Experimental modal analysis using blind source separation techniques

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Abstract

Recently, statistical and empirical signal processing techniques such as the proper orthogonal decomposition and the Hilbert-Huang transform have shown promise for structural system identification. In the present study, experimental modal analysis is carried out by employing blind source separation techniques and by interpreting the response of a mechanical system as a static mixture of sources. Specifically, it is shown under which circumstances the normal coordinates of the vibration modes may be interpreted as *virtual sources*. The advantages and limitations of the proposed method will be discussed, and the procedure will be demonstrated using numerical applications.

1 Introduction

Linear system identification is a discipline that has evolved considerably during the last thirty years. Modal parameter estimation, termed modal analysis, is indubitably the most popular approach to performing linear system identification in structural dynamics. The model of the system is known to be in the form of modal parameters, namely the natural frequencies, vibration modes and damping ratios. The popularity of modal analysis stems from its great generality; modal parameters can describe the behavior of a system for any input type and any range of the input. Numerous approaches were developed for this purpose: Ibrahim time domain method [**1**], eigensystem realization algorithm [**2**], stochastic subspace identification method [**3**], polyreference least-squares complex frequency domain method [**4**] to cite a few. A description of modal analysis is not within the scope of this paper; the interested reader may consult Ref. [**5**] for further detail.

Recently, statistical and empirical signal processing techniques have shown promise for experimental modal analysis. The relation between the proper orthogonal modes (POMs), extracted from the proper orthogonal decomposition (POD, also known as principal component analysis), and the normal modes was demonstrated in several studies [**6, 7, 8**]. Therefore, the POD was proposed as a means of computing the normal modes directly from the measured data [**9, 10**]. One of the intrinsic limitations is that the knowledge of the mass matrix is required. To address this issue, Chelidze and Zhou introduced a new multivariate data analysis method called smooth orthogonal decomposition (SOD) [**11**]. The Hilbert-Huang transform (HHT) has been shown to be effective for characterizing a wide range of non-stationary signals in terms of elemental components through what has been called the empirical mode decomposition [**12**]. The HHT has been utilized extensively, as it provides a concise basis for the analysis of nonlinear systems. As demonstrated in [**13, 14**], this technique is also useful for linear system identification.

The present study performs structural system identification using blind source separation techniques called independent component analysis (ICA) and second-order-based identification (SOBI). ICA is a relatively recent method [**15, 16**] and has already found several applications in structural dynamics, including damage detection [**17**], condition monitoring [**18, 19**] and discrimination between pure tones and sharp-pointed resonances [**20**]. A special issue dealing with ICA and blind source separation was also published in Mechanical Systems and Signal Processing in 2005 [**21**]. Several variants of ICA were also proposed, namely SOBI [**22**], joint approximate diagonalization of eigenmatrices (JADE) [**23**] and fastICA [**24**].

This paper first presents the BSS concepts, and a comparison with the POD method is also performed in Section 2. The proposed modal analysis procedure is then exposed, and the two methods (ICA and SOBI) are briefly explained in Section 3. Structural system identification using ICA and SOBI is performed on two simulated systems in Section 4.

2 Blind source separation techniques for modal analysis

2.1 Blind source separation concept and POD

Blind source separation (BSS) techniques were mostly developed during the last decade for information theory and signal processing. But their objective, which consists in revealing the underlying structure hidden in a set of measured data, is shared by many research fields. In fact, BSS techniques attempt to extract, from the only mixture of sources observed, independent excitation sources and relationships existing between these unknown inputs and the measured outputs.

Let us consider a linear system which is subjected to a set of unknown excitation sources $s(t)$. This system provides a set of responses $x(t)$ which are assumed to be linear combinations of sources. Noise, $n(t)$, can be considered and added to the response. Mathematically, this can be written as

$$x(t) = A \cdot s(t) + n(t) \tag{1}$$

Because the mixing matrix A is unknown, the estimation problem is considerably more difficult. Some additional hypotheses about the initial sources have to be taken into account for the purpose of finding the matrix W which will give the best sources approximation, noted $z(t)$.

$$z(t) = W^T \cdot x(t) \tag{2}$$

The necessary hypotheses, e.g., uncorrelation, non gaussianity or statistical independence, will be discussed below and in Section 3.

Scientists already considered the use of statistical signal processing techniques, such as principal component analysis (PCA), for the study of structural dynamics. The proper orthogonal decomposition (POD) is a variant of PCA for dynamical systems [**25**]. The POD and the BSS techniques share in fact the same objective. The basic idea of the POD is to reduce the large number of interdependent variables $x(t)$ to a much smaller number of uncorrelated variables $s(t)$ while retaining as much as possible of the variation present in the original variables. An orthogonal transformation to the basis of the eigenvectors of the sample covariance matrix is performed, and the data are projected onto the subspace spanned by the eigenvectors corresponding to the largest eigenvalues. The transformation gives uncorrelated signals and minimizes the average squared distance between the original signal and its reduced linear representation. In this sense, POD is optimal. The method is therefore suitable for variables with Gaussian distribution. This is illustrated in Figure 1 using a two-degree-of-freedom system. However, the limitation of the method may appear when the uncorrelated variables computed through the POD are not statistically independent. BSS methods such as independent component analysis (ICA) address this issue. The application of the POD and ICA is considered in Figure 2 for two variables with uniform distribution. The POD is clearly unable to recover the underlying structure in the data unlike ICA.

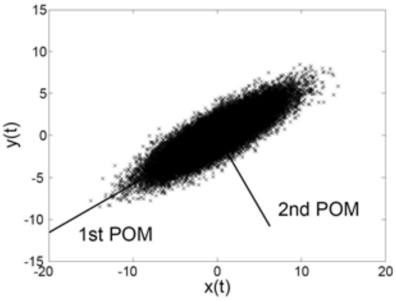

Figure 1 - Proper orthogonal modes in the case of two variables with Gaussian distribution

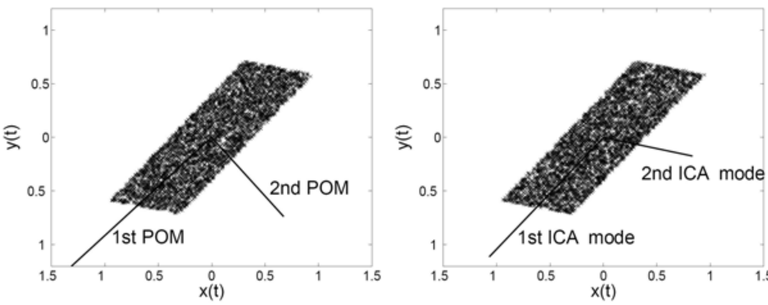

Figure 2 - Proper orthogonal and ICA modes in the case of two variables with uniform distribution

2.2 Normal coordinates as virtual sources

BSS techniques are able to separate the different excitation sources acting on a system. How could this be exploited in structural dynamics?

Let us consider a mechanical system governed by the equations of motion

$$M \cdot \ddot{x}(t) + K \cdot x(t) = f(t) \tag{3}$$

The system response $x(t)$ is the convolution product of the impulse response function $h(t)$ and the external force vector $f(t)$

$$x(t) = h(t) * f(t) \tag{4}$$

This relationship involves a dynamic mixture of sources. Unfortunately, the application of ICA to the convolutive mixture of sources is not yet completely solved and raises several problems [**21**].

The main idea of this paper, which will allow to bypass these difficulties, is to interpret the mechanical system as a static mixture of sources. Besides expression (4), the response of system (3) may be expressed through a modal expansion as

$$x(t) = \sum_{i=1}^{m} \mathbf{n}_{(i)} \cdot \eta_i(t) = \mathbf{N} \cdot \eta(t) \tag{5}$$

where $\mathbf{n}_{(i)}$ and $\eta_i(t)$ are the normal modes and the corresponding normal coordinates (i.e., the amplitude modulation of the modes), respectively; m is the number of degrees of freedom (DOFs) of the system. By definition, the normal modes provide a complete set for the expansion of an arbitrary vector. It turns out that the normal coordinates act as virtual sources on the system regardless the number and the type of the physical excitation forces. Under the assumption of independent normal coordinates, the application of BSS methods should therefore provide a straightforward identification of the eigenmodes of a structure through the computed mixing matrix.

In Ref. [**26**], a one-to-one relationship between the vibration modes and the ICA modes for free and random vibrations of weakly damped systems is demonstrated. The interest reader should consult this reference for further details.

3 Independent component analysis

Unlike the POD which simply assumes that the original sources are uncorrelated, statistical independence and non-Gaussianity are the guiding principles of ICA [**24**]. ICA methods solve the problem described in (1), i.e., the identification of the mixing matrix A and the sources $s(t)$, by assuming the statistical independence of the sources. The advantage is that higher-order statistics about the observed distribution are taken into account to extract the sources. Mathematically, statistical independence of the variables w_i means that the joint probability density function factorizes into the product of the probability density function of each variables

$$p(w_1, w_2, ...w_m) = \prod_k p_k(w_k) \tag{6}$$

In other words, the value of any variable cannot be inferred from the value of the others. For Gaussian distributions, ICA techniques will not perform better than decorrelation methods such as the POD, because high-order statistical cumulants vanish.

During the last few years, ICA has received increasing attention, and numerous methods for computing it were developed. They differ mainly by the contrast definition (i.e., the objective function) and/or by the algorithm resolution. Two of these methods are briefly described in this paper and used for output-only modal analysis. The first one exploits the mutual information minimization, and the other one uses the joint diagonalization of several covariance matrices.

3.1 Mutual information minimization

Since it is usually not possible to estimate sources that are perfectly statistically independent and since noise often perturbs the measurements (1), ICA consists in searching a linear transformation that *minimizes* the statistical dependence between its components. There exist numerous criteria for this purpose, and the method considered here is based on the mutual information concept.

The principal property of mutual information is that its minimum value appears if and only if the random variables are independent. Unfortunately it is extremely difficult to compute, and the alternative is to evaluate the negentropy through approximations. The negentropy is directly linked to mutual information and measures the distance to the Gaussianity. Comon proposed an expansion as function of cumulants of increasing orders [**16**]. The detailed description of ICA and the practical estimation of the independent components are beyond the scope of this paper. The interested reader may consult [**16, 24**].

3.2 Second-order-based identification

The other method, the second-order-based identification (SOBI), is based on joint diagonalization of several matrices [**22**]. It can be interpreted as an extension of the POD method for a set of covariance matrices characterized by different time lags. These matrices

$$\begin{aligned} R(0) &= E\left\{x(t) \cdot x^*(t)\right\} \\ R(\tau) &= E\left\{x(t+\tau) \cdot x^*(t)\right\} \end{aligned} \tag{7}$$

are evaluated from the observed data, and the basic idea is to find a matrix U, which jointly diagonalizes all the covariance matrices. It can be proven that the unitary matrix U corresponds to the mixing matrix.

The main feature of SOBI is that it makes an additional assumption regarding the sources in comparison with standard ICA. They must have different spectral contents; the algorithm therefore exploits the time coherence of the source signals. This is particularly appropriate in structural dynamics, because the normal coordinates are monochromatic (colored signals) for the free response and mostly monochromatic for the random response. Another advantage of the procedure is that being based on the joint diagonalization of a set of covariance matrices it only involves second-order statistics, which are easier to compute.

4 Numerical applications

To support the previous theoretical findings and to demonstrate the usefulness of BSS techniques for output-only modal analysis, numerical experiments are carried out in this section. A comparison of the results obtained using the two methods described above is achieved.

4.1 Identification of a discrete system

A three-degree-of-freedom (DOF) system is considered (Figure 3). This system is made of three masses connected in series through linear springs.

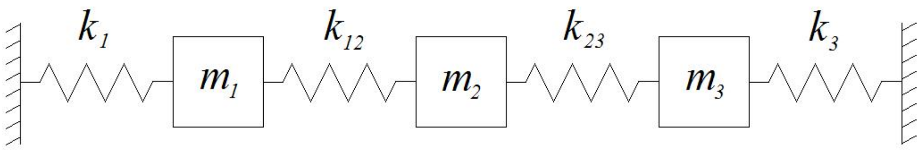

Figure 3 – Three-degree-of-freedom mass-spring model

Parameters of the system are $m_1 = 2$, $m_2 = 1$, $m_3 = 3$ and $k_1 = k_3 = k_{12} = k_{23} = 1$. Proportional damping is introduced through the α parameter. The homogeneous equation of motion is

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \cdot \ddot{x}(t) + \alpha \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \cdot \dot{x}(t) + \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \cdot x(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

Newmark's algorithm is used to compute the system response with a sampling frequency of 100 Hz. Data are then resampled to 10 Hz. In the first part, the *free response is considered* with zero initial conditions except for $\dot{x}_3 = 1$.

Theoretical modes and natural frequencies are calculated by solving the classical eigenvalue problem. The theoretical sources (i.e., the "true" normal coordinates) are determined by projecting the simulated response onto each eigenmode. Equation (5) gives

$$\eta(t) = s_{th}(t) = \mathbf{N}^{-1} \cdot x(t) \tag{9}$$

The quality of the identification using BSS techniques is assessed by comparison with these theoretical results. The modal assurance criterion (MAC) is used for the correlation of the identified and actual eigenmodes (a unitary value means a perfect correlation), and the normalized mean squared error (NMSE) is used for the sources correlation

$$NMSE = \frac{\left\langle (s_{th} - s_{BSS})^2 \right\rangle}{\left\langle (s_{th} - \langle s_{th} \rangle)^2 \right\rangle} \tag{10}$$

4.1.1 Identification results

This section presents a comparison of the two methods – ICA through mutual information minimization and its variant SOBI – for the three-DOF system described above.

For the SOBI method, the number of time lags τ and their values have to be fixed for the covariance matrices computation. The chosen delays correspond to a set of frequencies which were derived from an equidistant distribution of frequencies between the minimum and the maximum eigenfrequency of the studied system.

Now let us suppose that the system described above is such that $\alpha = 0.02$ (corresponding to damping ratios of 1.78, 1.09 and 0.63 %) and that its response is corrupted by white noise (the RMS amplitude of the noise equals 1 % of the signal RMS value). The simulated displacements are presented in Figure 4 together with the identified sources from ICA and SOBI methods. Note that results given by ICA and SOBI are identical in this case.

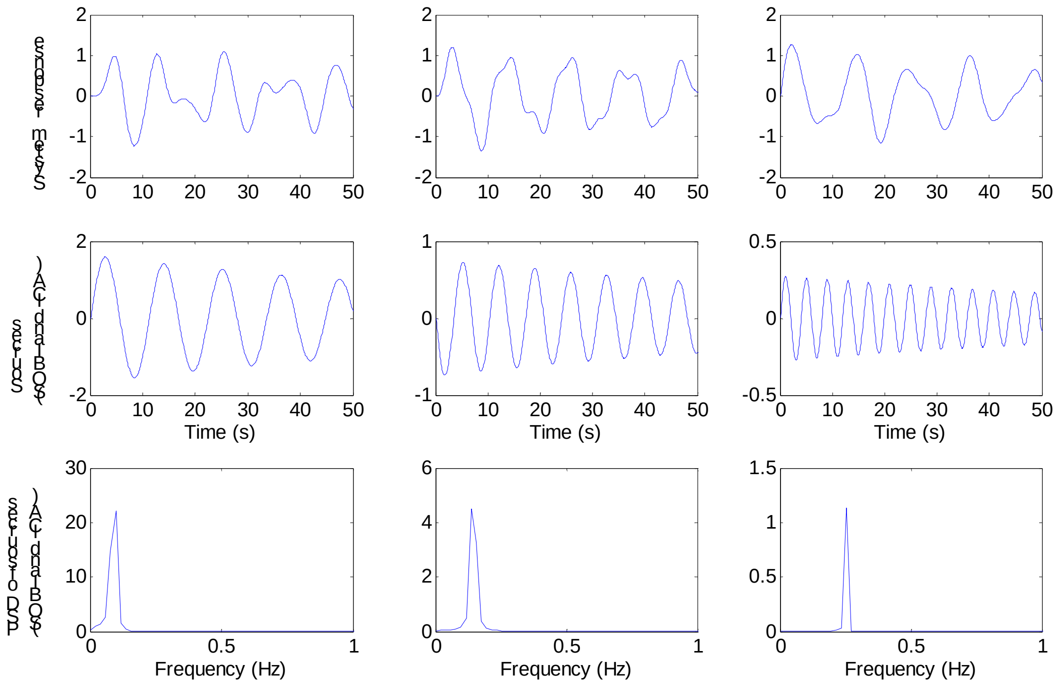

Figure 4 - Response and identified sources (with ICA and SOBI methods) for the 3DOF's system

The eigenmodes are accurately identified with the two methods. The MAC and the NMSE values for each mode are listed in Table 1. The damping is computed from the identified sources using logarithmic decrement.

		Mode 1	Mode 2	Mode 3
Frequencies (Hz)	**Theoretical**	0.0895	0.1458	0.2522
	SOBI	0.0892	0.1452	0.2509
	ICA	0.0892	0.1452	0.2509
Damping ratios (%)	**Theoretical**	1.78	1.09	0.63
	SOBI	1.75	1.05	0.60
	ICA	1.76	1.04	0.60

MAC	**SOBI**	1.000	1.000	0.999
	ICA	1.000	0.997	0.999
NMSE	**SOBI**	0.044%	0.071%	0.016%
	ICA	1.100%	0.215%	0.151%

Table 1 : Accuracy of identifications using ICA and SOBI (3 DOFs, free response, damping coefficient $\alpha = 0.02$, 1% noise)

4.1.2 Damping influence

Let us now consider the robustness of both methods to the amount of damping present in the system. Robustness to noise is discussed in the next section. The system considered here has its damping parameter α varying from 0 to 0.1. The corresponding damping ratios for $\alpha = 0.1$ are 8.89%, 5.46% and 3.15%. Only the first 500 sample points of the simulated displacements are taken into account for the identification. Figure 5 presents the quality of the results in terms of modes and normal coordinates identification.

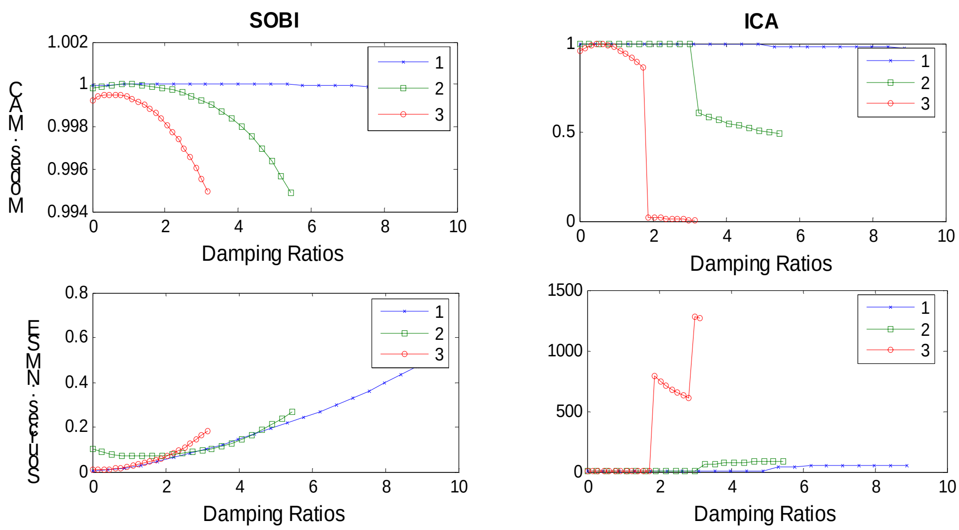

Figure 5 - Quality of modes and sources identified with ICA and SOBI method for the 3DOF's system regarding damping (3DOFs, free response, 0% noise)

Both methods seem to perform well for the weakly damped system. When damping increases beyond 1%, the identification using ICA fails, whereas the SOBI method continues to provide accurate results. For illustration, Figure 6 superposes the theoretical normal coordinates together with the sources identified through ICA and SOBI sources for the first mode of the system ($\alpha = 0.05$).

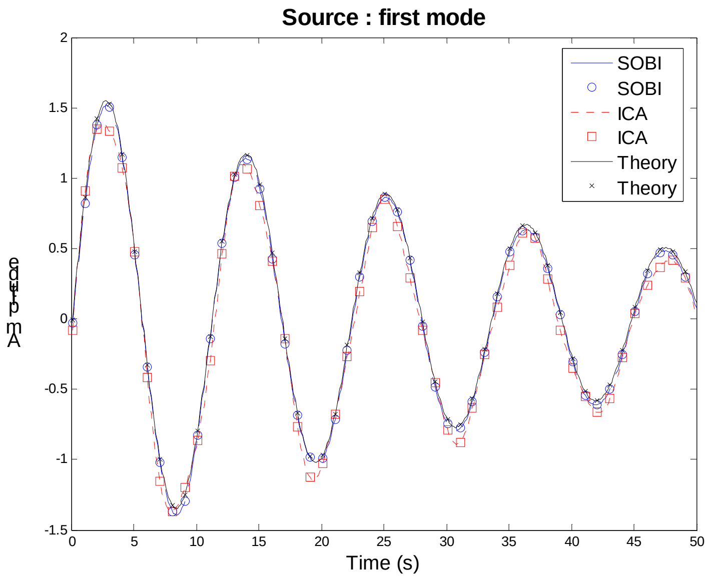

Figure 6 - Comparison of the theoretical, ICA and SOBI sources for the first mode of the 3DOF's system.

4.1.3 Noise influence

This paragraph studies the robustness of the identification to noise. The same system is considered with a damping coefficient $\alpha = 0.02$. The added white Gaussian noise is gradually increased from 0 to 20% of the RMS value. Figure 7 shows the signal distortion when 20 % of noise is added to the signals.

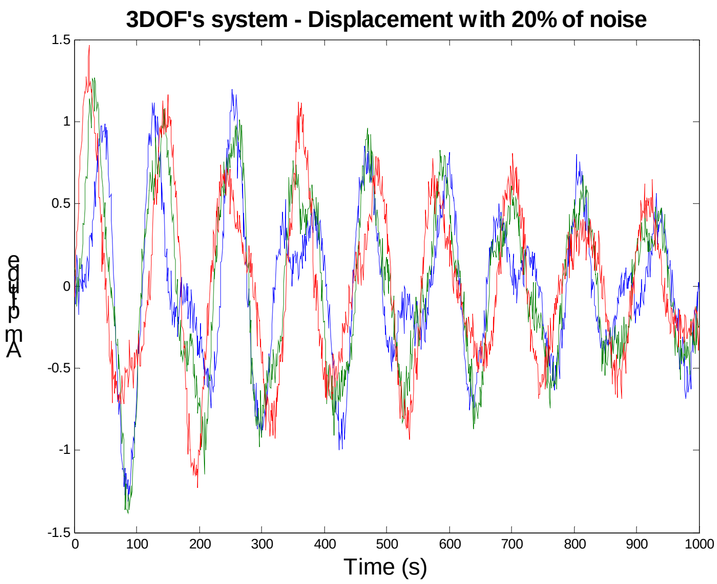

Figure 7 – Displacement of the 3DOF system when 20% of noise is added

As shown in Figure 8, the results seem to be fairly insensitive to noise, both for SOBI and ICA. Therefore, one can conclude that the methods behave remarkably well in the presence of noise.

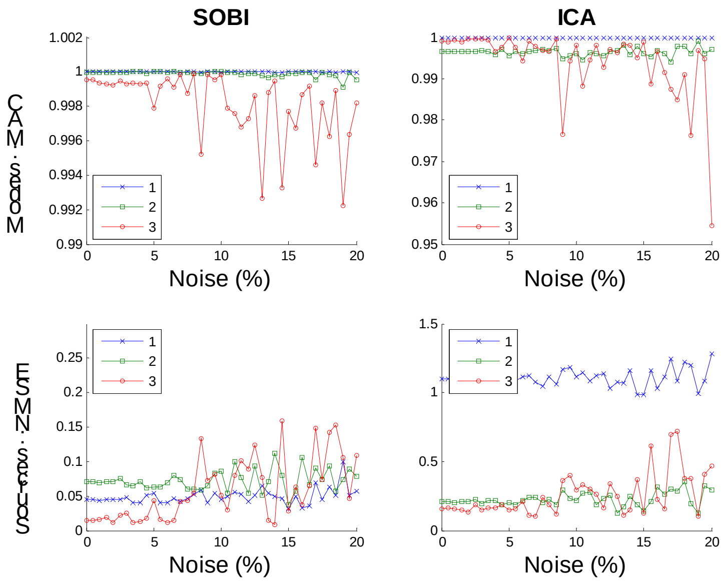

Figure 8 - Quality of modes and sources identified with ICA and SOBI method for the 3DOF's system regarding noise

4.1.4 Forced response

An advantage of BSS methods is that no a priori knowledge about the statistical distribution of the excitation signal is necessary. Moreover the knowledge of the time history of the applied force is not even required. Some numerical experiments were then realized on the previous system with a random excitation applied to mass m_1. This random excitation is characterized by a uniform distribution on the interval [-0.5 N ; 0.5 N]. 1000 sample points are taken into account for the identification after the transient response is damped out. The results were obtained with several values of the damping coefficient α. Noise (2% of the signal RMS value) corrupts the displacement signals.

For each case, 50 separate identifications resulting from 50 different samples of random applied force were carried out. An identification is considered as successful when the MAC value is higher than 0.98. The number of successful identification is given in between parentheses, and the mean values of the MAC and NMSE are listed in Table 2.

Damping coefficient and damping ratios		ICA			SOBI		
		Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
$\alpha = 0$	**MAC**	0.9989	0.9992	0.9929	0.9997	0.9994	0.9995
	NMSE	0.2466	0.1559	0.2326	0.055	0.037	0.017
		(49/50)	*(50/50)*	*(45/50)*	*(50/50)*	*(50/50)*	*(50/50)*
$\alpha = 0.002 \equiv \begin{cases} \xi_1 = 0.18\% \\ \xi_2 = 0.11\% \\ \xi_3 = 0.06\% \end{cases}$	**MAC**	0.9972	0.9984	0.9863	0.9996	0.9986	0.9990
	NMSE	0.298	0.182	0.366	0.052	0.057	0.030
		(49/50)	*(50/50)*	*(47/50)*	*(50/50)*	*(50/50)*	*(50/50)*
$\alpha = 0.02 \equiv \begin{cases} \xi_1 = 1.78\% \\ \xi_2 = 1.09\% \\ \xi_3 = 0.63\% \end{cases}$	**MAC**	0.9891	0.9937	0.9885	0.9984	0.9986	0.9972
	NMSE	2.764	1.527	0.729	0.396	0.268	0.088
		(35/50)	*(38/50)*	*(28/50)*	*(50/50)*	*(49/50)*	*(50/50)*

$\alpha = 0.1 \equiv$	$\xi_1 = 8.89\%$	**MAC**	0.9734	0.9788	0.9818	0.9926	0.9936	0.9889
	$\xi_2 = 5.46\%$	**NMSE**	7.562	6.943	1.170	1.484	1.056	0.456
	$\xi_3 = 3.15\%$		*(24/50)*	*(21/50)*	*(9/50)*	*(50/50)*	*(47/50)*	*(47/50)*

Table 2 : Accuracy of identifications using ICA and SOBI for several damping coefficients (3 DOFs, random response, 2% noise). Successful identification criterion: MAC>0.98

As it was observed for the free response, both methods seem to perform well for the weakly damped system. When damping increases beyond 1%, the identification using ICA fails, where as the SOBI method continues to provide accurate results. The signal response and the identified sources are represented in Figure 9 for $\alpha = 0.002$.

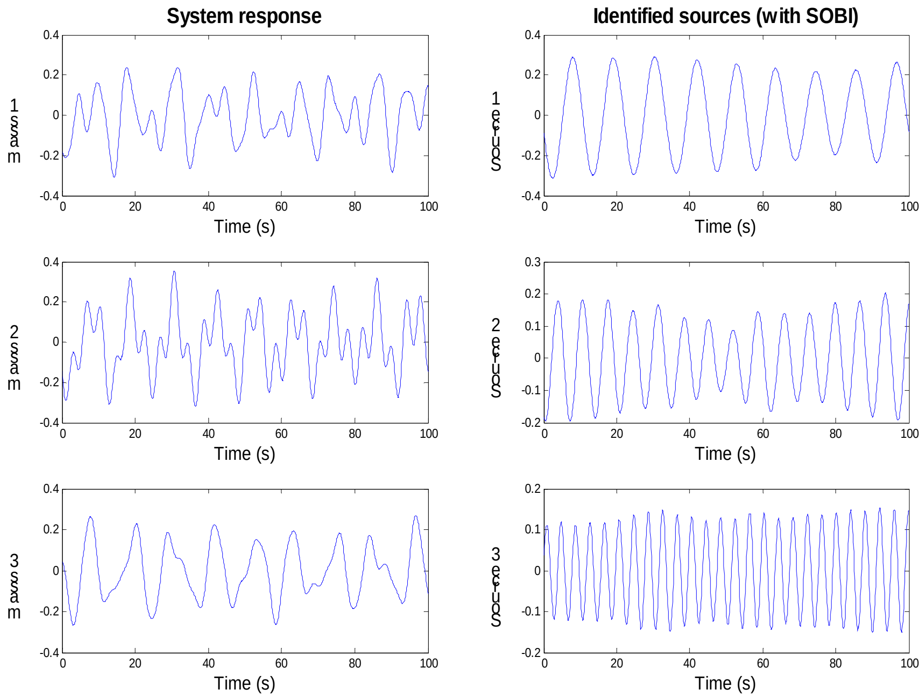

Figure 9 - Response and identified sources (with SOBI) for the random excitation

4.2 Identification of a distributed-parameter system

Another numerical experiment was also realized in order to demonstrate the applicability of the methodology. It consists of a cantilever steel beam modeled using the finite element method. The length of the beam is 0.7 m and the cross section is squared (*w*=*t*=0.014m). The theoretical results, namely the eigenfrequencies, the damping ratios and the eigenmodes, are also computed using the classical eigenvalue decomposition.

The system response is computed using Newmark's algorithm with a sampling frequency of 1e5 Hz. The data used for identification are the vertical accelerations at seven points which are uniformly spaced along the beam. The signals are resampled so that the frequency is 10000 Hz. The signals are corrupted with white Gaussian noise (5% of the signal RMS value).

4.2.1 Free response

The free response was obtained by suddenly releasing a vertical load of 100 N applied on the free extremity of the beam. Only the first 1000 sample points of the acceleration signals are taken into account for the identification. The SOBI and ICA algorithms are then directly applied to the acceleration signals, and the results are summarized in Table 3.

The SOBI algorithm clearly performs better than ICA. It identifies all the modes of the system accurately whereas the ICA algorithm retrieves only the first three modes. In fact, only the three first sources identified with ICA are purely monochromatic; the other sources mix the normal coordinates of the higher modes. Figure 10 presents the SOBI sources and the corresponding power spectral densities. As we can see, all the sources are monochromatic as requested by the SOBI algorithm. The shape of the theoretical, SOBI and ICA modes are also shown in Figure 11, which confirms the accuracy of the identification provided by SOBI.

		Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7
Frequencies (Hz)	**Theoretical**	23.6	147.8	414.3	814.0	1353.1	2038.2	2845.5
	SOBI	23.6	147.7	413.8	814.2	1352.5	2036.1	2839.4
	ICA	23.6	147.7	413.8	814.2	1352.5	2036.1	/
Damping ratios (%)	**Theoretical**	0.69	0.20	0.30	0.53	0.86	1.29	1.79
	SOBI	0.69	0.20	0.30	0.52	0.86	1.27	1.71
	ICA	0.76	0.20	0.30	0.58	0.9	1.4	/
MAC	**SOBI**	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995
	ICA	0.9996	0.9992	0.9991	0.8250	0.6066	0.5072	/
NMSE	**SOBI**	0.0019	0.0009	0.0005	0.0019	0.0169	0.0044	0.0098
	ICA	0.6833	0.6691	0.7024	27.8827	43.4520	45.1922	/

Table 3 : Accuracy of identifications using ICA and SOBI for the cantilever beam (free response, 5% noise).

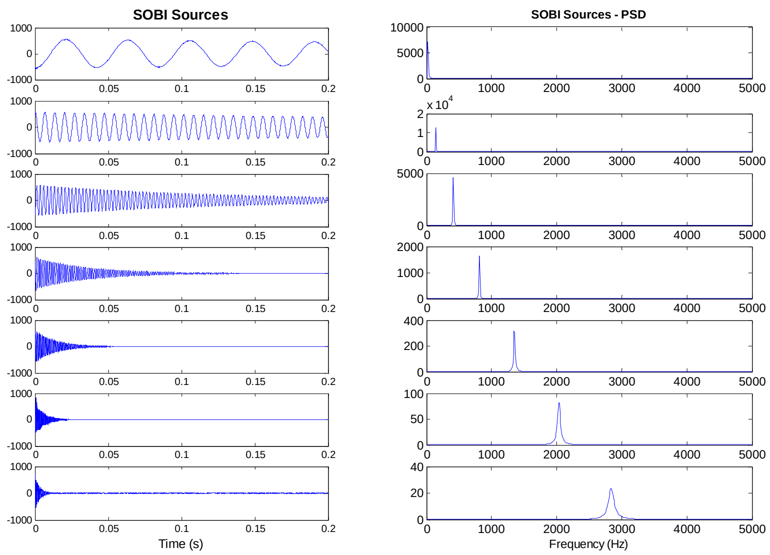

Figure 10 – Identified sources (with SOBI) and their power spectral densities

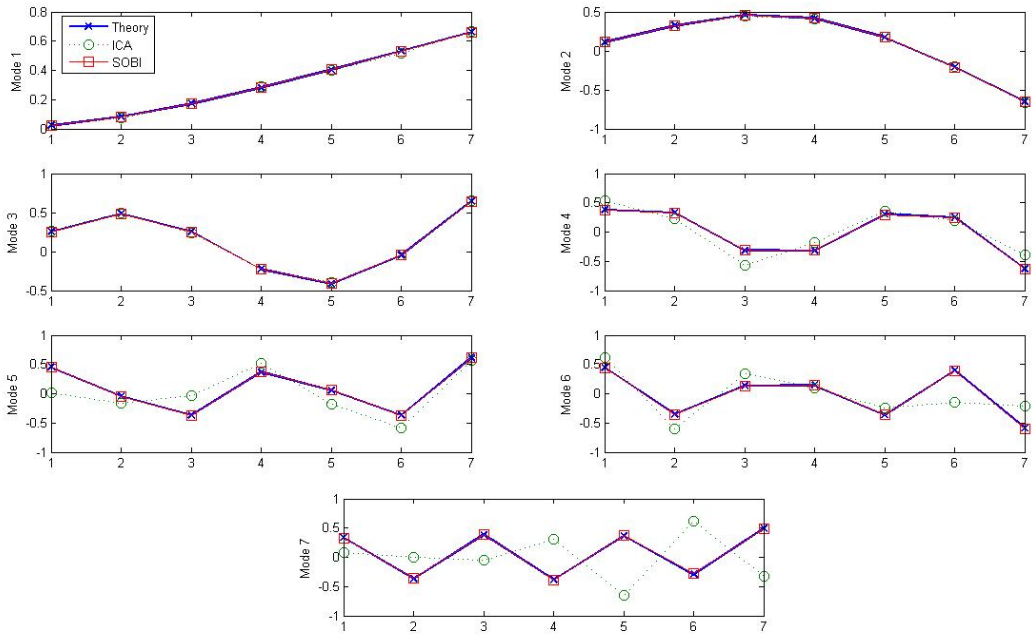

Figure 11 – Comparison of identified modes with the theoretical ones

4.2.2 Forced response

The two BSS methods were also applied to the response of the forced system to a random excitation (uniform distribution on the interval [-50 N ; 50 N]) applied vertically at the extremity of the beam. Because the results depend on the time history of the excitation signal, 50 separate identifications were realized. The results are presented in Table 4. Once again, SOBI performs well and is capable of identifying the majority of modes. ICA seems to be less robust.

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7
	SOBI						
MAC	/	0.9973	0.9989	0.9995	0.9997	0.9979	0.9992
NMSE	/	19.35	2.44	2.03	1.72	2.49	0.04
	(0/50)	*(39/50)*	*(45/50)*	*(45/50)*	*(45/50)*	*(45/50)*	*(45/50)*
	ICA						
MAC	/	0.9962	0.9953	0.9957	0.9900	0.9912	0.9894
NMSE	/	11.70	3.23	5.15	2.79	9.86	3.13
	(0/50)	*(5/50)*	*(7/50)*	*(6/50)*	*(7/50)*	*(3/50)*	*(8/50)*

Table 4 : Accuracy of identifications using ICA and SOBI for the cantilever beam (random response, 5% noise).

5 Conclusions

This paper proposes to exploit blind source separation techniques for experimental modal analysis. The key idea is to interpret the normal coordinates of a dynamic system as virtual independent sources. Under this assumption, there is a one-to-one mapping between the mixing matrix and the vibration modes of the structure.

Two methods were investigated (ICA and one of its variant SOBI) with two different numerical experiments, namely a discrete and a distributed-parameter system. Both methods are capable of performing an accurate identification of the modal parameters of weakly damped systems. We also showed that SOBI, which assumes that the sources have different spectral contents, is clearly more suitable for moderately or even highly damped systems.

These identification techniques possess several interesting features:

- They do not require the measurement of the applied force and can perform output-only modal analysis. In addition, in the random case, the knowledge of the statistical distribution of the applied force is neither necessary. This is particularly convenient in practical applications for which the external force cannot always be measured (e.g., vibrations of a bridge due to traffic and wind).
- A truly straightforward and simple identification is realized, because the vibration modes are merely the columns of the mixing matrix. The natural frequencies and damping ratios can be easily computed based on classical 1DOF techniques (e.g., the logarithmic decrement) applied to the computed sources.
- Compared to standard and efficient modal analysis techniques such as the stochastic subspace identification method, there is no need to specify a model order. As a result, computational modes are not an issue, and the use of stabilization charts, which always require a great deal of expertise, is avoided.
- According to the numerical simulations carried out herein, the methods seem to be fairly robust to the presence of measurement noise.

One potential limitation of the proposed modal analysis methodology is that the number of modes which may be computed cannot exceed the number of sensors. As a result, sensors should always be chosen in number greater or equal to the number of active modes.

Identification of experimental systems is an important step to pursue the validation of the methods. Results obtained to date demonstrate that the methods also perform well for practical applications. This will be discussed in a forthcoming paper [**27**].

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