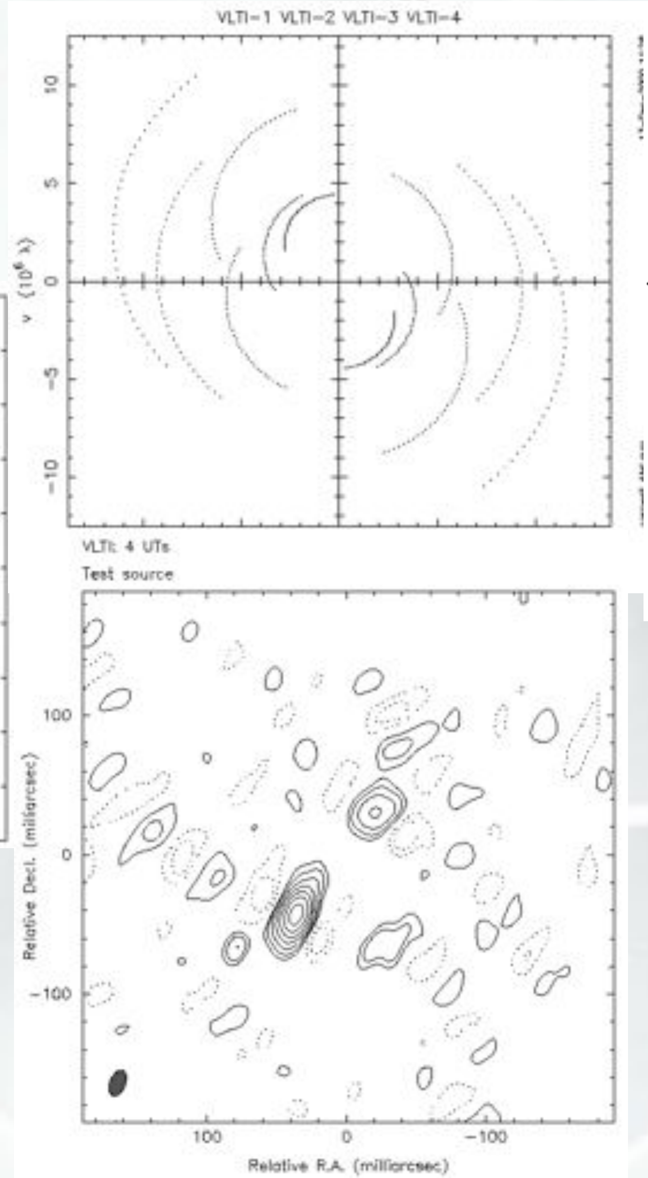


How does the *uv* plane coverage affect imagery?

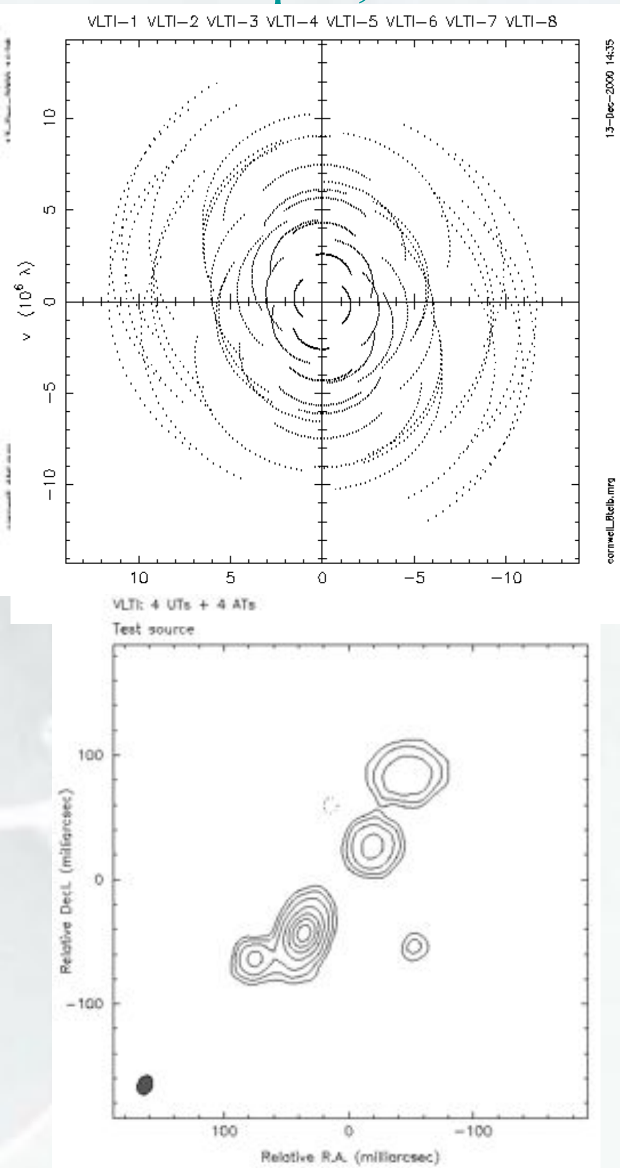
Model



4 telescopes, 6 hrs



8 telescopes, 6 hrs



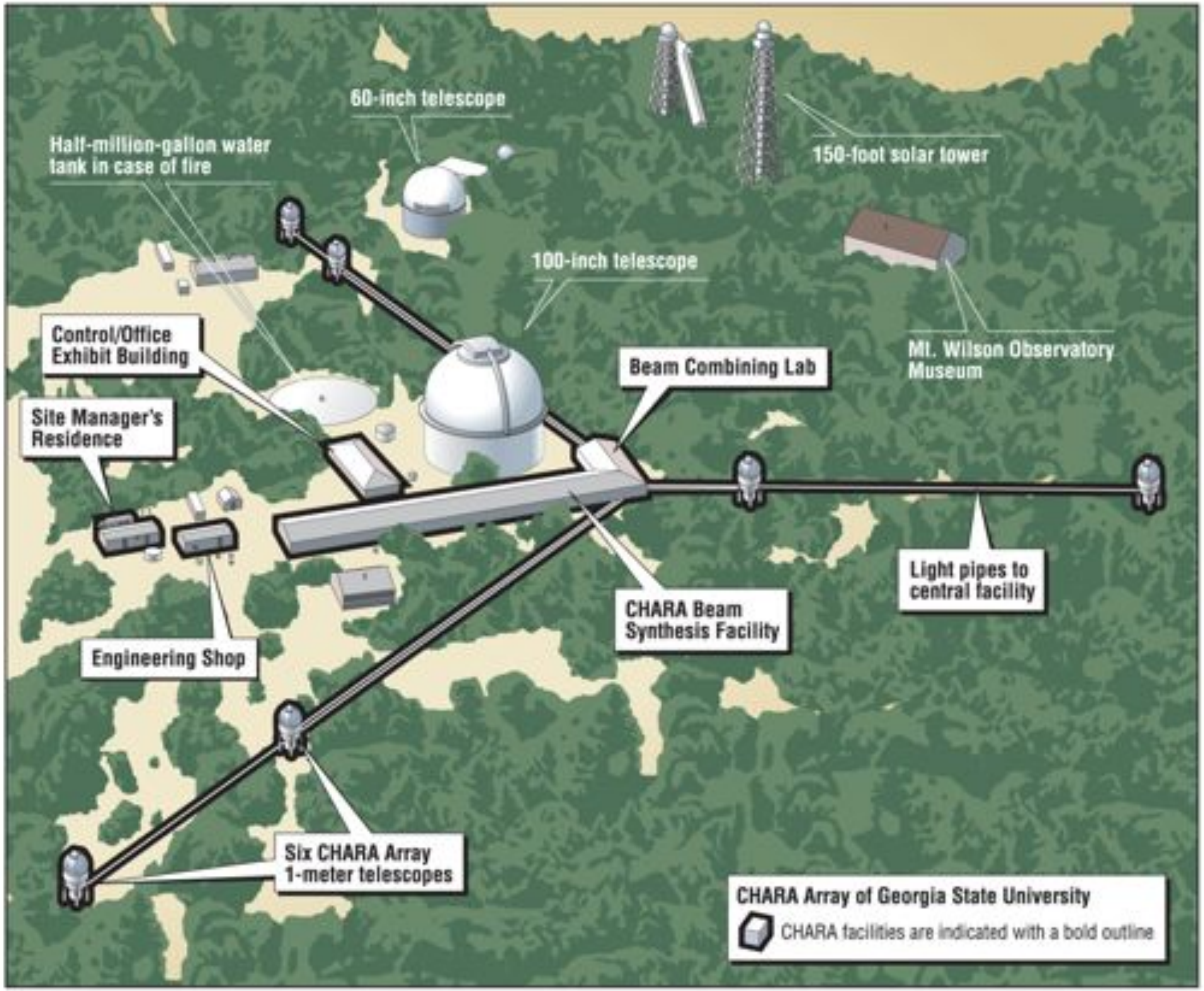
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day is also:

The CHARA interferometer

• 6 x 1m telescopes
• Max. Base: 330m


60-inch telescope
Half-million-gallon water tank in case of fire
150-foot solar tower
100-inch telescope
Control/Office Exhibit Building
Beam Combining Lab
Mt. Wilson Observatory Museum
Site Manager's Residence
Light pipes to central facility
CHARA Beam Synthesis Facility
Engineering Shop
Six CHARA Array 1-meter telescopes
CHARA Array of Georgia State University
CHARA facilities are indicated with a bold outline

An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day is also:

Palomar Testbed Interferometer (PTI)

• 3 x 40cm telescopes
• Max. Base: 110m

An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day

is also:

Keck interferometer

• 2 x 10m telescopes
• Base: 85m

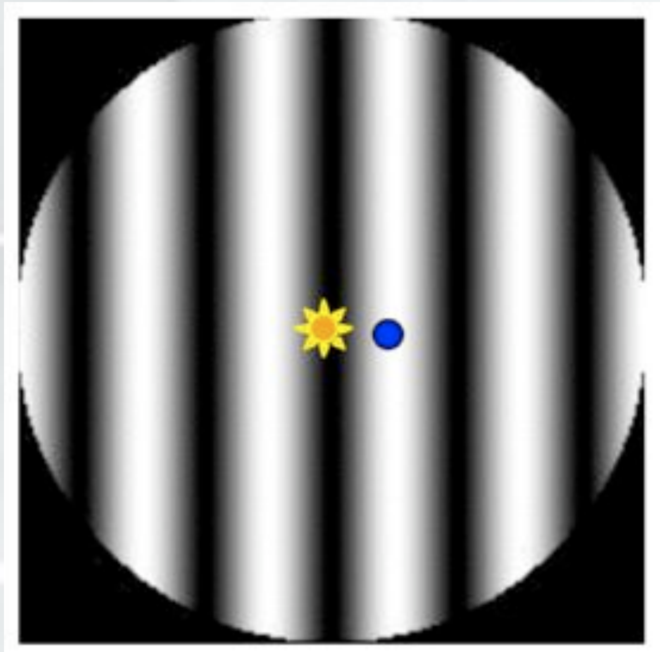
An introduction to optical/IR interferometry

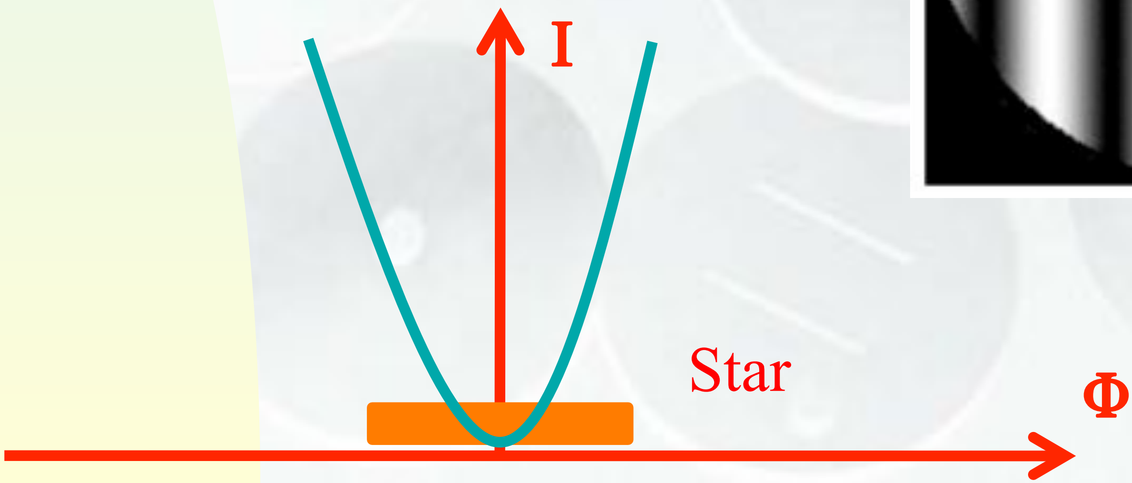
- 6 Some examples of optical interferometers

Interferometry to-day is also:

Nullin interferometry

- Measurement of « stellar leakage »
- Allow to resolve stars with a
  a small size interferometer

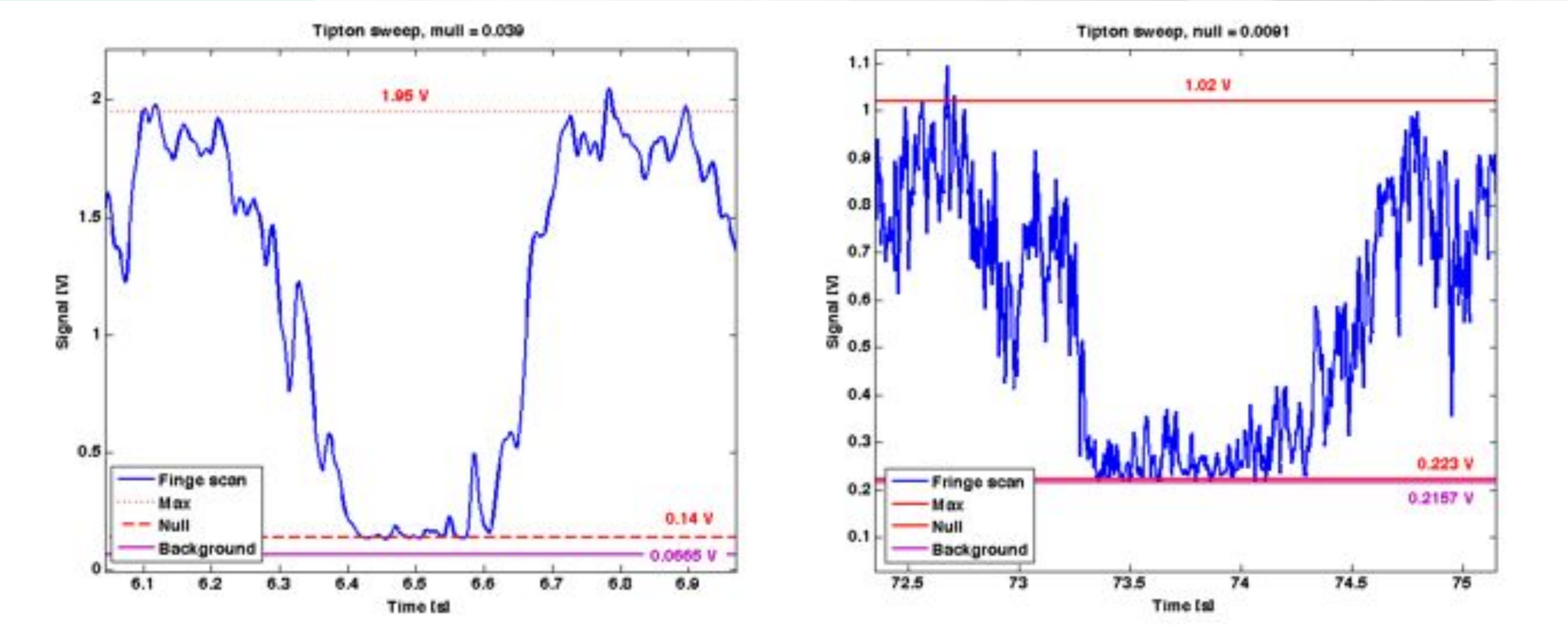




An introduction to optical/IR interferometry

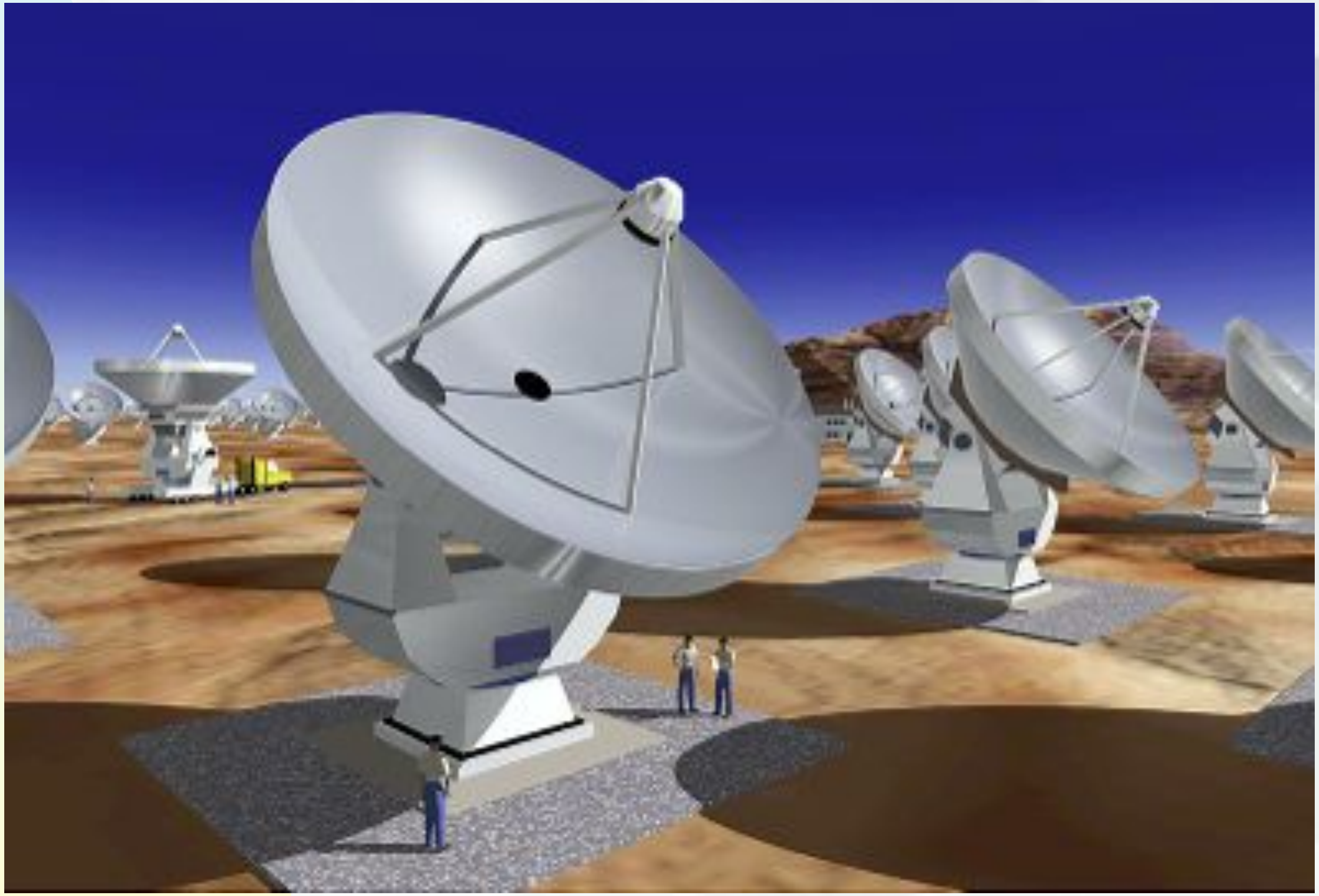
- 6 Some examples of optical interferometers

Interferometry to-day is also:



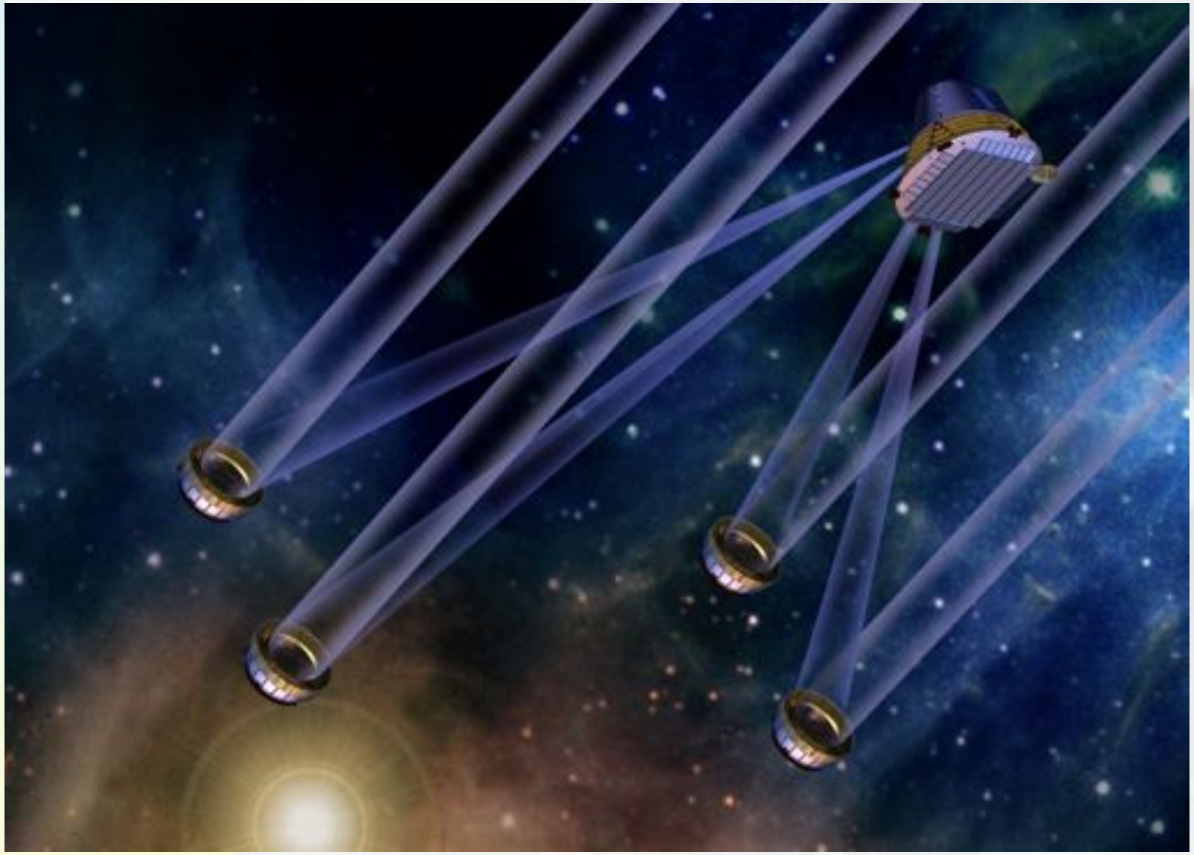
An introduction to optical/IR interferometry

- 6 Other examples of interferometers: ALMA



An introduction to optical/IR interferometry

- 6 Other examples of interferometers: DARWIN



An introduction to optical/IR interferometry

- 7 Some results

Star	Spectral type	Luminosity class	Angular diameter $\times 10^{-3}$ seconds of arc
α Boo	K2	Giant	20
α Tau	K5	Giant	20
α Sco	M1–M2	Super-giant	40
β Peg	M2	Giant	21
o Cet	M6e	Giant	47
α Ori	M1–M2	Super-giant variable	$34 \rightarrow 47$

Table 2.1. Stars measured with Michelson's interferometer. From Pease (1931).

An introduction to optical/IR interferometry

- 7 Some results

Table 2. Diamètres stellaires mesurés à l'I2T

NOM	SPECTRE	DIAMÈTRE λ = 0,55 µm en ms. d'arc	MESURÉ λ = 2,2 µm en ms. d'arc	R/R☉	TEMPÉRATURE EFFECTIVE λ = 0,55 µm en degrés Kelvin	λ = 2,2 µm en degrés Kelvin	DISTANCE en parsecs (1 pc = 3,26 al)
α Cas	K0II	5.4 ± 0.6		26 ± 8	4700 ± 300		45 ± 9
β And	M0III	13.2 ± 1.7	14.4 ± 0.5	33 ± 9	3800 ± 350	3711 ± 84	23 ± 3
γ And	K3II	6.8 ± 0.6		90 ± 14	4050 ± 250		70 ± 15
α Per	F5Ib	2.9 ± 0.4		66 ± 9	7000 ± 600		175 ± 9
α Cyg	A2Ie	2.7 ± 0.3		146 ± 45	8300 ± 600		500 ± 100
α Ari	K2III	7.6 ± 1		15 ± 5	4300 ± 350		23 ± 4
β Gem	K0III	7.8 ± 0.6		8 ± 2	4600 ± 230		11 ± 1
β Umi	K4III	8.9 ± 1		30 ± 9	4220 ± 300		31 ± 11
γ Ori	K5III	8.7 ± 0.8	10.2 ± 1.4	45 ± 10	4300 ± 230	3990 ± 270	59 ± 21
δ Ori	G8III	3.8 ± 0.3		15 ± 5	4530 ± 220		36 ± 8
μ Gem	M3III		14.6 ± 0.8	94 ± 30		3860 ± 56	60 ± 15
α Tau	K5III		20.7 ± 0.4	47 ± 7		3904 ± 34	21 ± 3
α Boo	K2III		21.5 ± 1.2	26 ± 6		4240 ± 120	11 ± 2
α Aur_A	G5III	8.0 ± 1.2		11.7 ± 2	5400 ± 200		13.7 ± 0.6
α Aur_B	G0III	4.8 ± 1.5		7.1 ± 2	5950 ± 200		13.7 ± 0.6
α Lyr	A0V	3.0 ± 0.2		2.6 ± 0.2			8.1 ± 0.3

An introduction to optical/IR interferometry

8 Three important theorems … and some applications

8.1 The fundamental theorem

8.2 The convolution theorem

8.3 The Wiener-Khintchin theorem

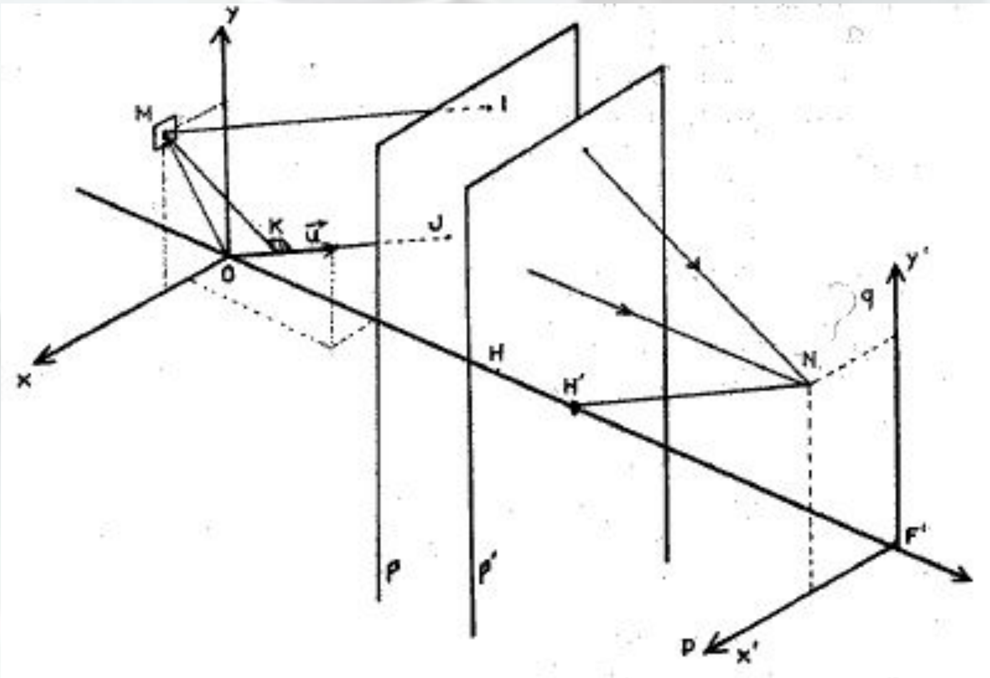
Réf.: P. Léna; Astrophysique: méthodes physiques de l'observation (Savoirs Actuels / CNRS Editions)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

a(p,q) = TF_(A(x,y))(p,q),

$$a(p,q) = \int_{R^2} A(x,y)\exp\left[-i2\pi\left(px+qy\right)\right]dxdy,$$



with

p = x' /(λ f)

q = y' /(λ f)

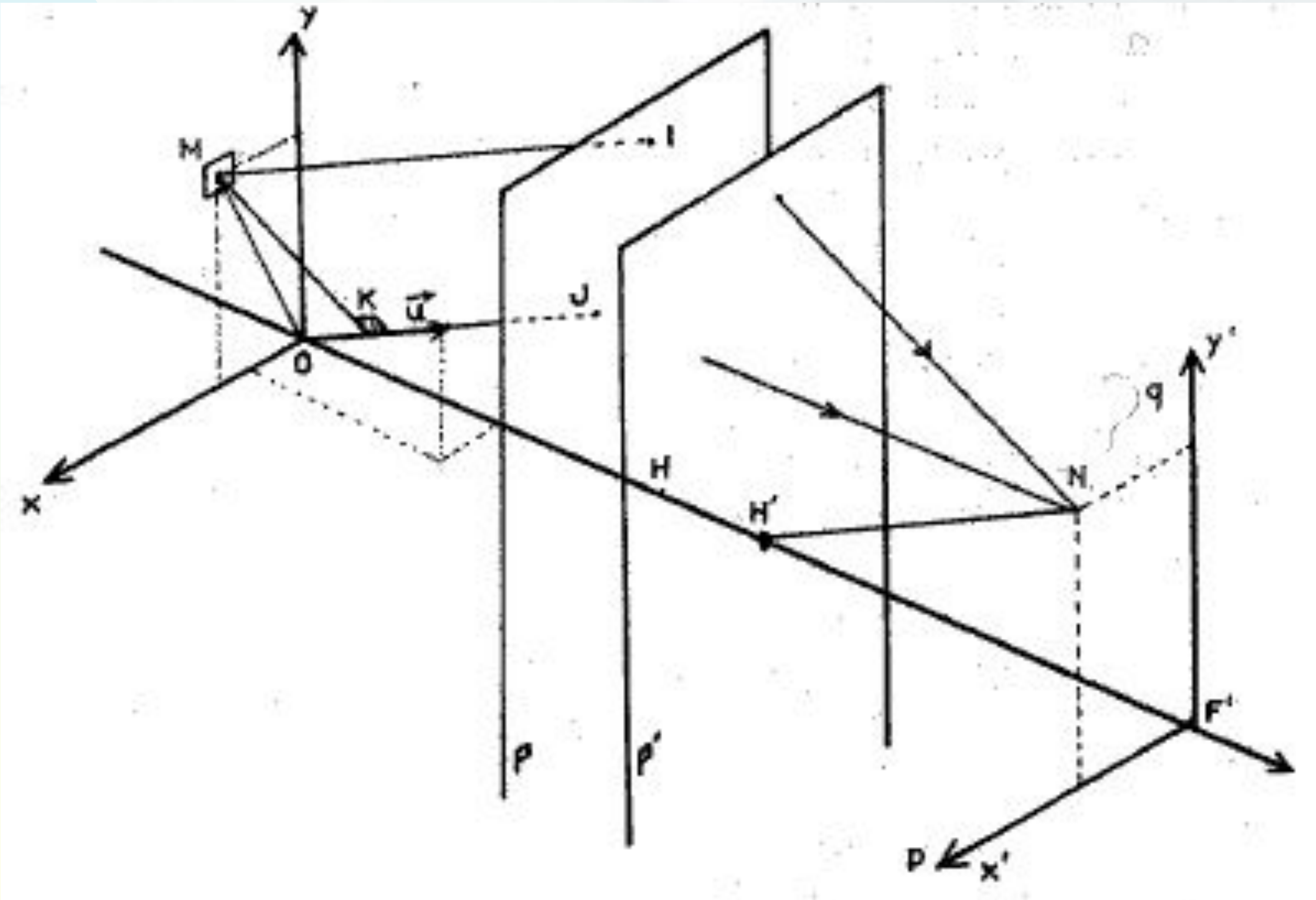
An introduction to optical/IR interferometry

8.1 The fundamental theorem

The distribution of the complex amplitude a(p,q) in the focal plane is given by the Fourier transform of the distribution of the complex amplitude A(x,y) in the entrance pupil plane.

An introduction to optical/IR interferometry

8.1 The fundamental theorem



An introduction to optical/IR interferometry

8.1 The fundamental theorem

- **Démonstration**

$A(x,y)\ \exp(i2\pi\nu t),$ (8.1.3.1)

$A(x,y) = \mathcal{A}(x,y)\ \exp(i\phi(x,y))\ P_0(x,y).$ (8.1.3.2)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

- **Démonstration**

$A(x,y)\exp(i2\pi\nu t + i\psi)$, (8.1.3.3)

$\delta = d(M\ I\ N) - d(O\ J\ N)$, (8.1.3.4)

$\psi = 2\pi\ \delta / \lambda$. (8.1.3.5)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

- **Démonstration**

$\delta = -d(O, K) = -|(\mathbf{OM\ u})|,$ (8.1.3.6)

$A(x,y)\ \exp(i2\pi(\nu t - xx'/\lambda f - yy'/\lambda f)).$ (8.1.3.7)

$p = x'/\lambda f,\ q = y'/\lambda f,$ (8.1.3.8)

$\exp(i2\pi\nu t)\ A(x,y)\ \exp(-i2\pi(xp + yq)).$ (8.1.3.9)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

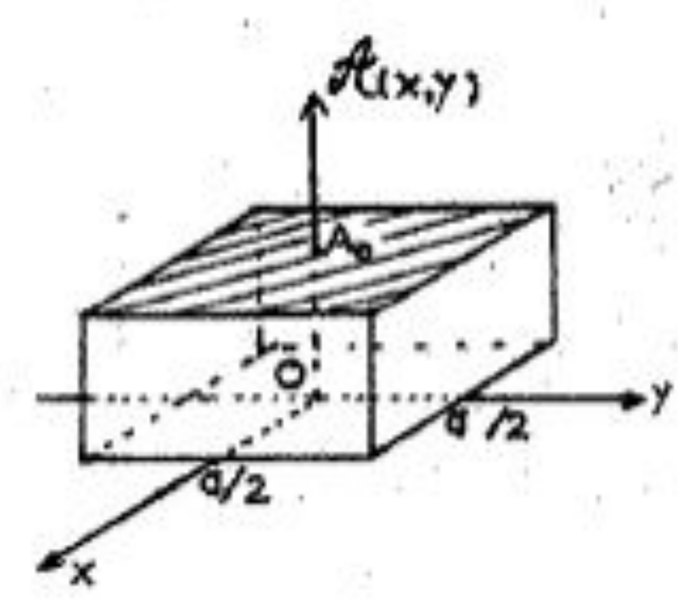
- **Démonstration**

$$a(p,q) = \int_{R^2} A(x,y) \exp\left[-i2\pi\left(px+qy\right)\right] dxdy, \qquad (8.1.3.10)$$

$$a\left(p,q\right) = TF\_\left[A\left(x,y\right)\right]\left(p,q\right) \qquad (8.1.3.11)$$

8.1 The fundamental theorem

Application: Point Spread Function determination



$$A(x,y) = A_0\,P_0(x,y), \qquad (8.1.1)$$

$$P_0(x,y) = \Pi(x/a)\,\Pi(y/a). \qquad (8.1.2)$$

An introduction to optical/IR interferometry

8.1 The fundamental theorem

$$a(p,q) = TF\_\left[A(x,y)\right](p,q) = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2} A_0 \exp\left[-i2\pi\left(px+qy\right)\right]dxdy \qquad (8.1.3)$$

$$a(p,q) = A_0 \int_{-a/2}^{a/2} \exp\left[-i2\pi px\right]dx \int_{-a/2}^{a/2} \exp\left[-i2\pi qy\right]dy \qquad (8.1.4)$$

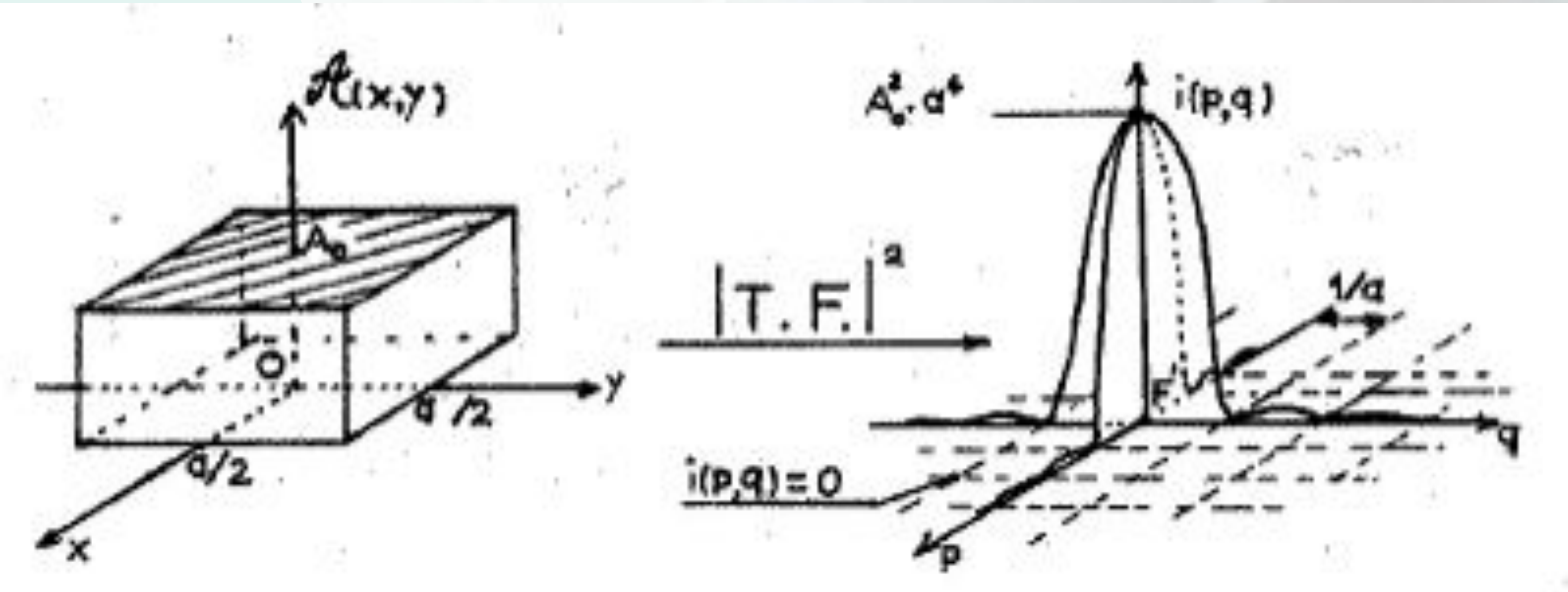
$$a(p,q) = A_0\, a^2\, [\sin(\pi pa) / (\pi pa)]\, [\sin(\pi qa) / (\pi qa)]. \qquad (8.1.5)$$

$$i(p,q) = a(p,q)\, a^*(p,q) = |a(p,q)|^2 = |h(p,q)|^2 =$$
$$= i_0\, a^4\, [\sin(\pi pa) / (\pi pa)]^2\, [\sin(\pi qa) / (\pi qa)]^2. \qquad (8.1.6)$$

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination



$\Delta p = \Delta x' /(\lambda f); \Delta q = \Delta y' /(\lambda f) = 2/a \Rightarrow \Delta\phi_{x'} = \Delta\phi_{y'} = 2\lambda/a$ (8.1.7)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination when observing a star along another direction

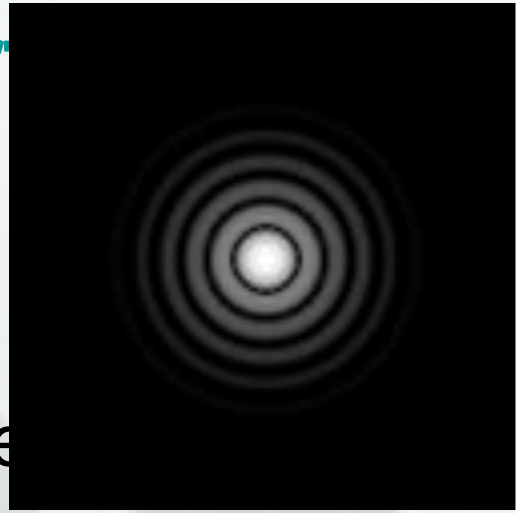
$\psi = 2\pi\, \delta / \lambda = 2\pi(xb/f + yc/f) / \lambda,$ (8.1.5.7)

$A(x,y) = P_0(x,y)\, A_0 \exp[2i\pi(xb/f + yc/f) / \lambda].$ (8.1.5.8)

$$a(p,q) = A_0 \int_{-a/2}^{a/2} \exp\left[-2i\pi(p - b/f\lambda)x\right] dx \int_{-a/2}^{a/2} \exp\left[-2i\pi(q - c/f\lambda)y\right] dy \quad (8.1.5.9)$$

$$a(p,q) = A_0 a^2 \left(\frac{\sin\left(\pi\left(p - b/f\lambda\right)a\right)}{\pi\left(p - b/f\lambda\right)a} \right) \left(\frac{\sin\left(\pi\left(q - c/f\lambda\right)a\right)}{\pi\left(q - c/f\lambda\right)a} \right) \quad (8.1.5.10)$$

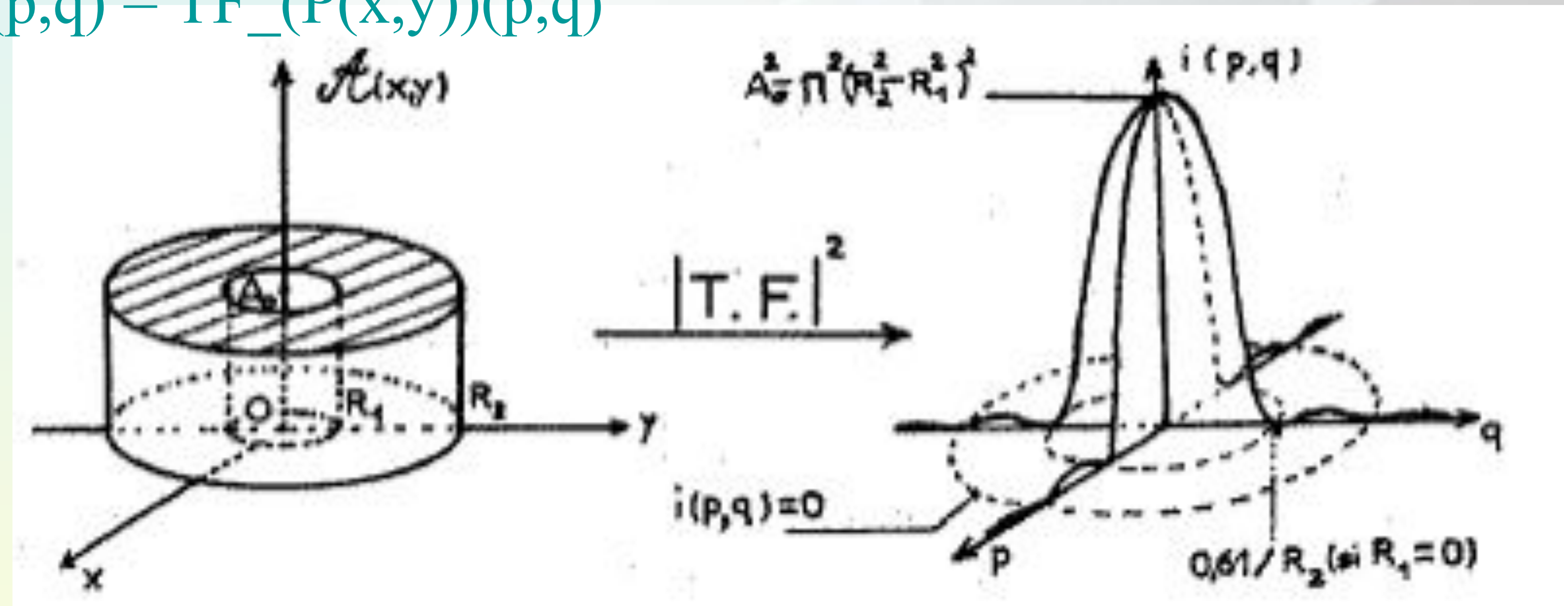
An introduction to optical/IR inter



8.1 The fundamental theorem

Application: Point Spread Function dete

h(p,q) = TF_(P(x,y))(p,q)



$$i(\rho') = |a(\rho')|^2 = (A_0\,\pi)^2\,[R_2^{\,2}\,2\,J_1(Z_2)/Z_2 - R_1^{\,2}\,2\,J_1(Z_1)/Z_1]^2, \quad (8.1.8)$$

with $Z_2 = 2\pi\, R_2\, \rho' / (\lambda f)$ and $Z_1 = 2\pi\, R_1\, \rho' / (\lambda f)$. (8.1.9)

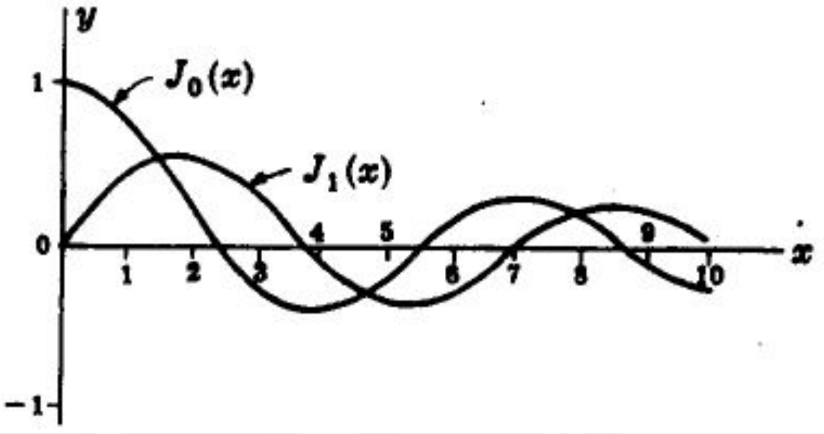
An introduction to optical/IR interferometry

BESSEL FUNCTIONS (REMINDER)

Integral representation of the Bessel functions

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos[x\sin(\vartheta)]d\vartheta \qquad J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos[n\vartheta - x\sin(\vartheta)]d\vartheta$$

Graphs of the $J_0(x)$ and $J_1(x)$ functions



Undefined integral

$$\int x' J_0(x')dx' = x\, J_1(x)$$

Series development (x ~ 0):

$J_0(x) = 1 - x^2/2^2 + x^4/(2^2 4^2) - x^6/(2^2 4^2 6^2) + \ldots$

$J_1(x) = x/2 - x^3/(2^2 4) + x^5/(2^2 4^2 6) - x^7/(2^2 4^2 6^2 8) + \ldots$

$J_n(x) = (2 / (\pi x))^{1/2} \cos(x - n\pi/2 - \pi/4)$ … and when x is large!

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination

x = ρ cos(θ), y = ρ sin(θ), p = ρ' cos(θ') / (λf), q = ρ' sin(θ') / (λf).

(8.1.5.13)

$$a(\rho',\theta') = A_0 \int_{R_1}^{R_2} \int_0^{2\pi} \exp\left[-2i\pi\rho\rho'\cos(\theta-\theta')/(\lambda f)\right] d(\theta-\theta')\rho d\rho$$

(8.1.5.14)

$$a(\rho',\theta') = a(\rho') = A_0\pi\left[\frac{2R_2^2}{Z_2}J_1(Z_2) - \frac{2R_1^2}{Z_1}J_1(Z_1)\right] \quad (8.1.5.15)$$

$$Z_2 = 2\pi R_2 \frac{\rho'}{\lambda f} \quad \text{et} \quad Z_1 = 2\pi R_1 \frac{\rho'}{\lambda f} \quad (8.1.5.16)$$

Pour le cas $R_1 = 0$ $\quad i(\rho') = |a(\rho')|^2 = 4(A_0\pi)^2 R_2^4 \left(\frac{J_1(Z_2)}{Z_2}\right)^2$ (8.1.5.17)

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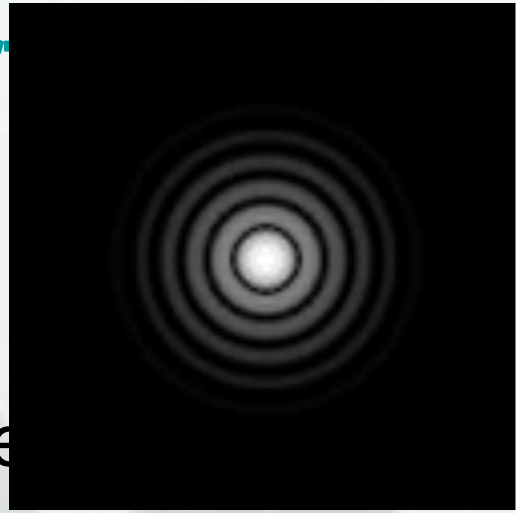
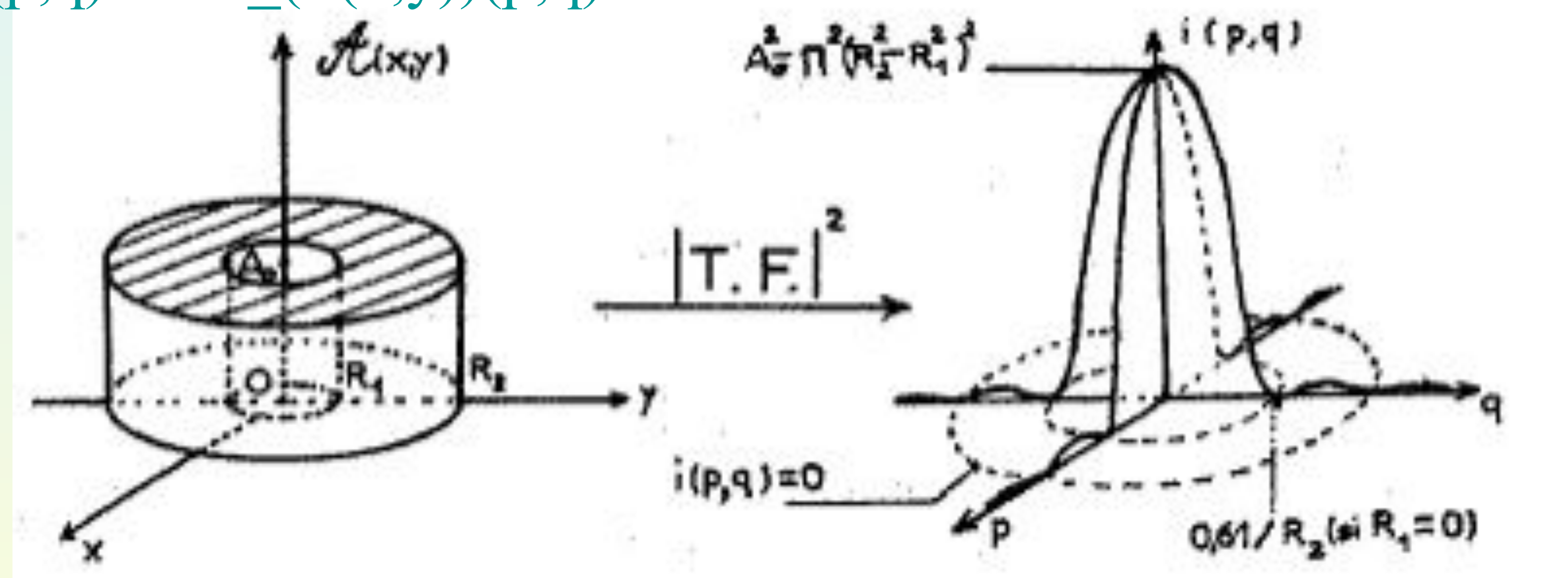


8.1 The fundamental theorem

Application: Point Spread Function dete

h(p,q) = TF_(P(x,y))(p,q)



$$i(\rho') = |a(\rho')|^2 = (A_0\,\pi)^2\,[R_2^{\,2}\,2\,J_1(Z_2)/Z_2 - R_1^{\,2}\,2\,J_1(Z_1)/Z_1]^2, \quad (8.1.8)$$

with $Z_2 = 2\pi\,R_2\,\rho' / (\lambda f)$ and $Z_1 = 2\pi\,R_1\,\rho' / (\lambda f)$. (8.1.9)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination

ρ' (=r) = 1,22 λ f / D (D = 2 R_2, R_1 = 0). (8.1.5.18)

$$\frac{2\pi\int_0^r i(\rho')\rho' d\rho'}{2\pi\int_0^\infty i(\rho')\rho' d\rho'} = 0{,}84$$ (8.1.5.19)

h(p,q) = TF_(P(x,y))(p,q). (8.1.5.20)