

# HYDRO-MECHANICAL MODELING OF DAMAGE DUE TO UNDERGROUND EXCAVATION IN HARDENED CLAY

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**ABSTRACT:** *A zone with significant irreversible processes and significant changes in flow and transport properties is expected to be formed after underground excavations. The perturbation of the excavation could lead to a significant increase of the permeability, related to diffuse and/or localized crack proliferation in the material. The main objective of the study is to model these processes at large scale in order to assess their impacts on the performance of radioactive waste geological repositories. This paper concerns more particularly the hydro-mechanical modeling of a long term dilatometer experiment performed in Mont Terri Rock Laboratory in Switzerland. The proposed model defines the permeability as a function of the aperture of the cracks that are generated during the excavation. With this model, the permeability tensor becomes anisotropic. These developments are validated with the results of the experiment.*

## 1 INTRODUCTION

The formation of an Excavation Damage Zone (EDZ) is a phenomenon that occurs in the most rock masses as a consequence of underground excavation. The EDZ appears as an area around the underground openings, where geotechnical and hydro-geological properties are altered. The purpose of this paper is the development of a hydro-mechanical model able to reproduce the evolution of permeability during the lifetime of the underground radioactive waste storage. Indeed, in order to propose a design of the geo-structure, the numerical models have to tackle the relevant processes and to predict the behavior of the engineered barrier system. The evolution of the permeability within the EDZ can be directly associated to strain localization or damage (Soulay et al., 2001). When a rock is damaged, crack networks are created, which constitute preferential flow paths. As a consequence, the rock hydraulic conductivity increases and generally becomes anisotropic (Shao et al., 2005). The permeability of crack is usually related to the crack aperture (Olsson and Barton, 2001) and the key issue is thus to propose a model providing the relationship between rock mass permeability and crack aperture, as well as the evolution of the crack aperture during the excavation. Different approaches exist to tackle this problem, from micro-macro theories (Dormieux and Kondo, 2004) to macroscopic (phenomenological) ones (Snow, 1969; Witherspoon et al., 1980). In this paper, we use the latter theories to relate the aperture evolution to the strain tensor (Liu et al., 1999; Chen et al., 2007).

As a starting point of our development, we describe first, in section 2, a long term dilatometer experiment, performed in Mont Terri Rock Laboratory in the context of radioactive waste storage studies. The dilatometer is supposed to sham the bentonite swelling

pressure on soil. This test exhibits the influence of the dilatometer on the axial transmissivity of the Excavation Damage Zone. The proposed developments on the permeability evolution are presented in section 3. In the section 4, a numerical study on the hydro-mechanical coupled effect in the EDZ observed in this experiment is proposed. Some comparisons with the results of the experiment show that this phenomenological approach permits to reproduce the behaviors observed in situ, like the decrease of the hydraulic conductivity in the EDZ with the dilatometer pressure increase. Finally, some conclusions and further perspectives of this study are given in section 5.

## 2 LONG TERM DILATOMETER EXPERIMENT

### 2.1 Loadings and hydraulic tests during the long term dilatometer experiment

A long term dilatometer experiment is performed in order to test the axial transmissivity of the EDZ and its evolution with time (Bühler, 2005). The general concept of this experiment is to combine dilatometer tests and numerous hydraulic tests with multi-packer system as illustrated on figure 1. A dilatometer probe is installed with two inflatable packers in a newly drilled borehole in Opalinus Clay. The two inflatable packers are inflated to a constant pressure which permits to isolate the deepest part of the borehole from the tunnel. The pressure in the dilatometer probe (modeling the effect of the bentonite swelling pressure) is increased stepwise and hydraulic tests are periodically performed under different dilatometer pressures. Active hydraulic tests are carried out periodically in the deepest interval of the borehole.

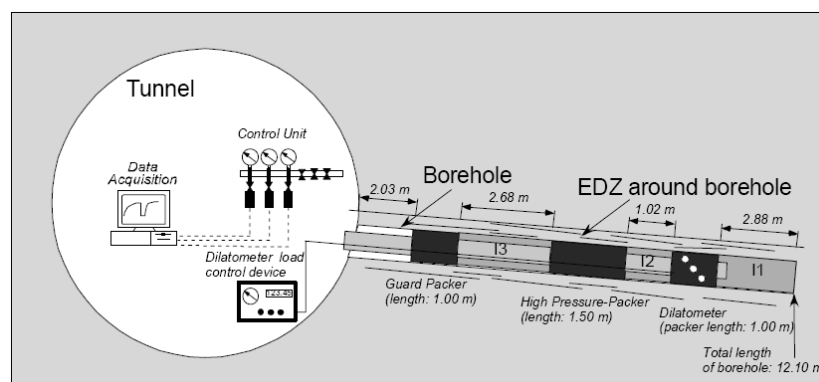


Fig. 1. Experiment layout long-term dilatometer experiment (Bühler, 2005)

Figure 2 summarizes the experiment process. It represents the evolution with time of the dilatometer pressure and the water pressures measured in I1, I2 and I3 intervals. This figure shows that the early pulse tests, realized in I1 interval, provide fast pressure reactions in the I2 neighboring interval between the dilatometer and the central packer. Since the dilatometer pressure is less than 3MPa, we observe in situ that I1 and I2 are hydraulically connected. This means that the contact between dilatometer and clay is not well performed and water can flow along the dilatometer from I1 to I2. As far as the dilatometer pressure increases (>3MPa), a hydraulic separation of the two intervals above and below the dilatometer is achieved. The contact between dilatometer probe and clay is well performed. I1 and I2 intervals are not anymore connected: during hydraulic test, the pressure in interval 1 increases, whereas the pressure in interval 2 gradually decreases. Transmissivity and hydraulic conductivity can be evaluated in the EDZ. When the dilatometer pressure is larger than 3MPa, we observe experimentally that transmissivity decreases with the increase of

dilatometer load on the borehole. It means that I2 time of reaction to the pressure modification in I1 is directly linked to the dilatometer load. The pressure in I1 increases with the dilatometer load. This is the consequence of the microcracks closure in the EDZ due to the dilatometer load. This leads to the conclusion, that at least some water flows parallel to the borehole along the dilatometer thanks to an excavation damaged zone along the borehole (Bernier et al., 2004).

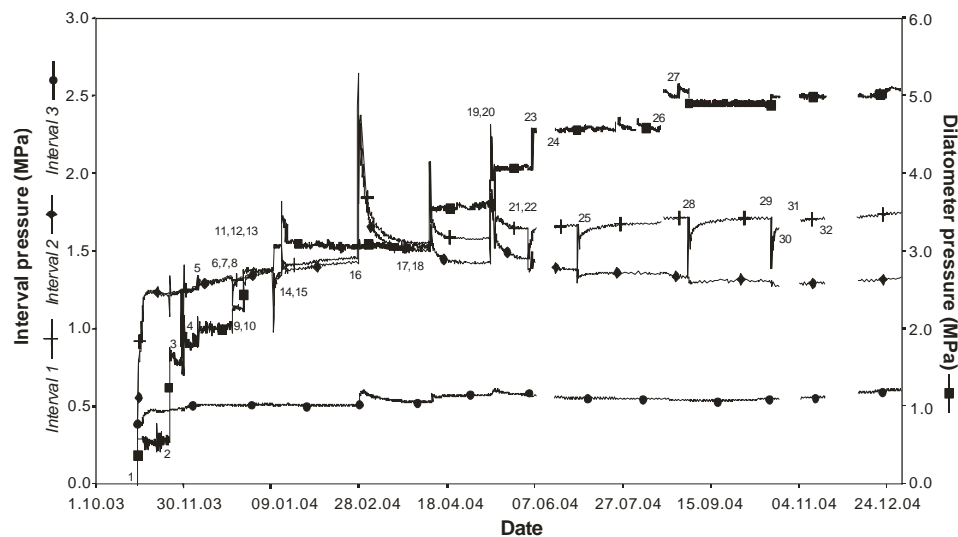


Fig. 2. Overview of the project progress (Bühler, 2005)

## 2.2 Opalinus clay

Hardened clay like Opalinus clay is an indurated clay that exhibit in its natural state very favorable conditions for the disposal of radioactive wastes. They generally have very low and uniform hydraulic conductivity, low diffusion coefficients and good retention capacity for radionuclides (Blüming et al., 2003). One concern regarding nuclear waste disposal is that the generally favorable properties of such formations could change and the host rock could loose part of its barrier function due to the disturbance and damage in the vicinity of the repository that result from the necessary excavations. Hardened clay behavior is transient between ductile and brittle. Therefore, discrete fractures are quite common in such materials. Field observations on hardened clay show that the EDZ contains discrete fractures and microfractures induced during excavation. The failure process can be of extensional or shear origin. According to Alheid et al. (2005) and Bossart et al. (2004), most discrete fractures in the EDZ in hardened clay are directly caused by unloading induced by the short-term excavation (undrained elasto-plastic response) and are of extensional nature. In fact, high deviatoric stresses closed to the tunnel wall open unloading joints (spalling) parallel to the tunnel wall without any sign of shear displacement. However, unloading joints are not isolated, but rather interconnected by shear fractures characterized by reactivated bedding planes. It results a more or less onion-like arrangement of fractures around the borehole where the hydraulic conductivity can be orders of magnitude higher than in the undisturbed clay.

## 3 HYDRO-MECHANICAL COUPLING MODELING AND PERMEABILITY EVOLUTION IN EDZ

A zone with higher transmissivity along the borehole, corresponding to the EDZ in the direct vicinity of the tunnel, is formed after drilling of the borehole due to the stress release and stress redistribution. This latter redistribution caused by underground excavation leads to

anisotropic permeability changes around this construction. As far as the low permeability is one of the desired properties for the host formation, all the processes influencing the diffusion property have to be understood and evaluated. In the literature, most of the studies concerning this topic propose to link the permeability tensor to stress or strain tensor. These developments are generally performed at the rock joint scale (Snow, 1969; Witherspoon et al., 1980; Oda, 1985; Liu et al., 1999; Chen et al., 2007; Giacomini et al., 2008). The objective of this paper is to extend these local scale models to larger scale in order to describe the global hydro-mechanical behavior in EDZ around tunnels.

### 3.1 Permeability tensor evolution with strain

Schematically, a crack can be seen as two parallel plates or a flat ellipsoid, like illustrated on figure 3. The flow modeling in this crack consists to consider an equivalent Darcy's media, in which the permeability value parallel to the crack plane (noted  $k^cr$  on figure 3a) is estimated by Poiseuille's solution:

$$k^{cr} = \frac{\lambda b^2}{12} \quad (1)$$

in which  $b$  corresponds to the crack width and  $\lambda$  is a roughness coefficient equals to one for an ideal flow and less than one when the head decreases because of crack roughness.

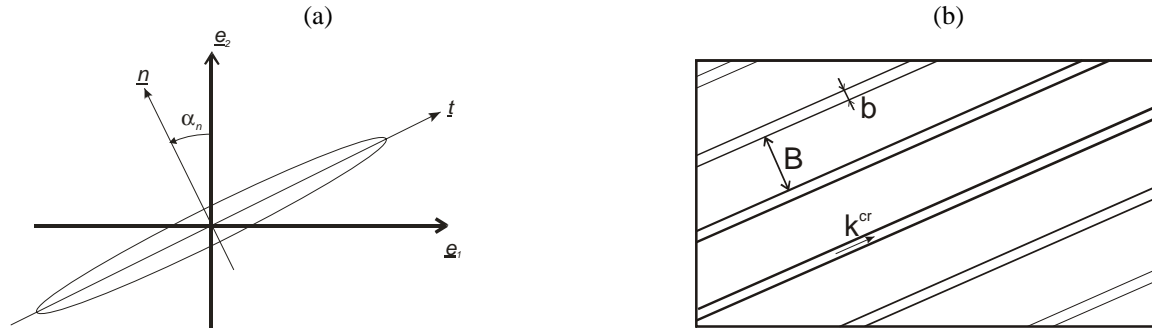


Fig. 3. Schematic representation of crack by an oriented ellipsoid (a), by parallel plates (b)

The permeability  $k_m$  of a medium with one set of cracks (figure 3b) can also be related to the ratio between the crack width  $b$  and the crack spacing  $B$  (Snow, 1969; Liu et al., 1999):

$$k_m = \lambda \frac{b^3}{12B} \quad (2)$$

$k_m$  is the permeability in the crack direction. Considering a general formulation in  $(e_1, e_2)$  axes, the permeability tensor becomes:

$$k_{ij} = \lambda \frac{b^3}{12B} [\delta_{ij} - n_i n_j] \quad (3)$$

with  $\underline{n}$ :  $\underline{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}_{(e_1, e_2)} = \begin{pmatrix} -\sin \alpha_n \\ \cos \alpha_n \end{pmatrix}_{(e_1, e_2)}$  the normal of cracks.

During excavation or borehole drilling, localized fractures are generated in the medium due to the stress release and stress redistribution. In this model, crack spacing  $B$  is assumed to remain constant while crack width  $b$  is assumed to evolve as:  $b = b_0 + \Delta b$ , with  $b_0$  the initial width and  $\Delta b$  the crack opening. Then, equation (3) becomes:

$$k_{ij} = \frac{\lambda}{12B} [b_0 + \Delta b]^3 [\delta_{ij} - n_i n_j] \quad (4)$$

$$k_{ij} = \frac{\lambda b_0^3}{12B} [1 + \Delta b / b_0]^3 [\delta_{ij} - n_i n_j] \quad (5)$$

in which the initial permeability in crack direction is equal to:  $k^0 = \frac{\lambda b_0^3}{12B}$

Localized fractures are mainly generated by traction in Opalinus clay. The crack opening over the initial width is assumed to be a linear function of tensile strain  $\varepsilon^T$  in the normal direction of the crack (Liu et al., 1999; Chen et al., 2007) and not to change during the loading:  $\Delta b / b_0 = A \varepsilon^T$ , with  $A$ , a non dimensional weight coefficient, which concentrates the crack properties, then:

$$k_{ij} = k^0 [1 + A \varepsilon^T]^3 [\delta_{ij} - n_i n_j] \quad (6)$$

This expression can be generalized for a medium with orthogonal sets of cracks oriented parallel to the principle strain directions. The global permeability tensor of the medium corresponds to the sum of the contributions of each n-crack:

$$k_{ij} = \sum_{n=1}^2 k_n^0 (1 + A_n \varepsilon_n^T)^3 \beta_{ij}(\alpha_n) \quad (7)$$

with  $k_n^0$  the initial permeability in the direction of the n-crack

$$\varepsilon_n^T = \langle \varepsilon_n \rangle = \begin{cases} 0 & \text{if } \varepsilon_n \leq 0 \\ \varepsilon & \text{if } \varepsilon_n > 0 \end{cases} \quad \text{with } \varepsilon_n \text{ the strain in the normal direction of the n-crack}$$

$A_n$  the weight coefficient associated to the n-crack properties

$$\beta_{ij}(\alpha_n) = \begin{pmatrix} 1 - \sin^2 \alpha_n & \cos \alpha_n \sin \alpha_n \\ \cos \alpha_n \sin \alpha_n & 1 - \cos^2 \alpha_n \end{pmatrix}_{(e_1, e_2)}$$

$\alpha_n$  the orientation of cracks, which is assumed to follow the same orientation during the loading and to correspond to the principle strain directions:

$$\alpha_n = \frac{1}{2} \arctg \left( \frac{2\varepsilon_{12}}{\varepsilon_{11} - \varepsilon_{22}} \right) + (n-2) \frac{\pi}{2} \quad \text{with } n = 1, 2$$

### 3.3 Hydro-mechanical finite element formulation

The hydro-mechanical finite element formulation follows the ideas of Lewis and Schrefler (2000), in which field equations are the mixture (solid skeleton and fluid phase) balance of momentum equation and the water mass balance equation. In this paper, the linearization of equations is not developed. The formulation can be found in Levasseur et al. (2009) and Collin et al. (2006).

## 4 NUMERICAL MODELLING OF DILATOMETER EXPERIMENT

### 4.1 Assumptions

The dilatometer experiment can be treated as a two dimensional problem in axisymetric condition. Even if Opalinus clay is known as an anisotropic porous medium, this initial anisotropy can be neglected compare to the drilling induced anisotropy of permeability tensor. So, in the model, clay is idealized as an isotropic medium in isothermal conditions. The 2D-axisymetric finite element model associated to this experiment is presented on figure 4.

The two main steps of the modeling are the excavation and long term dilatometer test. During the borehole drilling, the total stress and the pore pressure are decreased down to zero

at the borehole wall. During the dilatometer test, the boundary condition at the dilatometer probe and the packer corresponds to an impermeable condition for the flow problem and the dilatometer total pressure loading. During this second step, the two intervals are modeled by a highly deformable porous medium with a high permeability value. For the modeling, the Opalinus clay is supposed to be saturated. The initial conditions considered are listed in table 1. The Opalinus clay behavior is modeled by an elasto-plastic frictional model with a Van Eekelen criterion (Van Eekelen, 1980). The mechanical parameters of this model estimated from laboratory and in situ tests (Coll, 2005; Martin and Lanyon, 2003) are presented on table 2.

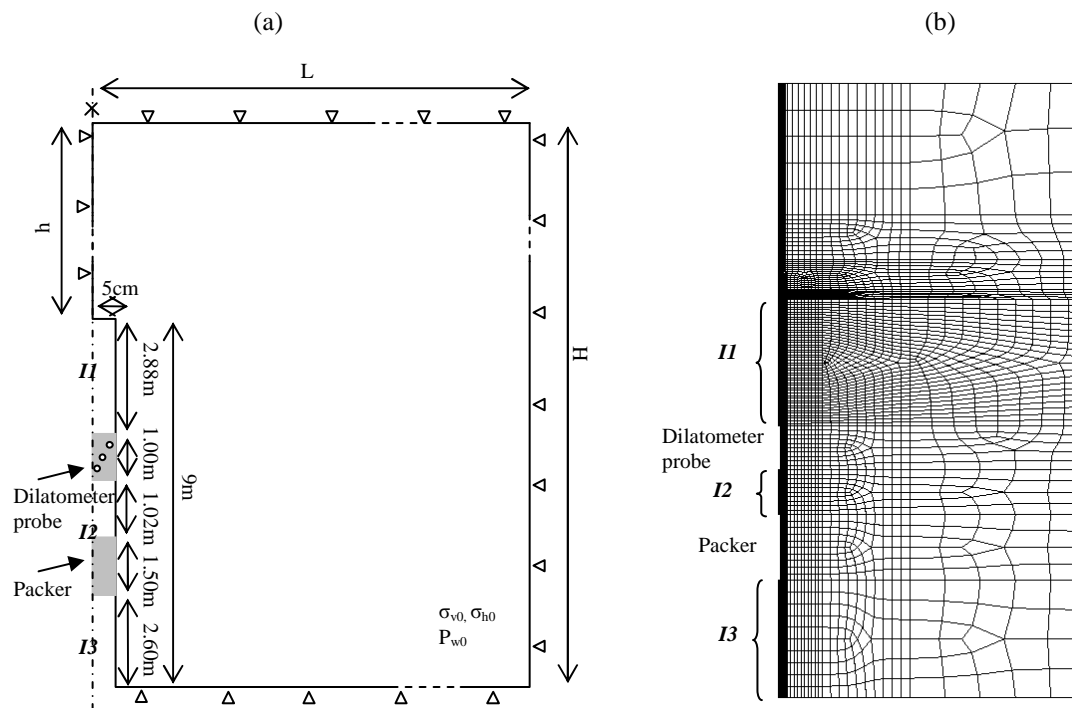


Fig. 4: Axisymmetric schematic representation of Mont Terri dilatometer test (a); associated 2D-axisymmetric finite element model (b – 4626 Elements; 13795 nodes)

Table 1. Initial state – stresses, pore water pressure

Initial state		Opalinus clay
Total stresses [MPa]	$\sigma_{v0}$	6.5
Pore pressure [MPa]	$P_{w0}$	2.0
Effective stresses [MPa]	$\sigma'_{v0}$	4.5

The weight coefficient  $A$  of Eq. (7) related to the coupling between permeability and strain is not directly known. This coefficient, which characterized the micro-structural properties of the medium, is obtained by trial and error in order to match the experimental measurement of transmissivity. Because of the hypothesis of isotropic medium, the orthogonal cracks are assumed to have the same properties. Coefficient  $A$  is the same for these both sets of cracks and has been estimated to  $6 \cdot 10^4$ . According to the numerical predictions of the tensile strains, these values impose a modification of around 5 orders of magnitude on axial hydraulic conductivity and quite no modification on radial hydraulic conductivity. This is in agreement with the hydraulic conductivity of EDZ fractures, which is many orders of magnitude higher in axial direction (5 to 8 orders), compare to undisturbed Opalinus clay (Bossart et al., 2004).

Table 2. Geomechanical and Hydraulic characteristics of Opalinus clay

Geomechanical characteristics		Opalinus clay
Young elastic modulus [MPa]	$E_0$	10 000
Poisson ratio [-]	$\nu$	0.270
Specific mass [kg/m <sup>3</sup> ]	$\rho$	2450
Initial cohesion [kPa]	$c$	2200
Initial friction angle [°]	$\phi_c$	25
Dilatation angle [°]	$\psi$	0
Biot's coefficient [-]	$b$	1
Hydraulic characteristics		
Initial porosity	$\phi_0$	0.137
Initial intrinsic permeability [m <sup>2</sup> ]	$k_0^{int}$	$2.10^{-20}$
Water specific mass [kg/m <sup>3</sup> ]	$\rho_f$	1000
Fluid dynamic viscosity [Pa.s]	$\mu_f$	$10^{-3}$
Liquid compressibility coefficient [MPa <sup>-1</sup> ]	$1/\chi_f$	$5.10^{-4}$

#### 4.2 Numerical results

The first step of this modeling is the borehole drilling. Figure 5 shows that the borehole drilling induces the development of tensile strains within the excavated damaged zone which impose an anisotropic permeability tensor in EDZ. During the second step of modeling, the dilatometer and the packer are installed in the borehole and their pressure increases to 3MPa. This operation has a direct impact on the radial strains and the axial conductivity: the tensile strains decrease and the axial permeability is reduced near the dilatometer with its loading. The effect of the dilatometer loads can not be noticed in such amplitude in other part of the borehole. The following phases of this modeling consist to alternate increases of dilatometer load with hydraulic tests. For each phase, the dilatometer load is increased by step of 0.5MPa in order to reach 5MPa at the end of the experiment. Then, hydraulic tests are performed from I1 interval. Because of injection, the pressure in I1 first increases and this overpressure progressively diffuses into the host rock. As permeability is larger in the direction parallel to the borehole than in the radial direction, the water flows out along the dilatometer to finally reach I2 interval and the pressure in I2 increases. Figures 5b characterizes the evolution of the axial permeability after each dilatometer loading along the borehole. We observe that the axial permeability value near the dilatometer is influenced by the dilatometer load. Larger is the dilatometer load, more the permeability decreases. This means that from a given load, the two intervals are disconnected.

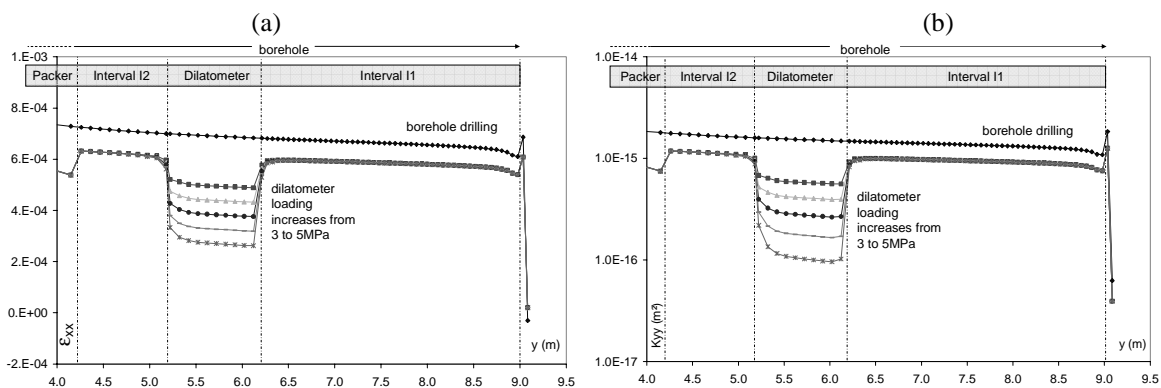


Fig. 5: Radial strains and axial permeability  $K_{yy}$  along the borehole after the borehole drilling and for different dilatometer load values.

Figure 6 presents the overpressure evolution in I2 interval after hydraulic tests. Experimentally, it has been demonstrated that the delay of this pressure reaction in interval 2 depends on the applied load onto the borehole wall (Bühler, 2005). Figure 6 shows that the proposed numerical model reproduces this effect: higher is the dilatometer load, later is observed overpressure in I2. The dilatometer load delays the pressure reaction in the adjacent test interval I2. Then, this modeling well characterizes the dependence between transmissivity and dilatometer loads. Finally, figure 7 compares the level of water overpressures measured in I1 and I2 during the experiment with the numerical predictions. This figure shows a good agreement between experiment and modeling. It proves that the link between transmissivity evolution and inflation pressure of the dilatometer is well reproduced in this model.

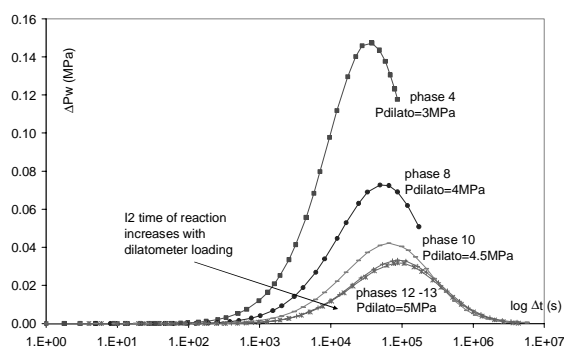


Fig. 6: Numerical results on pressure time of reaction in I2 to pulse test with dilatometer load

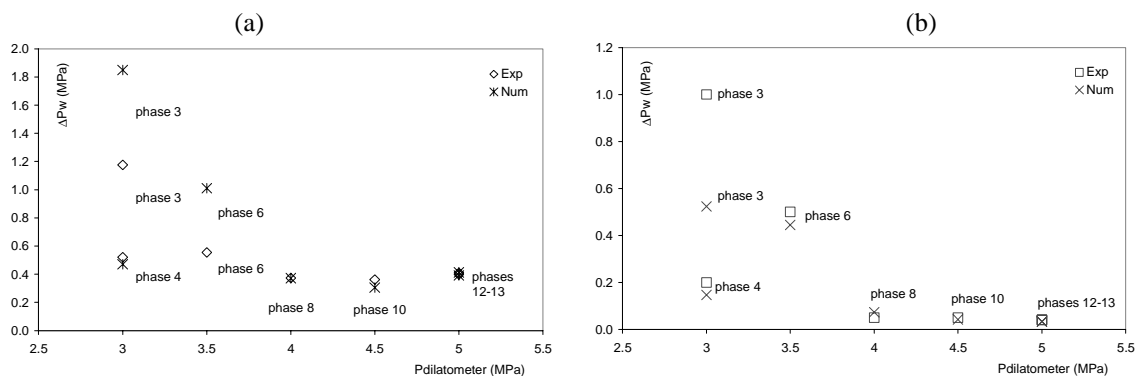


Fig. 7: Comparison between in-situ measurements and numerical responses of water pressure modifications in I1(a) and I2(b)

## 5 CONCLUSIONS

The excavation damage zone is a phenomenon that occurs in the most rock masses as a consequence of underground excavation. The EDZ appears as an area around the underground openings, where geotechnical and hydro-geological properties are altered. The numerical model should be able to predict the evolution of the permeability in the EDZ, in order to permit a correct design of the geo-structure. This paper presents first a laboratory test that evaluates the permeability changes occurring within the EDZ and that more specifically studies the influence of bentonite swelling pressure on the axial transmissivity of an excavation damage zone in the Opalinus clay. The experiment shows that the hydraulic conductivity of the EDZ is a function of the inflation pressure of the dilatometer. A constitutive model is then proposed to predict the evolution of the permeability within the



EDZ. In hardened clay, like Opalinus clay, EDZ is mainly characterized by extensional fractures and the proposed model relates the conductivity changes to the crack aperture in traction mode. Using an additional hypothesis on the link between crack opening and the principal strain tensor, our model is able to predict the anisotropic evolution of the permeability tensor during the excavation. Moreover, when dilatometer pressure increases, the axial permeability decreases. No significant water flow can evolve in EDZ behind the dilatometer. The comparisons between numerical predictions and measurements of the pressures in the intervals exhibit a good agreement and confirm that our model is able to catch the main hydro-mechanical processes occurring within the EDZ.

As EDZ in hardened clay, like Opalinus clay, is mainly characterized by extensional fracture, this proposed hydro-mechanical modeling of the excavation damage zone around underground excavation in hardened clay well reproduces the global behaviors observed in situ. However, in many geologic formations, shear cracks also play an important role. To take them into account, it will be necessary in the future to improve our approach by developing a homogenization model based on micromechanical concepts.

## ACKNOWLEDGEMENT

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