The Main Features and Characteristics of the 3-D Groundwater Finite Element Model in the Central Area of Shanghai

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ABSTRACT

According to the "two-step" method, the land subsidence model of Shanghai is composed of a flow part where the flow equation is solved in a "full" three-dimensional model, and a second part where the assumed one-dimensional deformations are computed by a coupled and non-linear flow-compaction model. After each time step, the results of the 3D flow model provide the time dependent boundary conditions of the 1D flow-compaction model. This last model is using the oedometric elasto-plastic law, coupled with vertical flow, assuring the variation of the storage coefficient in function of the changing void ratio and including linear and non-linear analysis of the vertical permeability. The main features and characteristics of the model are described and analyzed. Some results are presented and their reliability is discussed with regard to the chosen boundary conditions. As conclusions, the possible improvements are suggested.

PREVIOUS STUDIES

The city of Shanghai is situated in the vast low-lying coastal plain (usually called the Yangtze delta) characterized by the lower reach of the Yangtze river (Baeteman & Schroeder, 1990). Due to ground-water withdrawal, mainly from an aquifer situated between 60 and 80 m of depth, land subsidence occurred drastically. It was noticed as from 1921 but it reached 2.5 m to 3 m with a maximum annual rate of 98 mm between 1956 and 1959. The total cumulative subsidence given by Su (1984) and Shi & Bao (1979), shows a stabilization from 1963—1965 until 1985, due to the intensive recharge in the second aquifer in winter. Before the recharge has begun the pore pressure maps showed two plate-shaped depressions in the urban district, in accordance with the location of the main pumping. The lowest pore pressures in the subsoil of Shanghai were reached in 1960, before any recharge. After 1965, a relative stabilization of the subsidence is obtained after the small elastic rebound, since this date a residual subsidence about 3 mm / year is still recorded and a lot of measurement data have been collected by the Shanghai Geological Center confirming this fact. Although the total thickness of the loose sediments is about 300m, the observation data have indicated that 65% to 85% of the total subsidence occurred in the upper 70m portion.

NEW DATA

Intensive investigations of the Quaternary geology, the hydrogeology and the engineering geology were carried out simultaneously, collecting new data and numerous old data from Shanghai geologists and engineers. This work has been completed together by the Shanghai Geological Center (P. R. China), the Belgian Geological Survey and the Laboratory of Engineering Geology, Hydrogeology and Geophysical Prospecting of the University of Liège (Belgium). Very detailed data were thus available concerning the coastal lowland geology, the engineering geology and the hydrogeology, to provide the basic elements for the design of the mathematical model (Dassargues *et al.* 1990). The geological setting of the Shanghai area (until a depth of about 70 m) has been subdivided into significant lithological units (Fig. 1) on basis of environmental analysis (Baeteman & Schroeder, 1990). The main hydro-engineering geology characteristics are summarized in the papers of Dassargues *et al.* (1990) and Bateman & Schroeder (1990).

sedimentary environment		lithological unit	
fluvial	estuarine & coastal		
	intertidal and subtidal	1c unit	Holocene
	salt marsh (supra tidal)	2c unit	
flood basin and backswamp		DGSC unit	Upper
natural levee and channel		1A upper unit	
	channel and sand bars	1A lower unit	Pleistocene
	intertidal and subtidal	3C unit	
	channel and sand bars	2A unit	

Fig. 1 Stratigraphic sequence of the Pleistocene and Holocene deposits (from Baeteman & Schroeder, 1990)

MAIN CHARACTERISTICS OF THE MODEL

The land subsidence model of Shanghai includes a transient 3D flow model and a coupled non–

linear 1D flow–compaction model. The results of the flow model are the time dependent boundary conditions of the consolidation model. The models are implemented in the finite element code called LAGAMINE developed by the MSM department, University of Liège, since 1982. Large strains, both geometrical and material non–linear problems, mechanical problems, thermal conduction, seepage, ... can be modeled using this code (Charlier *et al.* 1988). It has been successfully applied in 1985—1987 to calculate the subsidence of the Ekofisk oil–field (Schroeder *et al.* 1988).

a) *The Flow Model*

The finite element method, based on the virtual power principle is applied. The internal virtual power (δW_i) and the external virtual power are expressed for a seepage problem (Charlier, 1987). The Gaussian numerical integration scheme is used on isoparamertic 8–nodes brick–like elements.

Two constitutive laws are used: Darcy's law gives (Charlier *et al.* 1988):

$$f = -\frac{K}{\gamma_w} grad\ p + grad\ z \tag{1}$$

where K is the permeability tensor, γ_w the specific weight of water, p the pore pressure and z the vertical coordinate.

For the 3D flow model, the law is considered to be linear: K is isotropic and assumed to be constant. We suppose also that the the volumetric strain variation f^v is equal to the volume cf water expelled during the compaction ('storage flow') :

$$f^v = \dot{V}/V = 3\dot{\varepsilon}_m \tag{2}$$

Using the Terzaghi principle expressed with mean stresses, $\sigma_m = \sigma'_m - p$ is written with tensions as positive stresses. If σ_m is supposed to be constant, we obtain $\dot{\sigma}' - \dot{p} = 0$ and and $\dot{\sigma}' = \dot{p}$. Equation (2) can be written:

$$f^v = 3\dot{\sigma}'_m/\chi = 3\dot{p}/\chi = C_p . \dot{p} \tag{3}$$

C_p is the storage coefficient expressed in terms of pressure (Dassargues *et al.* 1988), $C_p = 3/\chi$ with $\chi = E/(1-2\nu)$

Time integration must be realized in the transient flow problem; a Galerkin time scheme has been used for the present model .

b) *Coupled flow-compaction finite element model* (Charlier *et al.* 1990)

The settlement of a "plate–like" aquifer system can be considered as essentially a unidimensional vertical problem. Hereafter, we use the Cauchy stress tensor σ and the Cauchy conjugated strain tensor ε. For the 'oedometric' behaviour of the clayey soils, the effective stress tensor σ and strain tensor are reduced to:

$$\begin{gathered}\sigma'_{11} \neq 0,\ \sigma'_{22} = \sigma'_{33} \neq 0 \text{ and } \sigma'_{12} = \sigma'_{23} = \sigma'_{31} = 0 \\ \varepsilon_{11} \neq 0,\ \varepsilon_{22} = \varepsilon_{33} = 0 \text{ and } \varepsilon_{12} = \varepsilon_{23} = \varepsilon_{31} = 0\end{gathered} \tag{4}$$

The virtual internal mechanical power is easily obtained by:

$$\delta W_{im} = \int_v \sigma_{ij} \delta\varepsilon_{ij} dv = \int_v \sigma_{11} \delta\varepsilon_{11} dv \tag{5}$$

The axial strain rate of an element is: $\dot{\varepsilon}_{11} = \dot{L}/L_0$,

L represents the length variation rate, L_0 is the initial length of the "element". Only flow in the vertical directilon is allowed in oedometer tests, so that the expression of the internal virtual flow power (δW_{if}) is reduced in consequence. We suppose that the elements are only subjected to gravity and the external virtual power (δW_E) is written consequently. Based on the virtual power principle, equilibrium is obtained when $\delta W_{im}+\delta W_{if}=\delta W_E$ and nodal equivalent forces and flows are then easily got using the Gaussian numerical integration scheme (Charlier *et al.* 1990). A fully implicit time integration scheme has been adopted.

The following constitutive laws are concerned:

a) The Terzaghi principle (Terzaghi, 1943) : $\sigma'=\sigma+p$ (6)

b) The oedometer mechanical law (illustrated Fig. 2) can be transformed to an elastoplastic incremental form as follows:

$$\begin{array}{ll} \dot\sigma'_{11}=-\dot\sigma'_{11}\cdot A\cdot\dot\varepsilon_{11} & \text{if } |\sigma'_{11}|<|\sigma_c| \\ \left.\begin{array}{l}\dot\sigma'_{11}=-\sigma'_{11}\cdot C\cdot\dot\varepsilon_{11}\\ \dot\sigma_c=\dot\sigma'_{11}\end{array}\right\} & \text{if } |\sigma'_{11}|=|\sigma'_c| \\ |\sigma'_{11}|>|\sigma'_c| & \text{impossible} \end{array} \qquad (7)$$

where σ'_c is the extreme stress encountered by the soil through its whole history, σ'_{11} represents the yeld threshold, it is an internal variable .

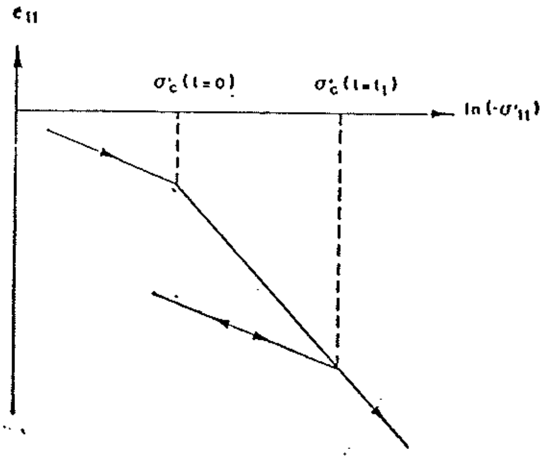

Fig. 2 Oedometer mechanical response'

c) The storage is equal to the volumic strain rate because water and soil grains are assumed to be incompressible (equation 3) .

d) The Darcy's law with a non-linear relation concerning K is inducted. The chosen variation law was developed originally by Nishida & Nakagawa, 1969. Its transformed from is written (Dassargues, 1990): $K = \exp(\alpha_N e+\beta_N)$ (K in m / s) (8)

where e is the void ratio, α_N and β_N are new material parameters related to plasticity index I_p and are obtained from laboratory tests providing I_p, e and K of the same samples.

We assume that volume variation is only due to void variation:

$$\dot V_{total}=\dot V_v=\dot\varepsilon(V_v+V_s)$$

$$\dot e = \dot V_v / V_s = \dot V_{total} / \dot V_s = \dot\varepsilon_{11}\cdot(V_v+V_s) / V_s = \dot\varepsilon_{11}(1+e) \qquad (9)$$

In summary, the relations mentioned above form, non-linear constitutive coupled laws. Where

A, C, α_N, β_N, γ_S, γ_w, are the parameters which can be determined from laboratory test and σ, σ' σ'_c and e are internal variables.

The Newton–Raphson technique is used to find the solution.

MODELING THE SHANGHAI SUBSIDENCE

The 3D flow model mentioned above with a complete discretization of the soil layers is implemented in the Lagamine code to simulate the spatial distribution of the pore pressure in function of time. The study area is divided into 10 layers of 205 8-nodes brick elements each. Fig. 3 shows the discretization pattern in one of the vertical cross-section.

The spatial distribution of the two parameters (K and S_s) is practically realized by using material classes for which the parameters are defined. Fourteen classes of materials are used in the model.

In our analysis, initial state of pore-pressure is chosen in geostatic equilibrium corresponding to the situation before 1920. The boundary are assumed impervious at the bottom and at the top of the model, and imposed varying pressure with time laterally. During "trial and error" calibration, the K and S_s values determined by various soil test data and several pumping test data, have been slightly modified.

The choice of the time step is mainly based on the measurements frequency for the pumping recharge data.

The subsidence computations are completed with a 1D coupled flow–compaction model. In our study area, 32 columns are chosen to caluclate the subsidence, each column containing 60 elements. The simulation is carried out with the water pressures (obtained by the flow–model), at the aquifer / aquitard boundaries. These prescribed water pressure are variable in time. Moreover, the non-linear variations of the permeability coefficient K and the specific storage coefficient S_s are taken into account in the model. Both K and S_s are actualized at each time step. The variation of K is performed by adapted Nishida relation and the S_s varies in the following way(Poland, 1984 and Dassargues, 1990):

$$S_s = \rho . g . \alpha \qquad (\alpha\text{: compressibility of the porous media}) \tag{10}$$

$$\text{and} \begin{cases} S_s = \gamma_w / A \cdot \sigma'_{11} \text{ (in the elastic part)} \\ S_s = \gamma_w / C \cdot \sigma'_{11} \text{ (in the plastic part)} \end{cases} \tag{11}$$

Unfortunately, we have only a small amount of "target points" available for the calibration before 1965, moreover the recorded subsidence is relative to the compaction of the 300 meters of loose sediments and the part due to the upper 70m is not known with accuracy. During the calibration, the computed subsidence are checked to be comprised in the ranges of the measured subsidences. Of course, the reliability of detailed results is affected by this fact. Some of the computed results are shown on Fig. 4.

A simulation of the future water pressure distribution is completed with pumping = 1.3× recharge, and computed future subsidences between 1988 — 2000 are found out. The computed additional compactions are comprised between 1.4 and 7.9 cm.

The only conceptual remark that could be formulate is concerned with the lateral boundary d σ, σ'

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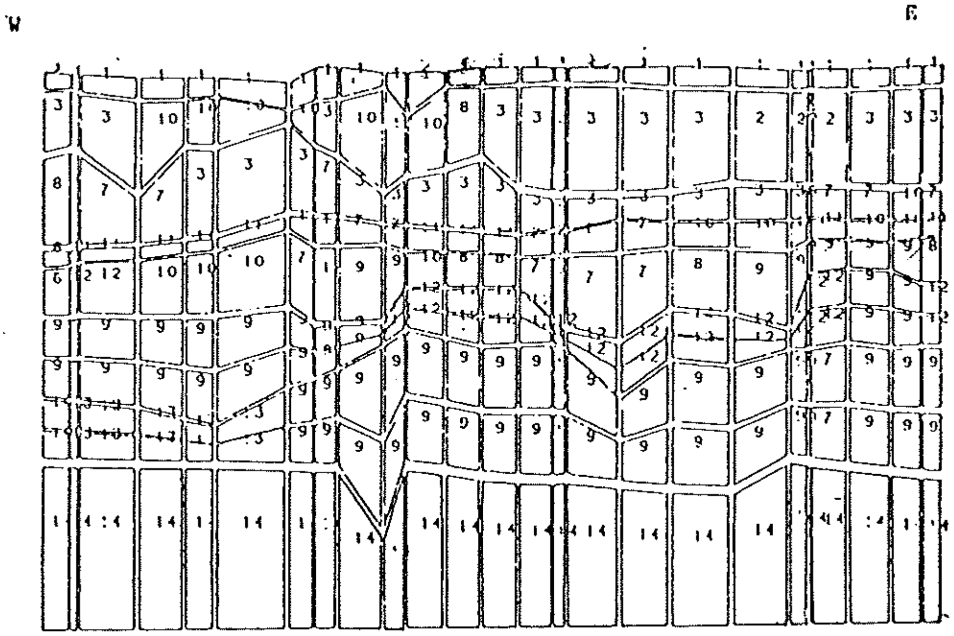
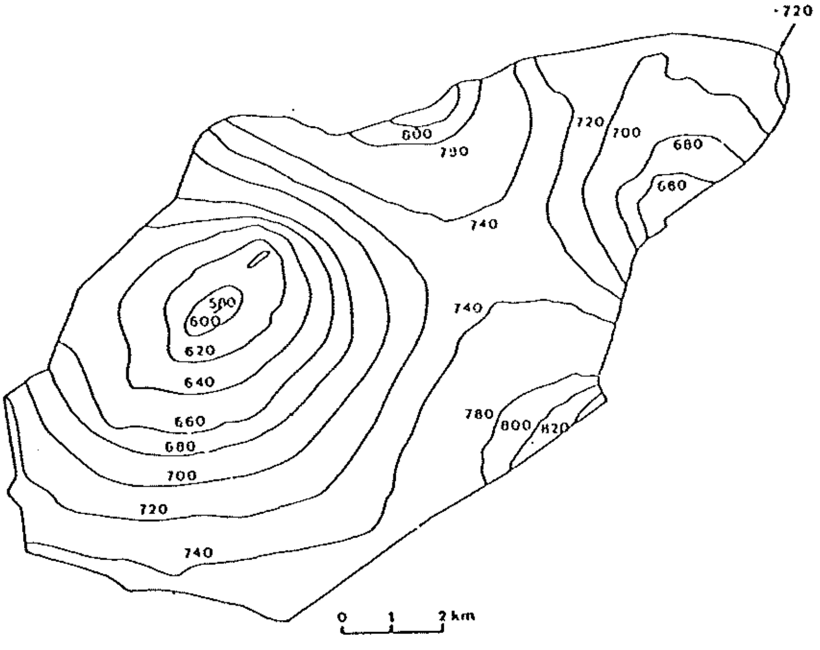


Fig. 3 One of the vertical cross-section in the 3D mesh.



Pore pressure in kPa

Fig. 4a Computed pore pressure map for the situation in 1960 conditions: how to take into account the actual lateral flow conditions ? It would be necessary to collect more informations about the hydrogeological parameters and stresses (pumping / recharge) in a large zone surrounding the studied one. Then a lateral extension of the model could be considered with prescribed potentials at lateral boundaries pushed away to the infinity in regard to the main stressed zone. Future works could be directed to this problems.

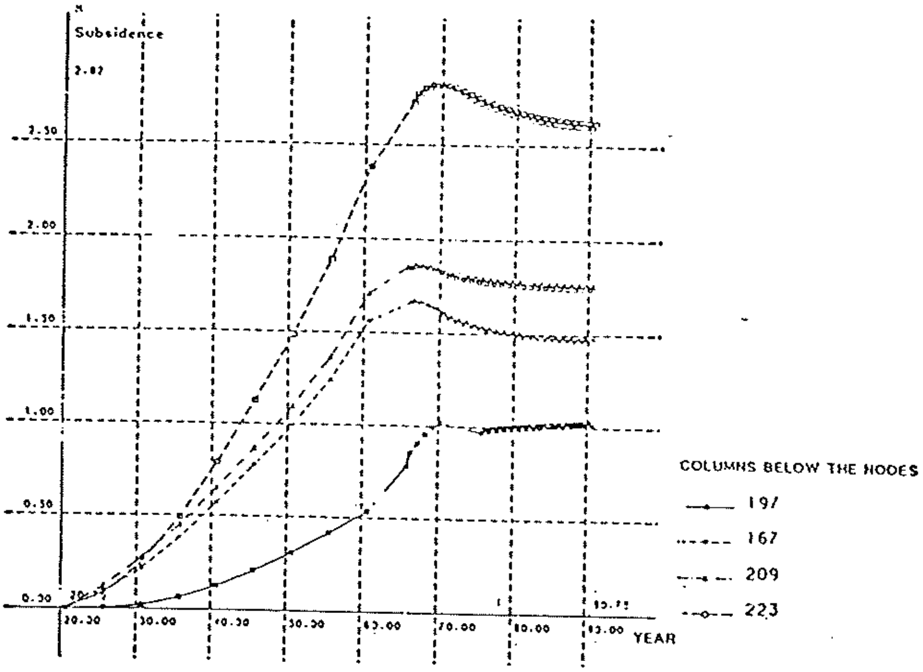


Fig. 4b Computed subsidence vs time in 4 of the 32 columns

ACKNOWLEDGEMENTS

This work has been carried out under an International Cooperation Agreement between the Ministry of Geology and Mineral Resources of the People's Republic of China and the Services of Scientific Policy and Planning of the Prime Minister of Belgium. The authors are grateful to the Chinese engineers of the Shanghai Geological Bureau for their participation in the works and discussions during the whole study. Thanks must also be expressed to Dr. Baeteman of the Belgian Geological Survey for the detailed Quaternary analysis.

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