Efficiency of Targeted Energy Transfers in Coupled Nonlinear Oscillators Associated with 1:1 Resonance Captures

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The aim of this work is to investigate conditions for optimal targeted energy transfer (TET) in a two – degree-of-freedom (DOF) nonlinear system under condition of 1:1 transient resonance capture (TRC) (Arnold, 1988; Quinn et al., 1995). In particular, we consider the following weakly damped system,

$$\begin{aligned} &\ddot{x}+\lambda_1\dot{x}+\lambda_2(\dot{x}-\dot{v})+\omega_0^2 x+C(x-v)^3=0 \\ &\varepsilon\ddot{v}+\lambda_2(\dot{v}-\dot{x})+C(v-x)^3=0 \end{aligned} \qquad (1)$$

that is, a linear oscillator (LO), described by coordinate *x*, coupled to a lightweight, essentially nonlinear attachment, termed nonlinear energy sink – NES, described by coordinate *v*. The small parameter of the problem, $0<\varepsilon<<1$, scales the mass of the NES.

This two – DOF system possesses surprisingly complex dynamics (Kerschen et al., 2006). Moreover, at certain ranges of parameters and initial conditions passive targeted energy transfer—TET—is possible, whereby vibration energy initially localized in the linear oscillator gets passively transferred to the lightweight attachment in a one-way irreversible fashion where it is locally dissipated without 'spreading back' to the LO.

In the first part of this work the topological features of the corresponding Hamiltonian dynamics of system (1) (i.e., with no dissipative terms) are discussed. Focusing on an intermediate-energy region close to the 1:1 resonance manifold of the Hamiltonian dynamics the topological changes of intermediate-energy impulsive orbits (IOs) are studied for varying energy. By IOs we denote periodic or quasi-periodic responses of the Hamiltonian system initiated with nonzero velocity for the LO and all other initial conditions zero. Specifically, it is found that above a critical value of energy, the topology of intermediate-energy IOs changes drastically, as these orbits make much larger excursions in phase space, resulting in continuous, strong energy exchanges between the LO and the NES, that appear in the form of strong nonlinear beats. It is also found that this critical energy of the Hamiltonian system may be directly related to the energy threshold required for TET in the corresponding weakly damped system. Hence, a direct link between the Hamiltonian and weakly damped dynamics is established.

In the second part of this work we revisit the intermediate-energy dynamics of the weakly damped system (1), in an effort to obtain conditions for realization of optimal TET from the LO to the NES. Since our study will be based on perturbation analysis, it will be necessarily restricted to the neighborhood of the 1:1 resonance manifold of the underlying Hamiltonian dynamics; hence, the damped dynamics will be studied under condition of 1:1 resonance capture. However, the ideas and techniques presented here can be extended to study optimal conditions for the more general case of $m:n$ subharmonic TET.

We start with the slow flow analysis of the governing equations through the complexification-averaging (CX-A) technique first developed by Manevitch (1999). This will be followed by qualitative analysis of the different mechanisms for TET in the system, and an analytical study of homoclinic perturbations in the weakly damped system. We will show that homoclinic perturbations yield an additional slow-time scale in the averaged dynamics which governs optimal TET from the LO to the NES occurring in a single 'super-slow' half cycle. Thus, we will prove that the existence of the homoclinic orbit is responsible for the jump of energy pumping efficiency as the initial energy of the system varies (Fig. 1). We will conclude this work by providing numerical results of TET efficiency for a wide range of system parameters, which verify the analytical findings.

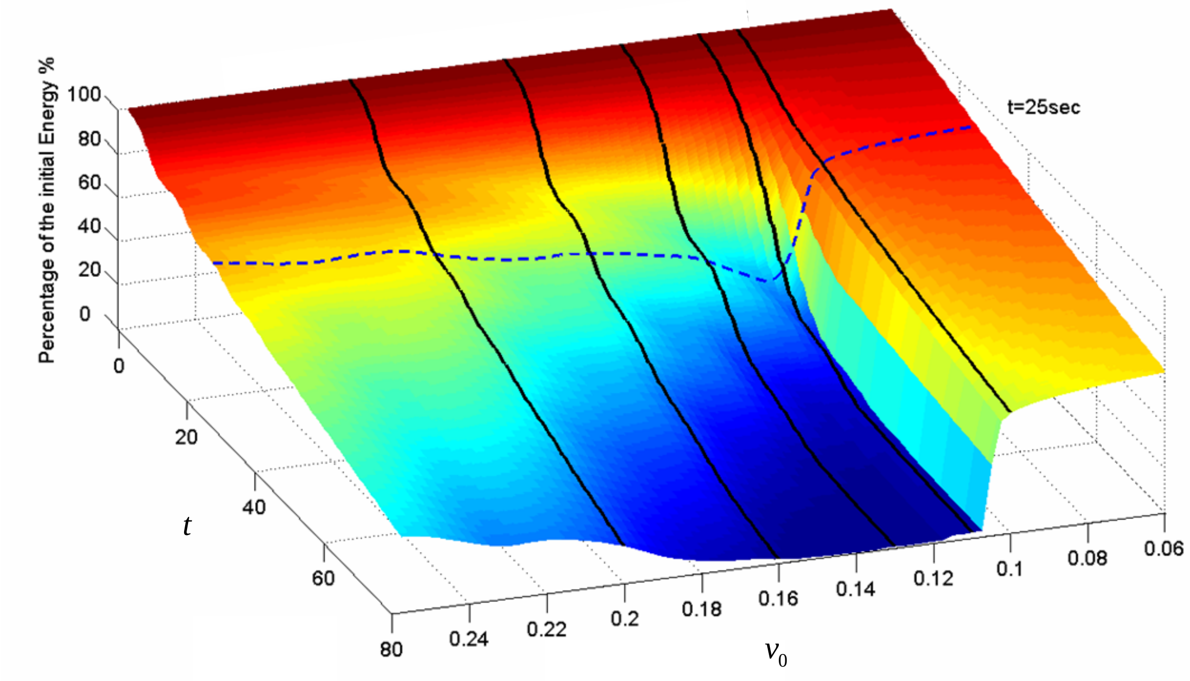

Fig. 1. Percentage of initial energy dissipated in system (1) when intermediate-energy damped IOs are excited ($\varepsilon = 0.05$, $C = 1$, $\omega_0 = 1$ and $\varepsilon\lambda_1 = \varepsilon\lambda_2 = 0.005$): solid lines correspond to excitation of specific periodic IOs, and the dashed line indicates the instantaneous energy at *t* = 25sec.

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