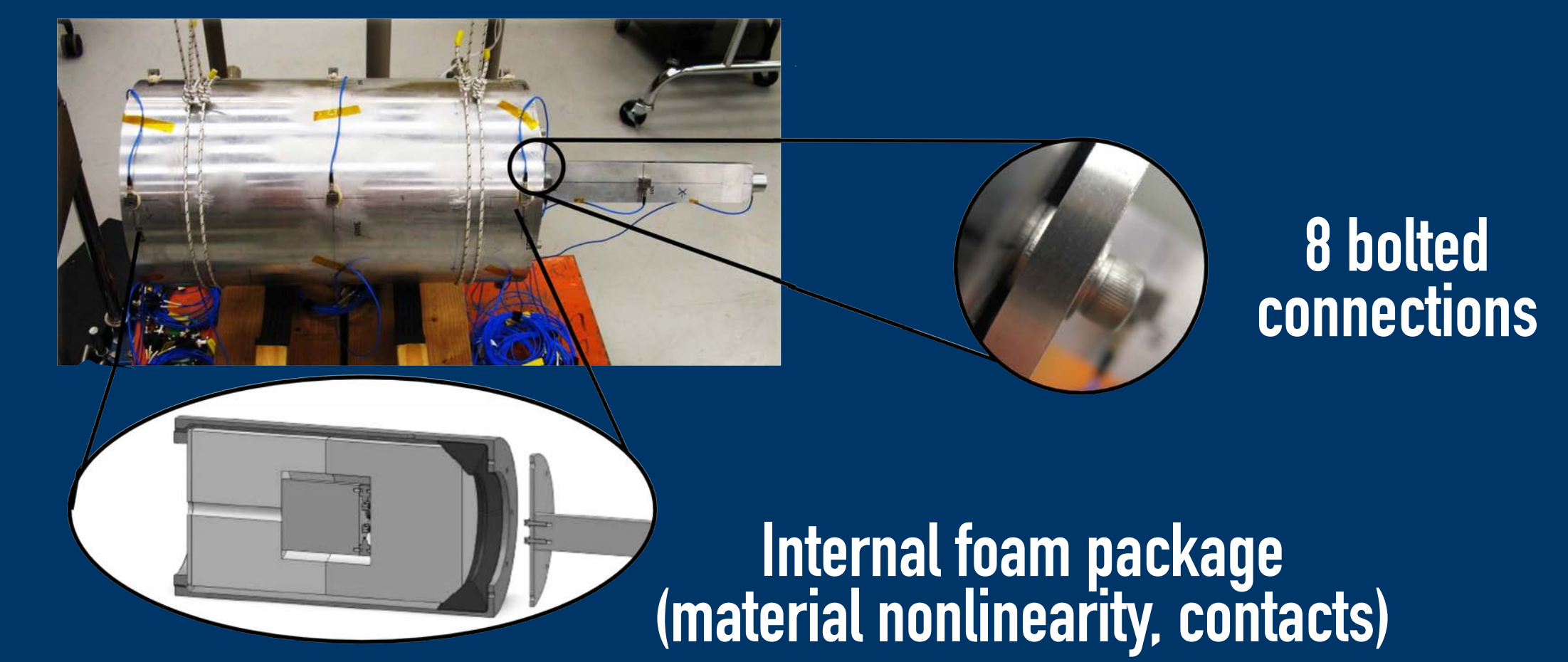


Reduced Order Modeling Techniques in Experimental Dynamic Substructuring



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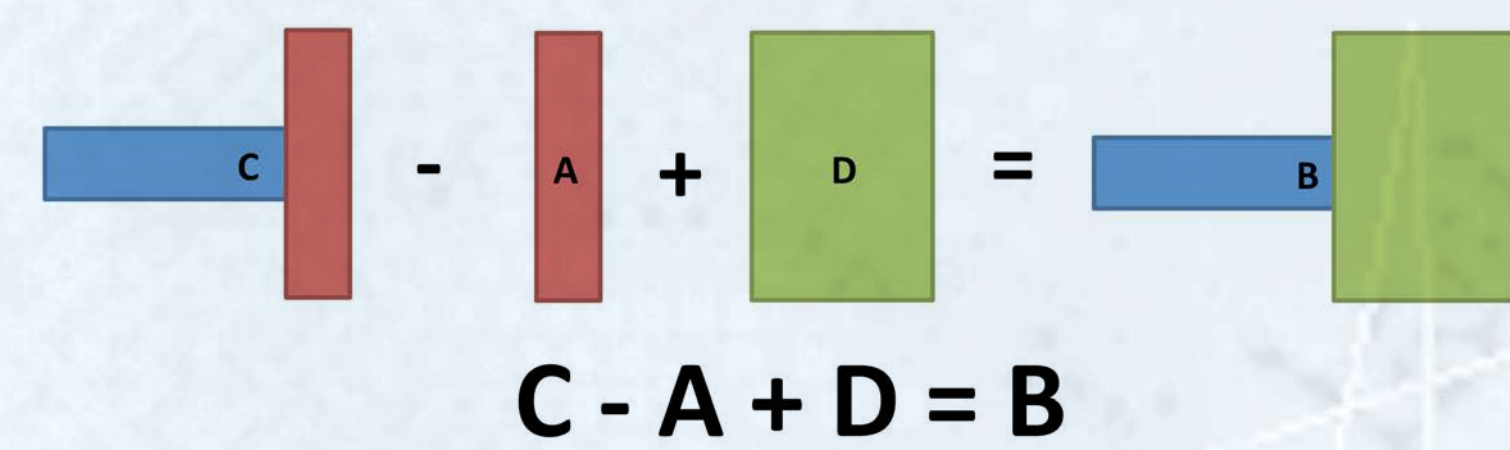
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Objective

- Compare different experimental substructuring techniques using two different systems
- Discover the best practices for experimental-numerical substructuring

Substructuring Theory

- Component Mode Synthesis (CMS) is used to combine two substructures to predict the dynamic response of the assembly
- This is useful when testing a full assembly is impractical or trying to analyze the effects of changing out different sub-assemblies
- These predictions can be very sensitive to interface errors where two substructures are joined
- In order to exercise the joints as seen in the assembly the experiment can be connected to known fixture or transmission simulator (TS)
- The Craig-Mayes method uses the transmission simulator theory to create an experimental Craig-Bampton like form of the experimental results



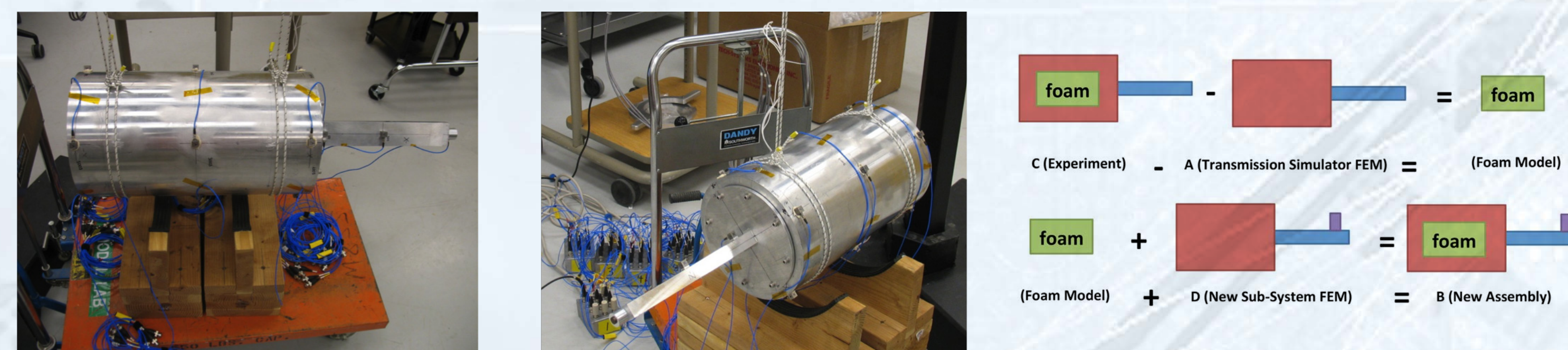
- In the transmission simulator method a known transmission simulator (A) is subtracted from the experimental subsystem (C)
- Next a new subsystem (D) is added to the result creating the target assembly (B)

$$\begin{bmatrix} \Phi_c & 0 & 0 \\ 0 & I_D & 0 \\ 0 & 0 & -I_A \end{bmatrix} \begin{bmatrix} \hat{q}_c \\ \hat{q}_D \\ \hat{q}_A \end{bmatrix} + \begin{bmatrix} 2Z_{cA} & 0 & 0 \\ 0 & 2Z_{cD} & 0 \\ 0 & 0 & -2Z_{cA} \end{bmatrix} \begin{bmatrix} \hat{q}_c \\ \hat{q}_D \\ \hat{q}_A \end{bmatrix} + \begin{bmatrix} \omega_c^2 & 0 & 0 \\ 0 & \omega_D^2 & 0 \\ 0 & 0 & -\omega_A^2 \end{bmatrix} \begin{bmatrix} \hat{q}_c \\ \hat{q}_D \\ \hat{q}_A \end{bmatrix} = \begin{bmatrix} \Phi_c^T F_c \\ \Phi_D^T F_D \\ \Phi_A^T F_A \end{bmatrix}$$

$$\begin{bmatrix} \Phi_c & 0 & 0 \\ 0 & \Phi_A \end{bmatrix} \begin{bmatrix} \hat{q}_c \\ \hat{q}_A \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Experimental System

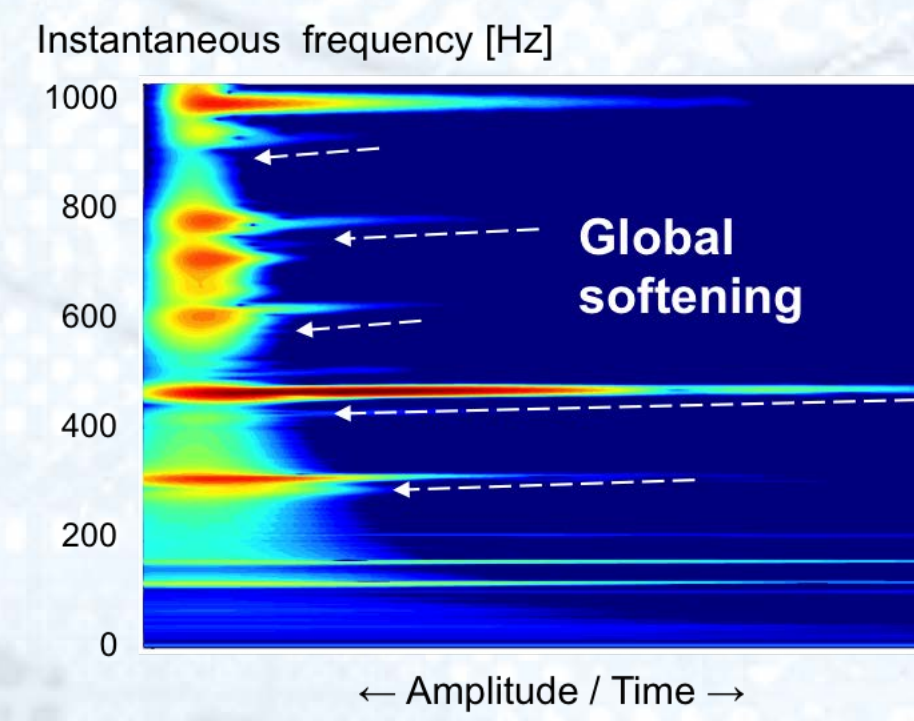
- Experimental system consists of the can-plate-beam system packed with foam and some internal instrumentation pieces
- Testing complete with low-level excitations to avoid nonlinearities in the system
- 14 Elastic modes extracted from experimental data



Nonlinearity Characterization

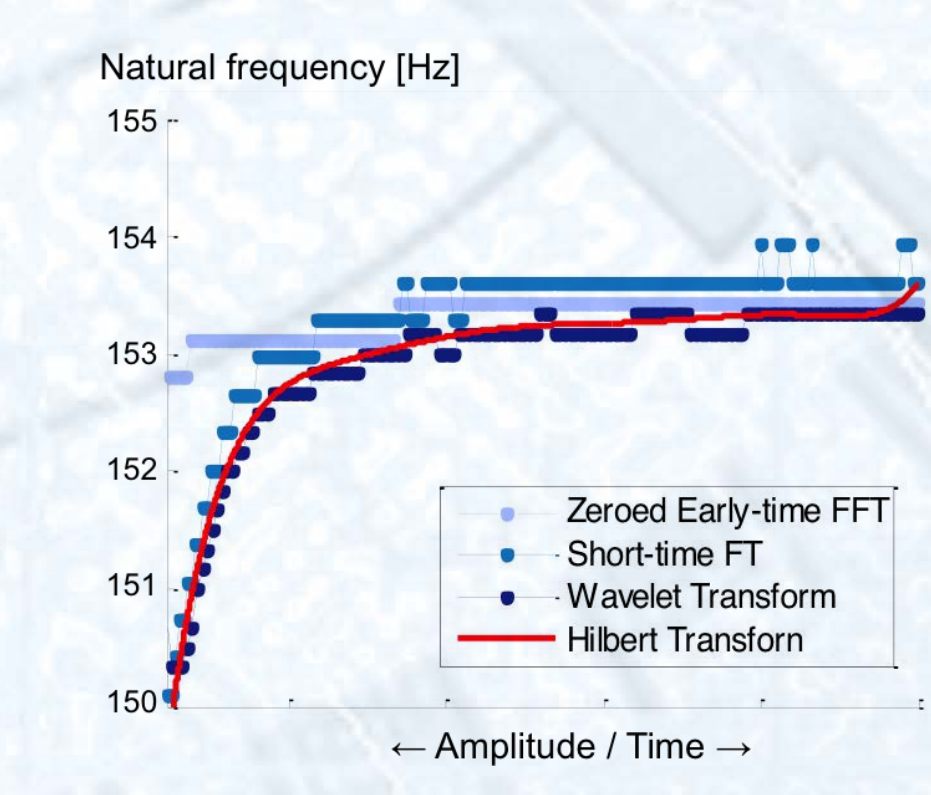
DETECTION - Wavelet Transform
Most nonlinear modes (exercising the joints)

- Mode 1: First bending mode of the beam in the horizontal plane
- Mode 2: First bending mode of the beam in the vertical plane
- Mode 6: Axial mode, beam and internals out-of-phase

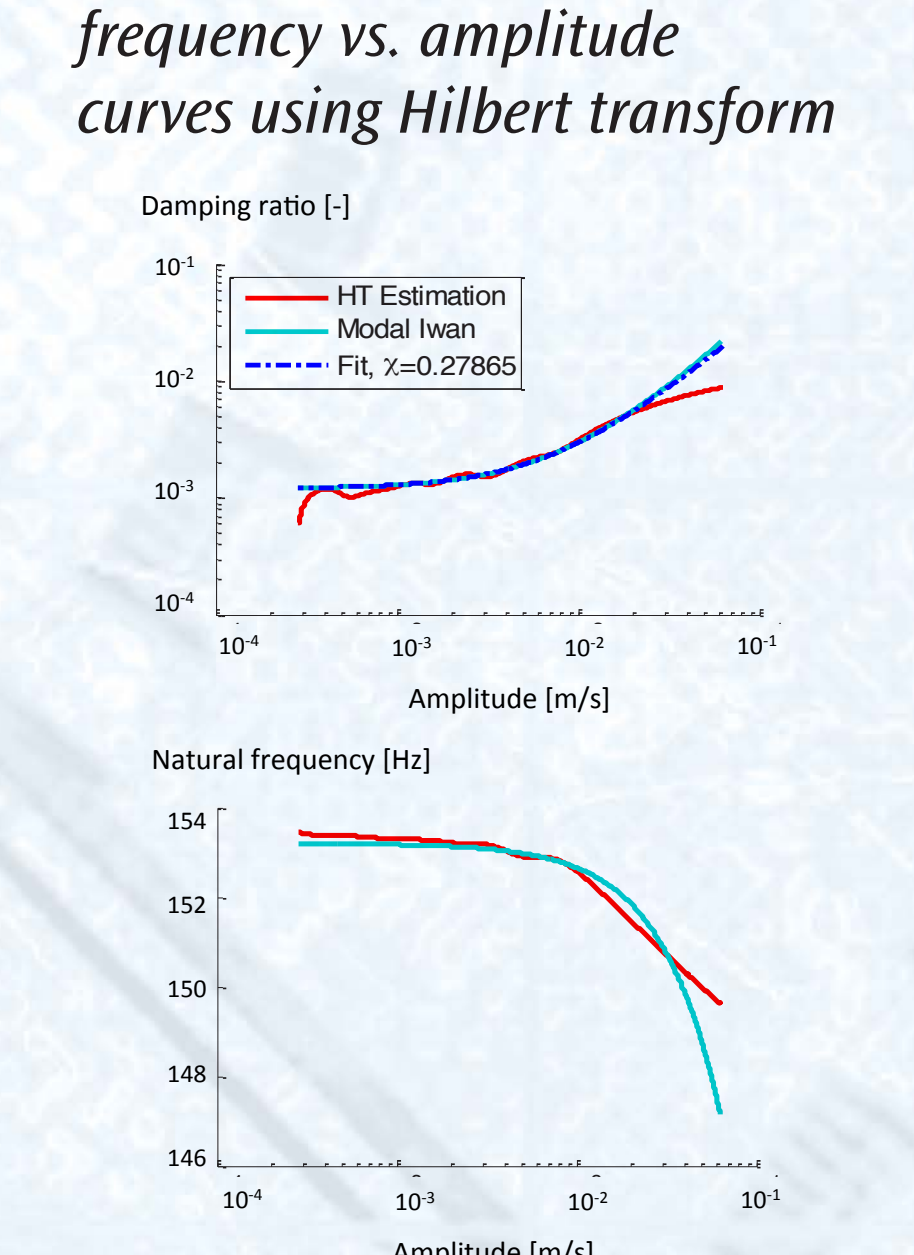


CHARACTERIZATION & QUANTIFICATION
Amplitude $\hat{\Lambda}$ \Rightarrow Natural frequency $\hat{\omega}$
Amplitude $\hat{\Lambda}$ \Rightarrow Damping ratio $\hat{\zeta}$
Natural frequency variations

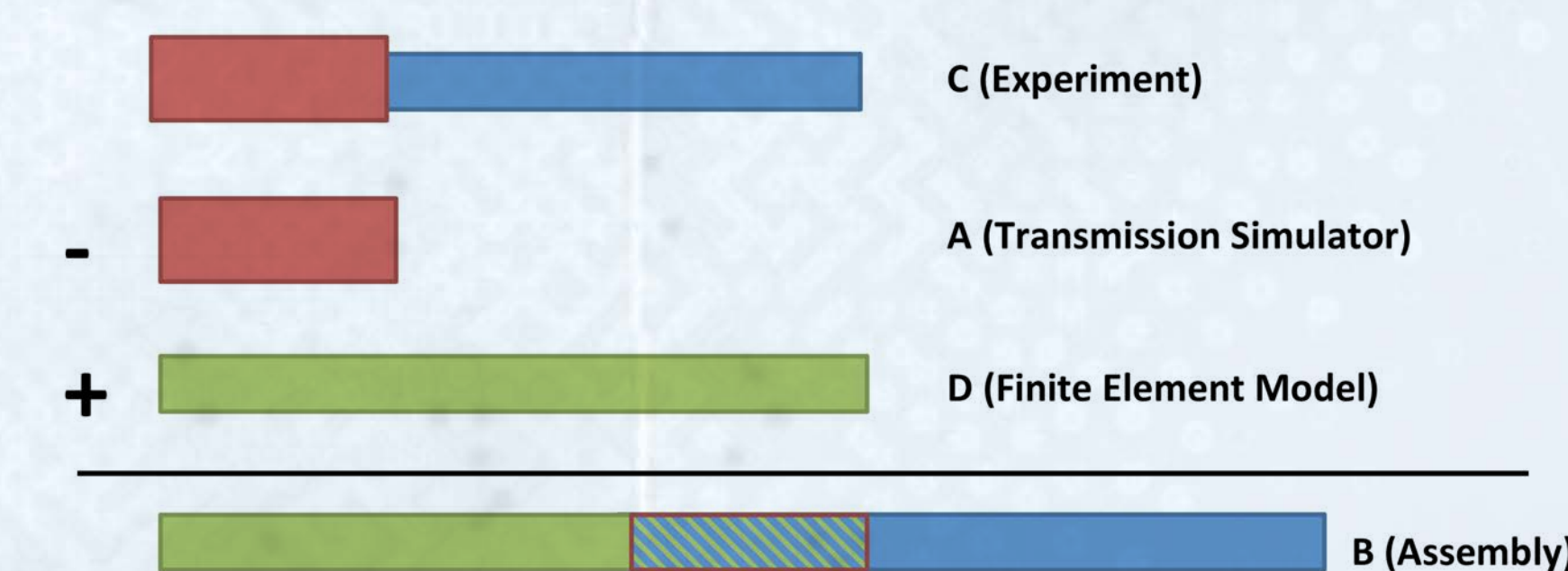
- Mode 1: -4%
- Mode 2: -3%
- Mode 6: -2.5%



PARAMETER ESTIMATION
Fitting a modal Iwan model on damping and natural frequency vs. amplitude curves using Hilbert transform



Beam Example

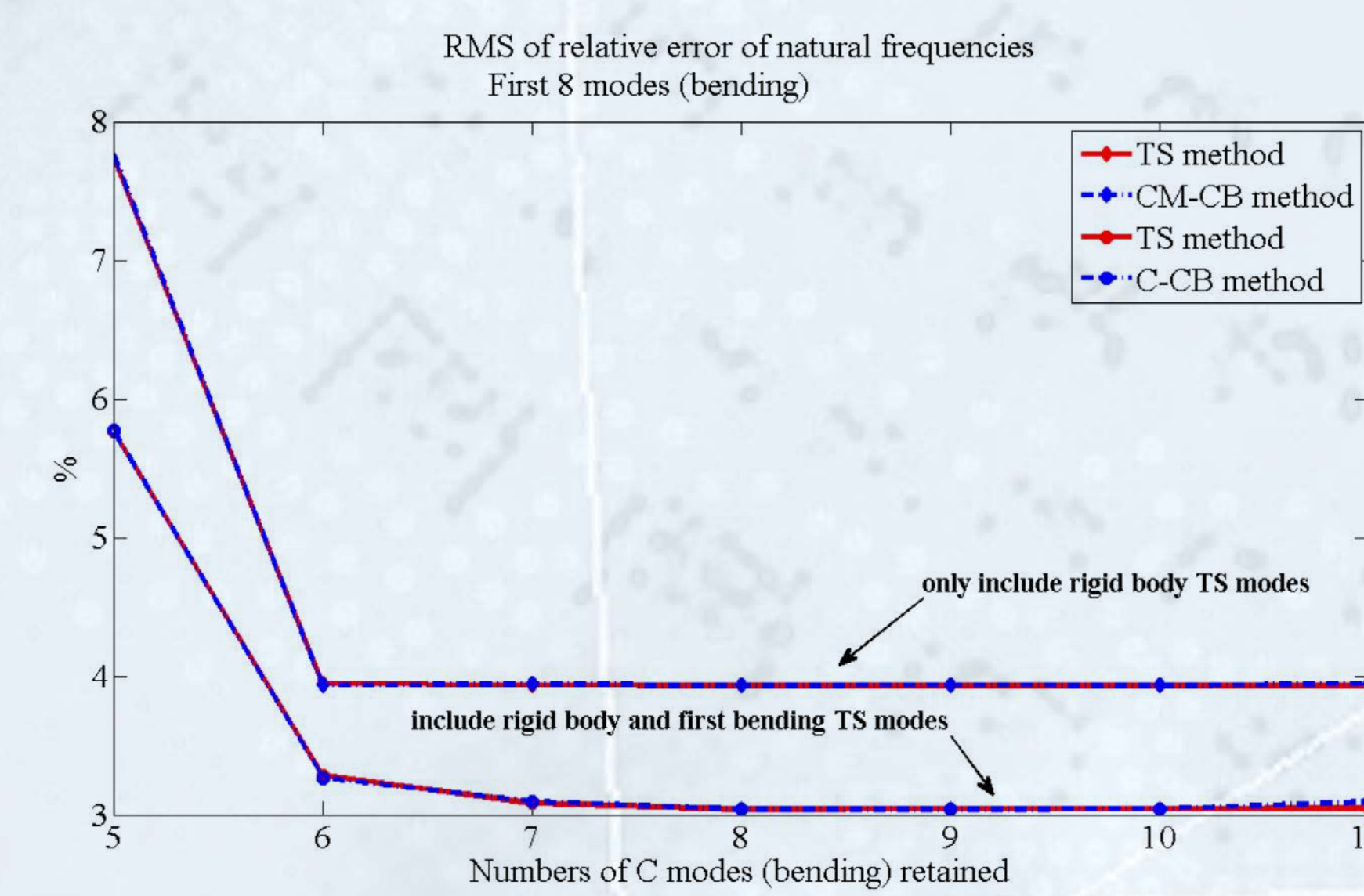


Substructuring $B=D+C-A$ by

- Craig-Mayes model of C-A attached to Craig-Bampton model of D (CM-CB) and traditional transmission simulator (TS)
- 2 degrees of freedom used at 3 node locations as connection

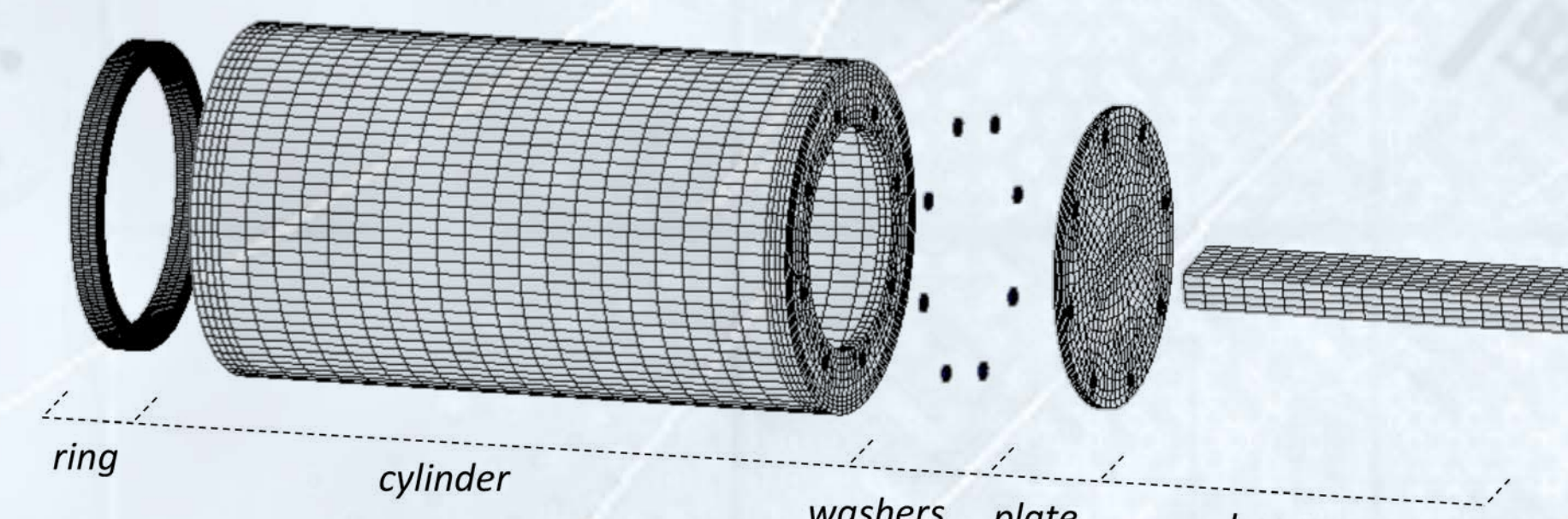
Convergence Test

- In this simulation, the TS and CM-CB method show similar convergence rates given the same TS and experimental modes
- The convergence improves when TS modes estimate the connection points motion more accurately
- After including sufficient (e.g., >6) experimental modes convergence rates show no significant improvement
- Further improvement is observed when the first bending mode of TS is included



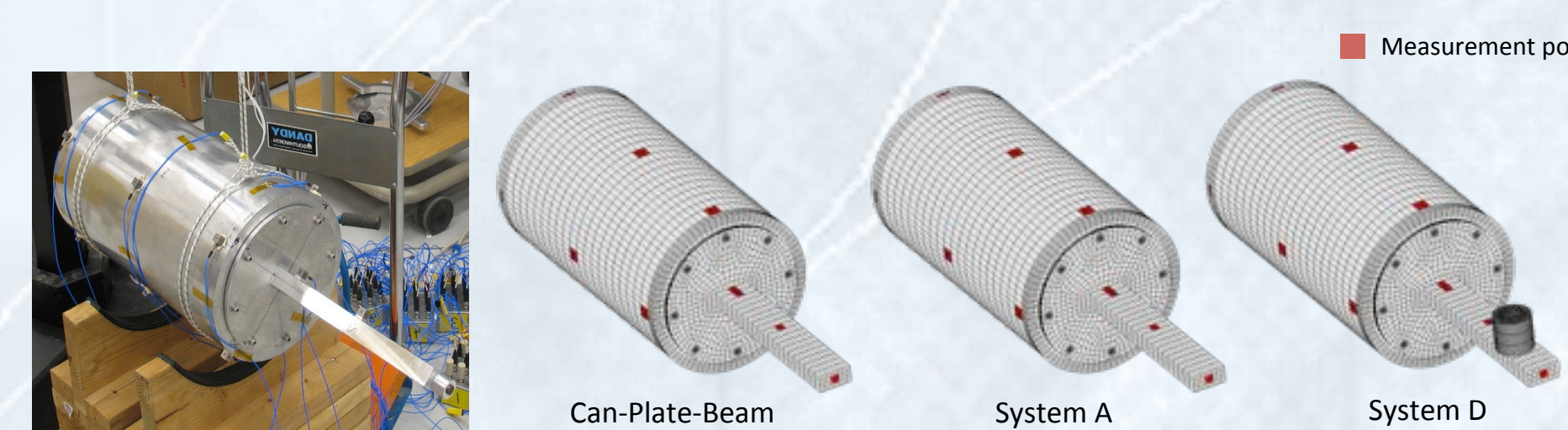
Finite Element Model

- Transmission components and properties



Components	Element type	Material
Beam, plate, cylinder, ring	Higher order 3-D 20 node solid	6061-T6 Aluminium
Washer	Higher order 3-D 20 node solid	Steel

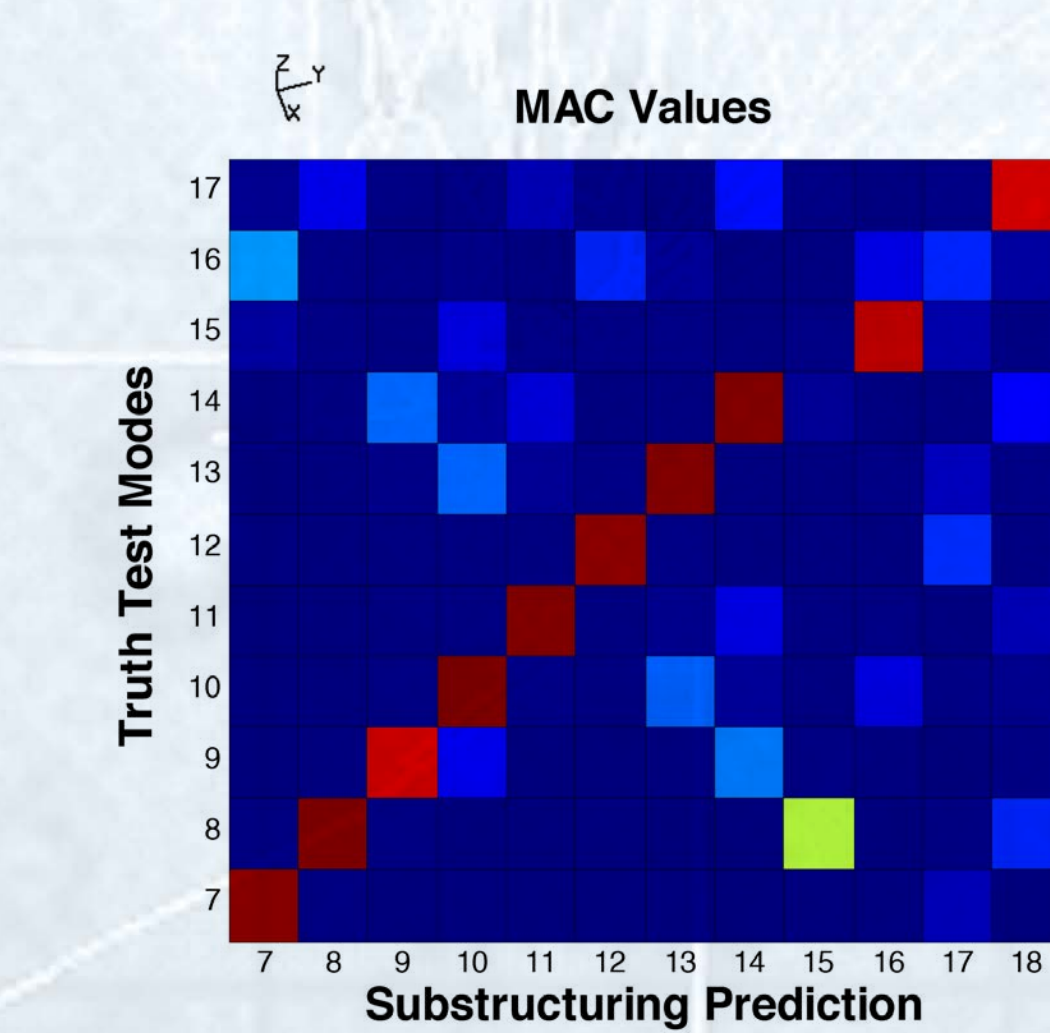
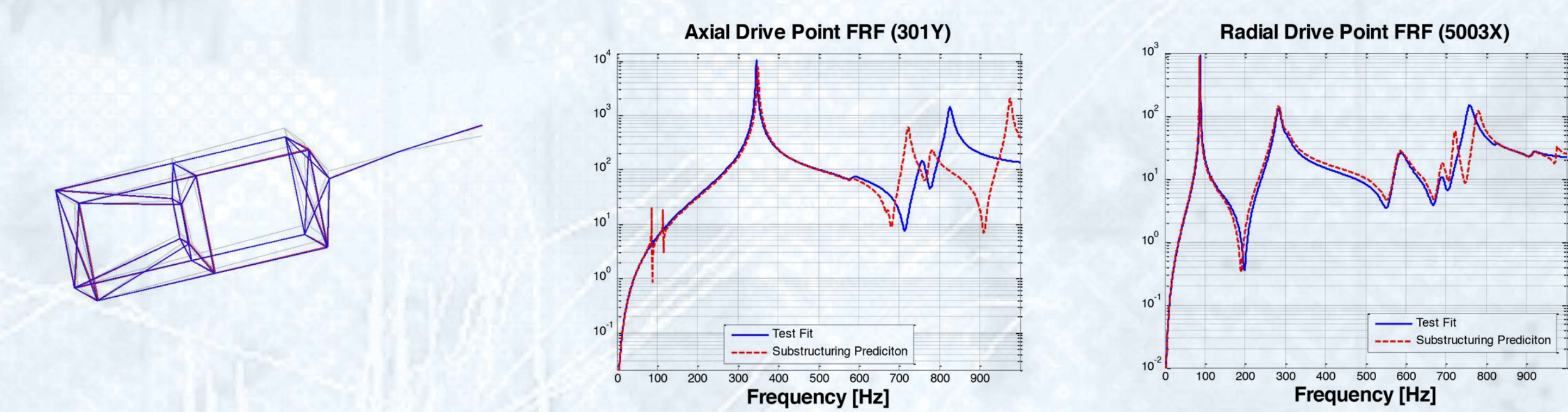
- System frequencies



Experiment [Hz]	Can-Plate-Beam Only		System A Can-Plate-Beam with Ring		System D Validation Structure
	[Hz]	% Difference	[Hz]	% Change	
134.2	136.8	1.90	137.2	2.23	101.9
171.2	176.8	2.80	177.5	3.68	133.9
490.0	422.8	-1.70	448.5	97.32	848.5
511.2	525.9	2.80	540.8	5.78	407.6
975.7	960.6	-1.50	993.8	1.85	993.8
1027	1025	0.19	1026	0.019	861.5
1312	1312	0.00	1549	18.04	1548.7
1528	1535	0.45	1598.5	4.62	1598.6
1637	1610	-1.68	1598.2	-2.37	1707.8
1801	1835.15	1.86	1834.2	1.85	1924.6
1833	1835.35	0.12	1835.3	0.13	2108.5

Substructuring Predictions

- Transmission Simulator approach completed using 13 modes from the transmission simulator (A) and 17 modes of the new subsystem (D)
- Modes of system A kept up to 989 Hz
- Modes of system D kept up to 1495 Hz



Mode #	$f_{\text{test}} [Hz]$	$f_{\text{prediction}} [Hz]$	$f_n \text{ Error } [\%]$	ζ_{test}	$\zeta_{\text{prediction}}$	$\zeta \text{ Error } [\%]$	MAC
7	88.33	87.09	-1.41%	0.00196	0.00214	8.83%	9798
8	115.80	115.13	-0.58%	0.00163	0.00207	26.75%	9925
9	275.97	276.21	0.09%	0.02468	0.02465	-0.14%	9116
10	283.32	283.45	0.05%	0.02151	0.02166	0.72%	9957
11	301.40	301.76	0.12%	0.02327	0.02290	-1.60%	9869
12	346.25	349.95	1.07%	0.00291	0.00358	23.33%	9683
13	584.71	583.33	-0.24%	0.02119	0.02138	0.92%	9963
14	635.16	634.96	-0.03%	0.02037	0.01897	-6.85%	9955
-	NA	671.02	NA	NA	0.00505	NA	NA
15	688.92	690.42	0.22%	0.01515	0.01367	-9.73%	9372
-	NA	721.79	NA	NA	0.00579	NA	NA
16	758.36	NA	NA	NA	0.01131	NA	NA
17	769.71	771.16	0.19%	0.01191	0.01203	1.09%	9121

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