Maximal fidelity between symmetric multiqubit states and entanglement classes

A. Neven,[1] P. Mathonet,[2] O. Gühne,[3] and T. Bastin[1]

[1]Institut de Physique Nucléaire, Atomique et de Spectroscopie, Université de Liège, 4000 Liège, Belgium
[2]Département de Mathématique, Université de Liège, 4000 Liège, Belgium
[3]Naturwissenschaftlich-Technische Fakultät, Universität Siegen, 57068 Siegen, Germany

Abstract

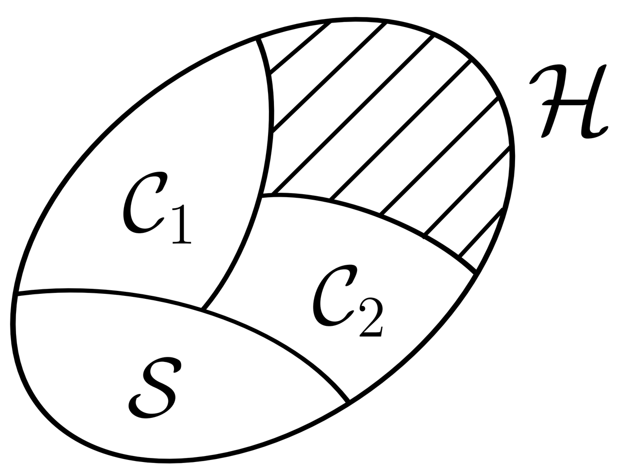
We present the results of our research concerning a conjecture about the maximal fidelity between a symmetric (permutation invariant) multiqubit state and the states belonging to a given entanglement class. The question raised by this conjecture is the following : "Is the maximum fidelity between a symmetric state $|\psi_s\rangle$ and the states belonging to an entanglement class $\mathcal{C}$ equal to the maximal fidelity between the state $|\psi_s\rangle$ and only the symmetric states of class $\mathcal{C}$" ? In our investigations, we considered two different types of classification : the SLOCC and LOCC (or LU) classifications. In both cases, we first performed numerical tests for a large set of symmetric states and entanglement classes. In light of the results we obtained, further analytical work enabled us to conclude about the validity of the conjecture.

Entanglement classification

All the entangled states of a system are not equivalent regarding their entanglement properties. They can indeed exhibit different kinds of entanglement and the point of entanglement classification is to regroup states having the same type of entanglement into entanglement classes.

➜ States of the same class can be used equivalently to perform certain tasks.

➜ The Hilbert state space $\mathcal{H}$ of the system is divided into several classes (the number of classes being not necessarily finite)



In this work, we considered two different kinds of classifications :

- LOCC (or LU) classification and
- SLOCC classification.

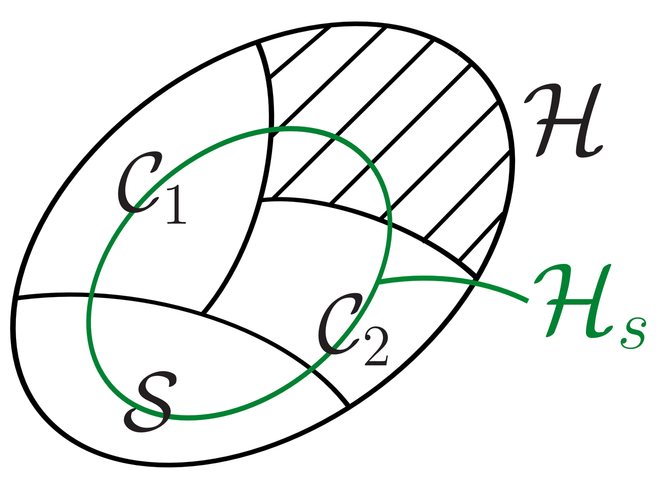
A (S)LOCC class regroups states that can be (Stochastically) obtained from each other using Local Operations and Classical Communication.

Symmetric states

Symmetric (or permutation invariant) states are interesting for several reasons :

➜ experimental interest (experiments using bosons for example)

➜ linear growth of the complexity instead of exponential for multiqubit systems, the symmetric subspace $\mathcal{H}_s$ being of dimension $N+1$ instead of 2^N for a system of N qubits.



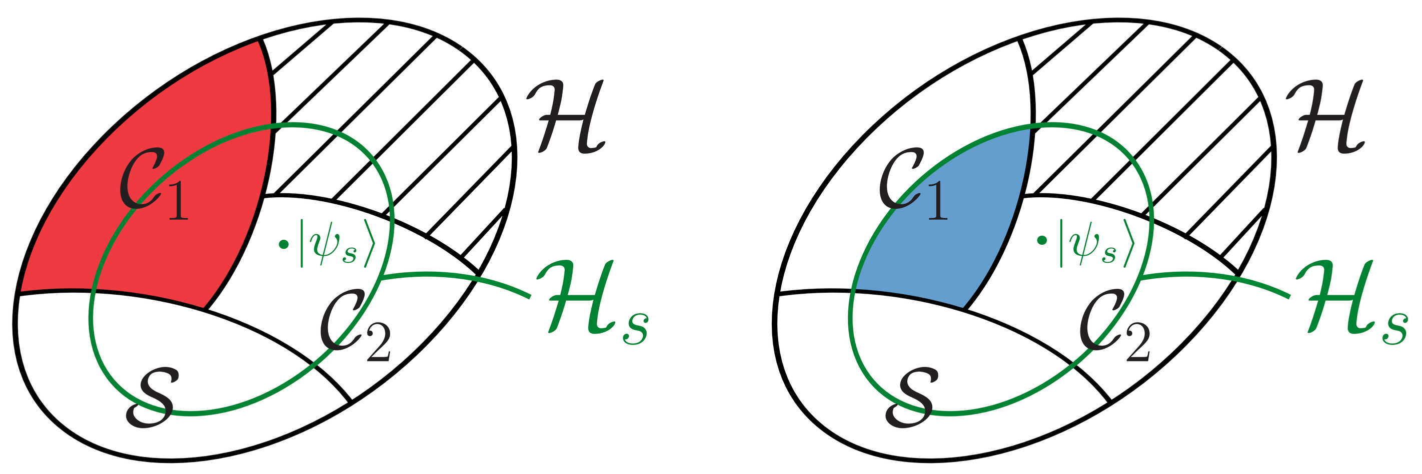
➜ further classification into a finite number of entanglement families [1] displaying hierarchic relations. For a system of N qubits, these families are denoted $\mathcal{D}_p$, p being any partition of N.

The conjecture

The conjecture we studied concerns the properties of symmetric states regarding entanglement classification. Considering an entanglement class $\mathcal{C}$ containing symmetric states and a symmetric state $|\psi_s\rangle$ belonging to another class, the conjecture can be stated as : "Is the maximal fidelity between the state $|\psi_s\rangle$ and all the states from the class $\mathcal{C}$ equal to the maximal fidelity between the state $|\psi_s\rangle$ and only the symmetric states belonging to class $\mathcal{C}$?" In mathematical terms, this conjecture reads

Given an entanglement class $\mathcal{C}$ containing symmetric states and a symmetric state $|\psi_s\rangle \notin \mathcal{C}$, do we have

$$\sup_{|\phi\rangle \in \mathcal{C}} |\langle\psi_s|\phi\rangle|^2 = \sup_{\text{symmetric}|\phi\rangle \in \mathcal{C}} |\langle\psi_s|\phi\rangle|^2 \ ?$$



➜ The conjecture is known to be true [2] for the particular class of separable states.

➜ Global validity of this conjecture would imply a great simplification, when dealing with symmetric states, in some discrimination strategies used to discriminate experimental states from a hole entanglement class [3] (those strategies requiring the computation of maximal fidelities).

Numerical results

From numerical estimations of maximal fidelities for a large set of state $|\psi_s\rangle$ and class $\mathcal{C}$ [4], we found

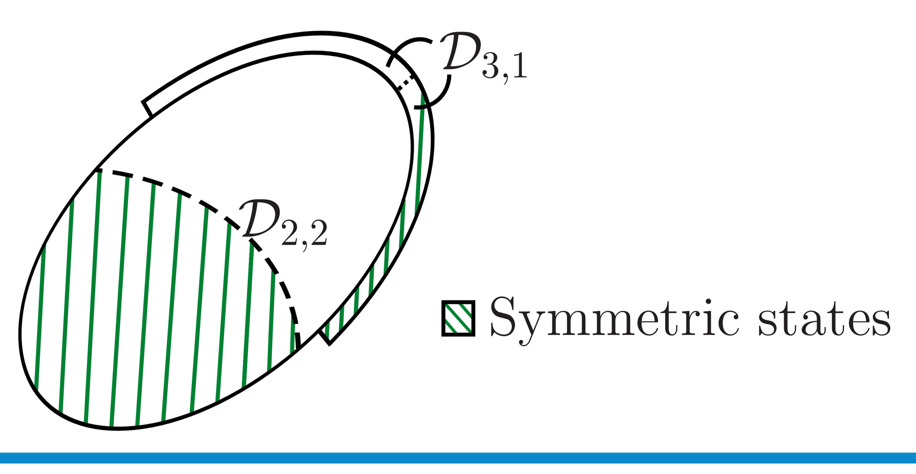
- no violation of the conjecture in the case of the LU classification,
- several violations of the conjecture in the case of the SLOCC classification, especially with classes from families of the type $\mathcal{D}_{N-k,k}$ ($k = 2, \ldots, [N/2]$).

In those cases, numerical estimations revealed unexpectedly high upper bounds :

$$\sup_{|\phi\rangle \in \mathcal{D}_{N-k,k}} |\langle\psi_s|\phi\rangle|^2 \approx 1 \qquad (1)$$

for N=4, 5, 6 , k=2, ... , $[N/2]$, $|\psi_s\rangle \in \mathcal{D}_{N-1,1}$.

➜ These upper bounds close to unity imply that for any state of the entanglement family $\mathcal{D}_{N-1,1}$, there exists a really "close" state in each family of the type $\mathcal{D}_{N-k,k}$. Taking $N = 4$, this suggests a class topology of the form



Analytical results

In light of the numerical results, we investigated the conjecture from an analytical point of view and could prove

- that the conjecture is valid in its whole generality for the LU classification of multiqubit states,
- that the upper bound in (1) is strictly 1 (even though this value cannot be reached) and greater than the corresponding value for the symmetric subspace, proving that the conjecture is not correct in its whole generality for the SLOCC classification.

➜ The topological consequence of this result is confirmed for the families $\mathcal{D}_{N-k,k}$ for each $N \geq 4$.

References

[1] T. Bastin *et al.*, PRL **103**, 070503 (2009)
[2] R. Hübener *et al.*, PRA **80**, 032324 (2009)
[3] S. Niekamp *et al.*, PRA **82**, 022322 (2010)
[4] A. Neven, Master thesis, Université de Liège (2014)

Acknowledgement

We thank the FRS-FNRS for financial support.

contact : a.neven@ulg.ac.be