FEATURE EXTRACTION FOR NON-LINEAR MODEL UPDATING

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SUMMARY: Feature extraction problems aims at extracting from multivariate data sets representative structures of lower dimension. For linear dynamics, the simplest and most popular features are the resonant frequencies, mode shapes and modal damping ratios. In the field of non-linear systems, other features must be defined because mode shapes are no longer effective to represent the system dynamics. Proper orthogonal decomposition (POD) investigates the use of the proper orthogonal modes (POMs) as features to better characterise non-linear mechanical systems. The method is based on an optimisation procedure which minimises the difference between the experimental and simulated POMs. However, the main shortcoming of POD is that it only considers second order statistics and its effectiveness is limited by its global linearity. The purpose of this paper is to introduce a method which takes higher order statistics into account and which can eliminate dependencies not removed by POD. This paper concludes by observing on an illustrative example that the system dynamics is better captured using the features proposed by this technique than using the POMs. In this context, it represents potentially a more effective tool than conventional POD in the field of identification and updating of non-linear mechanical systems.

KEYWORDS: proper orthogonal decomposition, principal component analysis, non-linear systems updating, feature extraction, local linear models.

INTRODUCTION

Proper orthogonal decomposition (POD), also known as principal component analysis (PCA), is a multivariate statistical method and can serve two purposes, i.e., data compression by projecting high-dimensional data onto a lower-dimensional space and feature extraction by revealing, through the proper orthogonal modes (POMs), relevant but unexpected structure hidden in the data. A very appealing property of the POD is its optimality. Among all possible decompositions of a random field, POD is the most efficient in the sense that for a given number of modes, the projection on the subspace used for modelling the random field will on average contain the most energy possible. POD is now increasingly exploited by the structural dynamicists. For instance, it has been applied to build reduced order models [1, 2] or to control torsional vibrations in long strings [3].

In the field of non-linear systems, features such as the resonant frequencies, mode shapes and modal damping ratios do no longer provide important insight into the system dynamics. POD investigates the use of the POMs as features to better characterise non-linear mechanical systems. The method proposed in references [4, 5] is based on an optimisation procedure which minimises the difference between the experimental and simulated POMs. Although it can be applied to non-linear systems, it is important to emphasise that POD is a linear procedure and that its optimality holds only with respect to other linear representations.

Recognising the shortcomings of PCA (or POD), researchers in the field of statistics and neural networks have developed non-linear extensions of PCA. A few global non-linear variants of PCA have been proposed [6, 7] but an alternative paradigm is to capture data complexity by a combination of local linear PCA projections. The aim of this paper is thus twofold. On the one hand, we wish to introduce the structural

dynamicist to a local non-linear extension of PCA, denoted as VQPCA, where VQ stands for vector quantisation. On the other hand, the performance of PCA and VQPCA are compared on an illustrative example.

PRINCIPAL COMPONENT ANALYSIS

PCA involves a mathematical procedure that transforms n possibly correlated variables collected in a data vector $\mathbf{x}$ into r ($r < n$) uncorrelated variables called principal components. This is realised by finding r directions $\mathbf{e}_i$ with $i = 1, ..., r$ onto which the retained variance under projection is maximal. These directions, denoted as principal directions or principal axes (or POMs when the terminology POD is used), are obtained as the r leading eigenvectors (i.e., associated with the r largest eigenvalues) of the covariance matrix

$$\Sigma = E[(\mathbf{x} - \boldsymbol{\mu})(\mathbf{x} - \boldsymbol{\mu})^T] \tag{1}$$

where $E[\cdot]$ is the expectation and $\boldsymbol{\mu} = E[\mathbf{x}]$ is the mean of the data. If the principal directions are collected in a matrix $\mathbf{E} = [\mathbf{e}_1 \; ... \; \mathbf{e}_r]$, then $\mathbf{z} = \mathbf{E}^T(\mathbf{x} - \boldsymbol{\mu})$ is a reduced r-dimensional representation of the observed vector $\mathbf{x}$. The components of $\mathbf{z}$ are called the principal components. Among all linear techniques, PCA provides the optimal reconstruction $\hat{\mathbf{x}} = \boldsymbol{\mu} + \mathbf{E}\mathbf{z}$ of $\mathbf{x}$ in terms of the quadratic reconstruction error $\|\mathbf{x}^2 - \hat{\mathbf{x}}^2\|$.

From the foregoing developments, it appears that PCA builds a global linear model of the data, i.e., an r-dimensional hyperplane spanned by the r leading eigenvectors of the data covariance matrix. If the data lies on a non-linear manifold, one might reasonably expect that PCA will incur a higher error than a non-linear technique. This simple realisation has prompted the development of non-linear alternatives to PCA.

A LOCAL PCA APPROACH: VQPCA

A possible non-linear alternative to PCA lies in the construction of local PCA projections. Indeed, if a non-linear surface is smooth, then each local piece looks more and more linear under magnification. The first application of a local PCA method dates back to 1971 and is due to Fukunaga and Olsen [8]. Since then, it has been applied for the identification of intrinsic dimension of data [9], for handwritten character recognition [10] and for dimension reduction of speech [11] and images [11, 12].

This section aims to present a local alternative to PCA, called VQPCA, where VQ stands for vector quantisation. The VQPCA algorithm involves a two-step procedure:

1. the partition of the input space into a set of regions by vector quantisation;
2. the construction of separate low-dimensional co-ordinate systems in each local region using PCA.

Vector quantisation

Vector quantisation (VQ) is a classical technique for signal coding and data compression [13]. Let $\mathcal{X}$ be a set of observed n-dimensional data points $\mathbf{x}_i$ with $i = 1, ..., m$. An q-level vector quantiser is defined by a codebook $\mathcal{C} = (\boldsymbol{\mu}_1, ..., \boldsymbol{\mu}_q)$, a partition $\mathcal{S} = (S_1, ..., S_q)$ and a distortion function $d(\mathbf{x}, \boldsymbol{\mu})$. It is a mapping f that approximates each point $\mathbf{x}_i$ in the set $\mathcal{X}$ by a component $\boldsymbol{\mu}_j$ of the codebook $\mathcal{C}$: $f(\mathbf{x}_i) = \boldsymbol{\mu}_j$ if $\mathbf{x}_i \in S_j$. An q-level quantiser is said to be optimal if it minimises the averaged distortion $D = E[d(\mathbf{x}, \boldsymbol{\mu})]$.

In reference [11], Kambhatla makes use of VQ to define the regions for the local PCA. The algorithm to design the vector quantiser is based on an approach of Lloyd [14] and is referred as the generalised Lloyd algorithm [13]. The codebook vectors $\boldsymbol{\mu}_j$ and the regions S_j satisfy Lloyd's optimality conditions:

1. each region S_j (with its corresponding codebook vector $\boldsymbol{\mu}_j$) corresponds to all $\mathbf{x}_i$ that lie closer to $\boldsymbol{\mu}_j$ than to any other codebook vector (nearest neighbour mapping). Mathematically, $S_j = \{\mathbf{x}_i \mid d(\mathbf{x}_i, \boldsymbol{\mu}_j) < d(\mathbf{x}_i, \boldsymbol{\mu}_k), \forall \, k \neq j\}$.

2. each codebook vector $\boldsymbol{\mu}_j$ is placed at the centroid of the corresponding region S_j.

For a distortion function based on Euclidean distance, the regions are convex sets called Voronoi cells and the centroid of a region is the mean of the data points in this region $\boldsymbol{\mu}_j = E[\mathbf{x}_i \mid \mathbf{x}_i \in S_j]$.

Accordingly, the generalised Lloyd algorithm is as follows:

1. given q a number of regions, initialise the codebook $\mathcal{C}$ from randomly selected points in the data set $\mathcal{X}$;
2. compute the corresponding optimal partition following the first optimality condition;
3. compute the corresponding optimal codebook following the second optimality condition;
4. iterate steps 2 and 3 until convergence.

The convergence is achieved when the fractional change in the averaged distortion D between the kth and $(k+1)$th iterations is below some specified threshold. It can be argued that each iteration of the algorithm either reduces the distortion or leaves it unchanged.

It is worthwhile noticing that several variants of this algorithm exist, e.g., tree-searched VQ and multistep VQ [13]. These variants aim to reduce the computation or memory requirements but may compromise the performance relative to what could be achieved with a standard VQ. Since the computational aspects are not an issue in this work, the basic Lloyd algorithm is considered throughout the paper.

By way of illustration, the generalised Lloyd algorithm, with a distortion function based on the Euclidean distance and a number of regions equal to 20, is applied on a set of 5000 two-dimensional random vectors chosen from a normal distribution with mean zero and variance one. The distribution of the random vectors is displayed in Fig. 1(a). Fig. 1(b) depicts the Voronoi cells S_j together with their corresponding centroids $\boldsymbol{\mu}_j$.
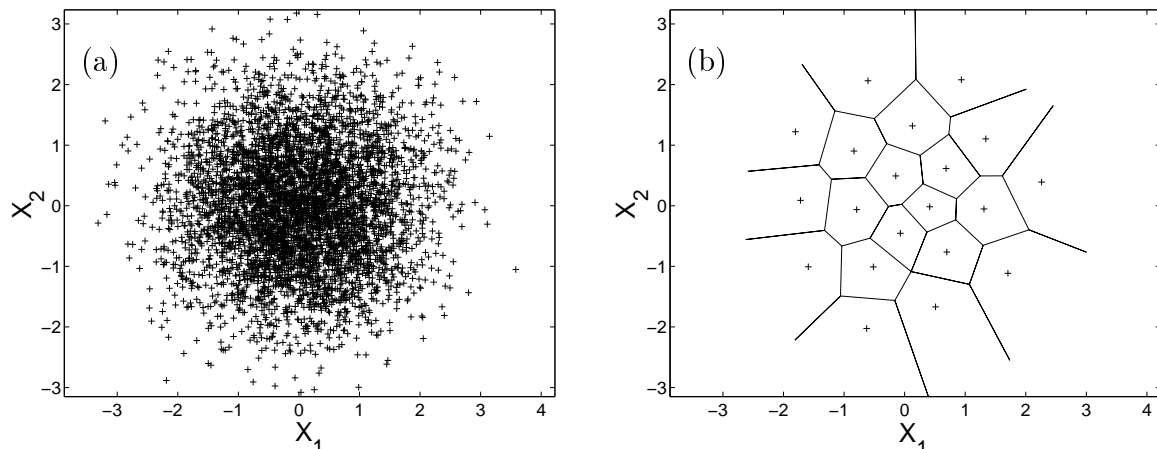

Fig. 1: (a) Distribution of the data; (b) clustering of the data. —, Voronoi cells; +, centroids.

Dimension reduction by local linear models

Consider again a set of observed n-dimensional data points $\mathbf{x}_i$ with $i = 1, ..., m$. In order to reduce the dimensionality of the vector $\mathbf{x}_i$, we need to determine an encoding function $f : \Re^n \to \Re^r$ such that $\mathbf{z}_i = f(\mathbf{x}_i)$ is a compact r-dimensional representation of $\mathbf{x}_i$. Similarly, a decoding function $g : \Re^r \to \Re^n$ has to be calculated such that $\hat{\mathbf{x}}_i = g(f(\mathbf{x}_i))$ is the reconstruction of the initial vector $\mathbf{x}_i$.

In the present work, the purpose is to build low-dimensional co-ordinate systems in the q local regions defined by the vector quantiser. If the local regions are small enough, the data manifold is not curved much over the extent of the region and it may be locally approximated as a hyperplane. In other words, a separate PCA model in each of the q regions should be adequate. Instead of having single encoding and decoding functions, a collection of functions $f_j(\cdot)$ and $g_j(\cdot)$ with $j = 1, ..., q$ is obtained.

VQPCA algorithm

The VQPCA algorithm is an extension of a standard vector quantiser. VQPCA partitions the input space into a set of regions and approximates each region by a hyperplane defined by PCA, while a standard vector quantiser approximates each region by a codebook vector. The VQPCA algorithm is as follows.

1. Partition $\Re^n$ into q disjoint regions $S_1, ..., S_q$ using the generalised Lloyd algorithm with Euclidean distance as the distortion function.

2. For each Voronoi cell S_j and its corresponding centroid $\boldsymbol{\mu}_j$, estimate the local covariance matrix

$$\boldsymbol{\Sigma}_j = \frac{1}{N_j} \sum_{\mathbf{x} \in S_j} (\mathbf{x} - \boldsymbol{\mu}_j)(\mathbf{x} - \boldsymbol{\mu}_j)^T \tag{2}$$

   where N_j is the number of vectors mapped to S_j. Next, compute the eigenvectors $(\mathbf{e}_{j1}, ..., \mathbf{e}_{jr})$ of each matrix $\boldsymbol{\Sigma}_j$.

3. To reduce dimension of any vector $\mathbf{x}_i$, determine the cell S_j which contains the vector and project $\mathbf{x}_i$ onto the r leading eigenvectors to obtain the local linear coordinates

$$\mathbf{z}_i = f_j(\mathbf{x}_i) = [\mathbf{e}_{j1} \ ... \ \mathbf{e}_{jr}]^T (\mathbf{x}_i - \boldsymbol{\mu}_j) = \begin{bmatrix} \mathbf{e}_{j1}^T(\mathbf{x}_i - \boldsymbol{\mu}_j) \\ \vdots \\ \mathbf{e}_{jr}^T(\mathbf{x}_i - \boldsymbol{\mu}_j) \end{bmatrix} \quad \text{if } \mathbf{x}_i \in S_j \tag{3}$$

   The compressed representation of $\mathbf{x}_i$ consists of the index j of the Voronoi cell in which $\mathbf{x}_i$ lies and the r-dimensional vector $\mathbf{z}_i$. The data is reconstructed from this representation according to

$$\hat{\mathbf{x}}_i = g_j(f_j(\mathbf{x}_i)) = g_j(\mathbf{z}_i) = \boldsymbol{\mu}_j + [\mathbf{e}_{j1} \ ... \ \mathbf{e}_{jr}] \mathbf{z}_i \tag{4}$$

   The accuracy of the compressed representation is assessed using the normalised mean square error (MSE)

$$MSE = \frac{E[\|\mathbf{x} - \hat{\mathbf{x}}\|^2]}{E[\|\mathbf{x} - E[\mathbf{x}]\|^2]} \tag{5}$$

AN ILLUSTRATIVE EXAMPLE

The effectiveness of the VQPCA approach may be demonstrated with the following example. It shows how the modes extracted from the VQPCA algorithm can capture more accurately the dynamics of a non-linear beam (see Fig. 2) than the principal directions given by PCA. The beam is modelled with seven beam elements and the local non-linearity k_{nl} is a spring that exhibits a cubic stiffness. The free vibration of the beam is simulated with an initial displacement given by a static force F_0 applied at the end of the beam.

The data set consists of seven vertical accelerations measured along the beam. PCA which is equivalent to VQPCA with a single Voronoi cell is first applied to the data. In a second step, the data is modelled with VQPCA using a number of regions varied from 5 to 30. Table 1 summarises the relative performance of PCA and VQPCA in terms of compression accuracy measured by the MSE. Fig. 3 shows the acceleration at the fourth node reconstructed from one- and two-dimensional representations generated both by PCA and VQPCA (20 cells).

It is clear from this figure that VQPCA provides a more accurate representation of the system response than PCA. This advantage is also reflected in the errors reported in Table 1. For instance, for a unimodal representation, VQPCA with 20 cells attains about 80% lower error than PCA and is still a better approximation than a bimodal PCA representation. It is worthwhile noticing that for a fixed number of regions, the VQPCA algorithm was run with different initialisations of the codebook. In this way, the sensitivity of the generalised Lloyd algorithm to the starting point is tested. The results were never significantly influenced by the starting point.

In reference [15], it is also shown that when the response of a non-linear system is dominated by a single synchronous non-linear normal mode, the first mode given by the VQPCA algorithm is a much closer approximation to the non-linear normal mode than the corresponding PCA mode.
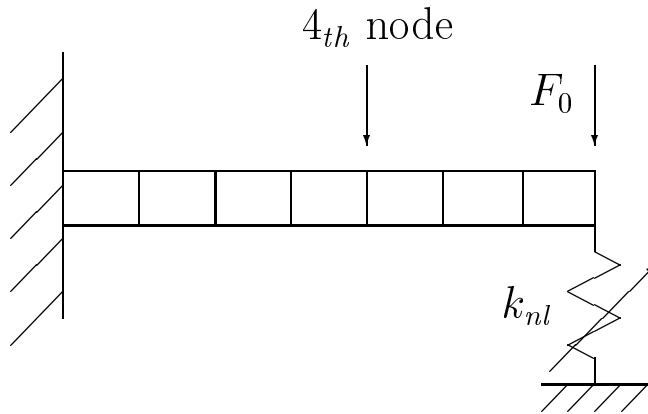
Fig. 2: Model of the non-linear beam.

Table 1: PCA and VQPCA applied to the non-linear beam example

Number of regions	MSE (%) 1 mode	MSE (%) 2 modes	MSE (%) 3 modes	MSE (%) 4 modes
1 (PCA)	31.75	9.15	3.53	0.010
5	17.45	7.30	1.52	0.009
10	12.34	5.09	1.18	0.008
20	6.72	2.85	0.83	0.006
30	5.69	2.08	0.60	0.004

CONCLUSION

Modelling complexity in the data by a combination of simple linear models is an attractive paradigm. Accordingly, a local non-linear variant of PCA, denoted VQPCA, has been proposed in this paper. The method first exploits vector quantisation to cluster the data space into disjoint regions. Then, a standard PCA model is built in each region defined by the vector quantiser. Thus, VQPCA approximates the data distribution with a set of local hyperplanes. The location and the distribution of this set capture the large scale, non-linear structure of the data, while coordinates on the hyperplanes capture the local variations.

VQPCA provides insight into the structure of a data set that PCA could not. Its superiority over PCA has been demonstrated on an illustrative example. It is well suited for dimensionality reduction and for estimation of the intrinsic dimensionality. There are other applications that deserves attention. PCA was used to monitor the condition of signals during manufacturing [16]. The non-linear models provided by VQPCA should provide more accurate models of the undamaged data and hence improve the sensitivity and specificity for fault detection. PCA was also applied to identify and update non-linear mechanical systems [4, 5]. In this context, the modes given by VQPCA may be viewed as promising features to analyse the behaviour of the non-linear system. This will be studied in further work.
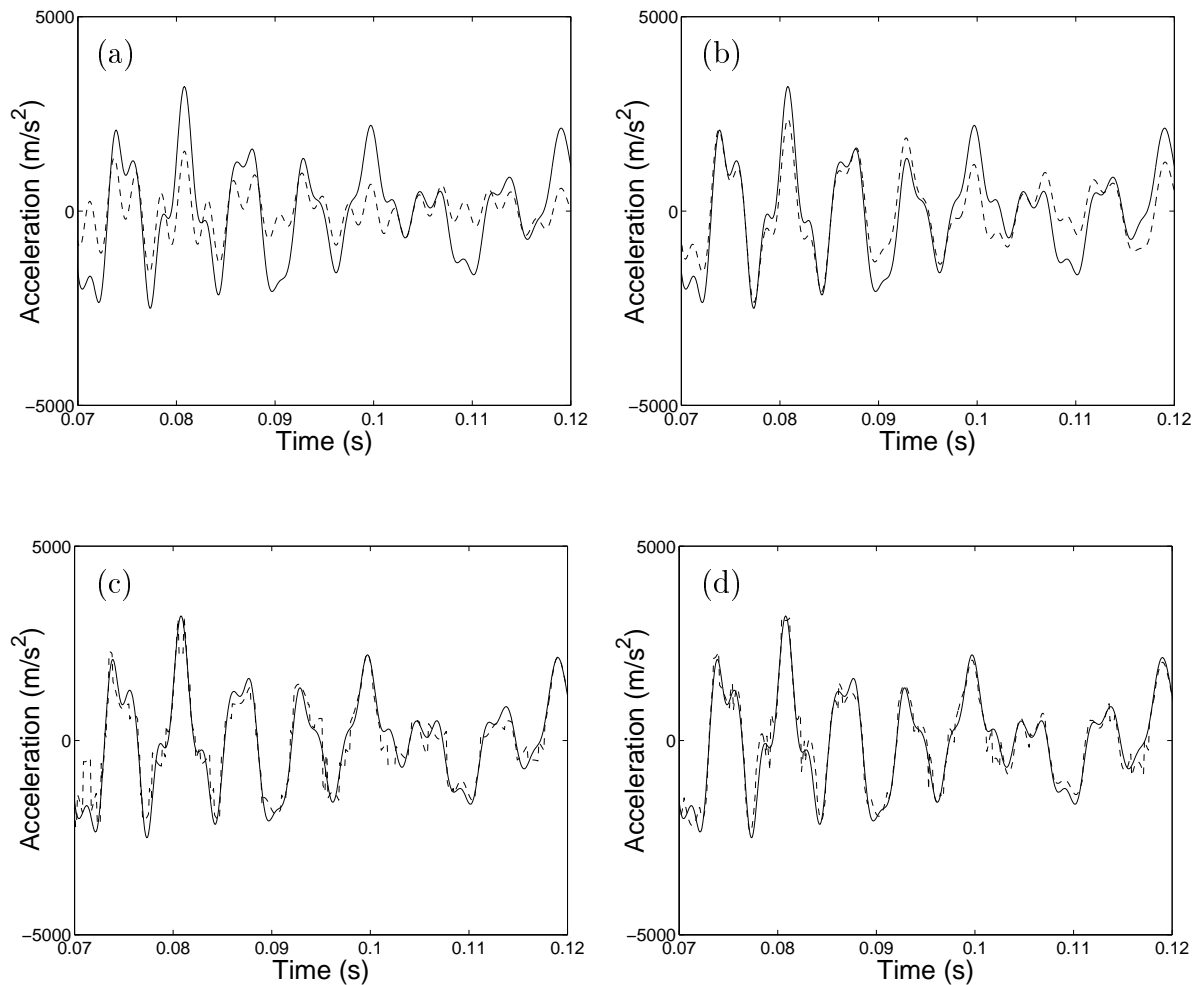
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Fig. 3: Reconstruction of the dynamical response of the non-linear beam. ———, System response; - - -, reconstructed response. (a) PCA 1 mode; (b) PCA 2 modes; (c) VQPCA 1 mode, 20 cells; (d) VQPCA 2 mode, 20 cells.

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