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Bilevel optimization in the context of intermodal pricing: state of art

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Abstract

With the rising interest to stimulate intermodal transport, determining pricing strategies can be intrinsically challenging. We provide a review on the current state of research in intermodal pricing, through which we identify a peculiar gap in optimization approaches. A suggestion to exploit the bilevel optimization technique is presented, as well as an account of its widely successful application to price setting problems. The different approaches to express the network users' behavior, regarded as the lower level problem, are highlighted together with the particular modeling aspects of intermodal networks.

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Keywords: intermodal transport; pricing problems; bilevel optimization; Stackelberg games; network equilibrium

1. Introduction

The European conference of ministers of transport (1997) defined *intermodal transport* as the movement of goods, in one and the same loading unit (or vehicle), by successive modes of transport without handling the goods themselves when changing modes. Generally, rail or inland waterways are used for most of the traveled route, known as the *main haulage*, and road for the shortest possible initial and final parts of the transport chain, known as the *pre- and post-haulage* or *drayage operations*.

External effects of intermodal transport are significantly lower than those of the all-road transport, which hence makes the former a more sustainable and ecologically preferred choice in most cases. This conclusion is solid enough even in the presence of uncertainty factors and scientific debates on the subject (Kreutzberger et al., 2003). Triggered by the growing world-wide interest to confine the harmful effects of transport on the environment, intermodal transport has gained a special interest among researchers and policy-makers, e.g. the roadmap set by the

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European Commission's White Paper (2011) aims to shift 30% of road freight over 300 km to other more environment-friendly modes by the year 2030, and more than 50% by 2050.

However, despite the above motivation and the considerable opportunities to generate economies of scale through freight consolidation and higher load factors (Kreutzberger, 2003), intermodal transport is facing a stiff competition from all-road transport in Europe. Indeed, 44.9% of the freight transport takes place via road (European Commission, 2013).

In addition to a number of qualitative and level-of-service related factors, intermodal transport competitiveness, and modal choice for that matter, is greatly sensitive to the determination of the right services tariff, known as the *pricing strategy* (Bontekoning et al., 2004). Generally speaking, pricing strategies are distinguishable in the way they handle the interplay between profitability and competitiveness. A service price has to be high enough to cover its costs, and hence generate a profit, and low enough to remain attractive to the target customers.

In the intermodal transport context, service prices are to be considered as tactical decisions. Bontekoning et al. (2004) identify two levels, at which the pricing strategy operates. First, at the level of the individual actor in the intermodal chain, where in order to devise his own strategy, each actor must be aware of his own market postion and the cost structure of the other actors. Previous studies belonging to this class are mainly concerned with calculating opportunity costs and providing educated pricing guidelines in this light, mostly from the prespective of the network (mainhaul) and the drayage operators. Second, a pricing strategy can be considered at the whole door-to-door level, where pricing decisions are taken for the intermodal service from the perspective of the service providers (carriers), while accounting for the potential competition and the target customers' (shippers') choices.

In addition to acknowledging the previous view, we suggest a further classification of the latter category: simulation versus optimization approaches. While several examples of decision support systems simulating service prices and pricing-related policies can be noted, the literature is almost silent on optimization methods for intermodal pricing except for a few attempts, either directed to limited case studies or relying basically on market research and customers segmentation methods to capture the network users' reactions to their pricing decisions. Driven by the limitations imposed by these approaches in what concerns the service and customer classes, and the demanding need for handling more generalized and realistic cases, we aim through this paper at justifying a motivation to exploit a modeling concept that was found successful in the domain of pricing problems in general, and the spinned-off network-based variants in specific: namely bilevel optimization. The paper will be organized as follows: we start by reviewing intermodal pricing-related approaches following the suggested classification in section 2. Section 3 covers the main categories of bilevel optimization problems both in the general transportation as well as the pricing context, including the previous bilevel attempts applied to intermodal networks. In section 4, we consider the design possibilites in applying bilevel optimization to intermodal pricing, mainly for the lower-level problem, as well as the modeling of multi-modal networks. Our discussion is finally concluded in serction 5.

2. Intermodal pricing review

The first developed multi-modal network models that were able to handle intermodal flows appeared in the early 1990s (Caris et al., 2013). The most notable decision problems are terminals location-allocation, internalizing external costs, bundling strategies and pricing strategies. The last include the models that consider pricing decisions as simulated parameters, as well as those that consider them as explicit decision variables. We are interested by the studies that directly deal with service pricing questions, as well as by those that tackle decision factors influencing the intermodal market position and its promotion such as: introducing taxes/subsidies and changing the fuel prices.

2.1. Simulation models

The Geographic Information Systems (GIS) technology has introduced new opportunities to model large and complex intermodal networks. One example is the GIS-based location analysis model entitled "LAMBIT" (Macharis and Pekin, 2009). The authors aim to make ex-ante and ex-post analysis of policy measures in Belgium to stimulate the intermodal transport market. In addition to assessing the efficiency of adding new terminals, the model simulates policy options of price scenarios. The model consists of multiple layers depicting each mode of transport, and nodes depicting the location of the intermodal terminals and the port of Antwerp. The transport prices of each mode are estimated according to the real market price structures. The Dijkstra algorithm is then applied to find the shortest path and the intermodal transport costs from the port of Antwerp to each Belgian municipality. The total transport costs are compared for all-road against intermodal transport using inland waterways or rail for the main haulage, and the market areas of each inland terminal are consequently determined. The simulations suggest that different policy measures to stimulate inland waterways and rail transport should be more coherently integrated to avoid the undesirable modal shift between the intermodal transport options.

The LAMBIT model is further exploited in (Macharis et al., 2010). Several fuel price increase scenarios are simulated on the model to visualize their impact on the market area, with the reference scenario being the current market prices. The most interesting situation is the case of a substantial fuel price increase; this forms a stronger price advantage for intermodal transport on the long haul, pushing the break-even distance, after which intermodal transport offers a more competitive alternative, to become significantly smaller. Nevertheless, fuel price increases cannot compete in effect with the policy instrument of *internalizing external costs*, even when they reach their double prices. The fact that a full internalization of external costs is still the subject of strong debate is noted.

Santos et al. (2015) discuss the impact on the promotion of intermodal freight transport of adopting three different transport policies: subsidizing intermodal transport operations, internalizing external costs and assuming a system perspective when optimizing the location of inland intermodal terminals. The study is concerned with the Belgian case. A mixed integer programming model is introduced, with the decisions being related both to the location of railroad terminals and to the allocation of freight flows between the modes, and the objective to minimize the total transport costs from the shippers perspective. The model is based on the *p*-hub location problem, where the main contribution lies in the novelty of using this formulation to address a real intermodal freight problem, as well as the depiction of the concept of economies of scale in non-linear transport cost functions. The authors deduce the significant impact of the subsidies decisions on the volumes of freight migrating to intermodal transport, as well as the relevance of the subsidies provided by the Belgian government to the success of intermodal transport.

The decision support models concerning pricing-related policies are not limited to the above examples. A thorough review of policy support studies for intermodal transport is also available through Caris et al. (2013).

2.2. Optimization models

To the best of our knowledge, very few examples can be mentioned where the prices or the subsidies policies for intermodal services are tackled as explicit decisions through an optimization model. Tsai et al. (1994) consider the problem of finding an *optimal price* for intermodal service in competition with the all-road truck service. Two competition market models are introduced for that purpose. On the supply (carriers) side, it is assumed that the market is shared between a single, profit-maximizing intermodal service provider and several highly-competitive trucking companies, all charging the same prices and none aiming to generate a profit. A key characteristic of the study is its explicit consideration of the unbalanced flow factor, which is reflected in the trucking and intermodal cost structure. On the demand (shippers) side, however, two mode choice methods are defined for each of the presented models.

For the first model, the minimum logistics costs principle is used, stating that, in a certain demand situation, all shippers will choose the mode with lower logistics costs. The characteristics of the optimal solutions of this model, as well as the searching strategies for the maximum profit are analyzed. On the other hand, the second model depicts the shippers' mode choice behavior through a logit demand function, in terms of the intermodal and truck service differences. The special interactions between the variables suggest the representation of the model in a Stackelberg leader-follower game, where the intermodal company is the market leader and the truck companies are the followers who respond to the pricing initiatives of the leader. Two approaches are introduced to search for the optimal solution, with the more simplified among them applied on approximate data and demonstrating a reasonable behavior. The authors conclude with the importance of developing more efficient solution approaches, as well as deeply investigating into the theoretical and empirical aspects of the extended network model to consider realistic cases.

Wang (2002) presents a framework that is initially motivated to assist in a Port Authority's investment decisions, in the frame of an intermodal freight network. It illustrates the interaction, as well as the mutual decision impact between three levels of players: the Port Authority, carriers (terminal operators) and shippers. In a first step, a carriers' (oligopolistic) market is considered, in terms of their pricing and routing decisions. The equilibrium of the carriers' behavior follows the *Nash equilibrium*, based on their knowledge of the operating cost function, their forecast of the service demand function, and their monitoring of each other's strategy. In a second step, the interaction between the carriers and the shippers is cast into a Stackelberg game, formulated as a bilevel programming problem. On the higher level, the carriers' previous problem is represented, while on the lower level, the shippers' reaction is formulated using a general *Spatial Price Equilibrium* (SPE) model. A heuristic algorithm, based on a sensitivity-analysis method, is developed. And, in a final step, the bilevel approach is applied to evaluate the impact of several alternative investment strategies of the Port Authority on the carriers and the shippers, through a net social benefit formulation. A small-scale case study is presented to demonstrate the efficiency and the applicability of the proposed methods.

In the Li and Tayur (2005)' paper, the problem jointly tackles pricing and operations planning for a special case of an intermodal service company, while satisfying service constraints and maximizing profit. For the pricing sub-problem, unlike the above examples, the authors choose to follow the traditional marketing research approaches in order to capture the potential customers' decisions and to model how the demands change with the prices. In this approach, customers select a service according to their evaluation of it, beside the service's price, and the selection logic is translated into a reservation price concept. Next, a probability density function is designed and fit through regression on a surveyed reservation prices data set. Once the function is found, the demand may be represented in terms of the prices and a mathematical programming problem with concave objective function and linear constraints is obtained. The problem is considered for the case of two service classes. However, a limitation to the proposed approach is the high complexity to obtain the demand (and price) function through analytical methods, when the number of customer or product classes becomes large. The authors acknowledge the necessity to investigate numerical solution procedures in that case.

A common point of interest among the above examples is the manner in which they account for the reaction of the target network users to their price decisions, forecasting in turn the demand volumes to their services and their respective profit. Some of them already touch upon the Stackelberg game concept, expressed in bilevel optimization, for that purpose due to its natural depiction of the hierarchical decision process and its efficient implementation in the pricing domain. Nevertheless, the inflexibility and limitation remain the dominant aspects of the presented approaches so far. If anything, this suggests the peculiar possibility for intermodal service providers to fall into suboptimality. This is specially alarming as to the future of intermodal transport in the market and the subsequent application of the target EU transport policies; a risk confirmed by the previously noted modal split figures.

3. Bilevel optimization

Bilevel optimization problems; introduced in Bracken and McGill (1973), give the mathematical programming formulation of the (static) Stackelberg game-theoretical concept. The problem involves two sequential layers of players, commonly referred to as: the leader and the follower(s). In the game, the leader, given a precedence privilege and an ability to anticipate the follower's decision logic, plays first and decides on a most advantageous strategy, taking into account the follower's, supposedly, optimal reaction to his/her strategy. In more mathematical terms, a subset of the variables of the leader's optimization problem is constrained to assume an optimal solution to the follower's optimization problem, which is, in turn, parameterized by the remaining variables, however not restrained to the leader's constraints. By denoting the leader and follower's decision vectors respectively as x and y, the objective functions as F and f, and the constraints as G and g, we get the following problem formulation (Colson et al., 2007):

$$\min_{x \in X, y} \quad F(x, y) \tag{1}$$

$$s.t. \quad G(x, y) \leq 0, \tag{2}$$

$$\min_{y} \quad f(x, y) \tag{3}$$

$$s.t. \quad g(x, y) \leq 0, \tag{4}$$

where constraints *G* involve variables from both levels, in contrast to the feasible set defined by *X*, and must be indirectly enforced in order to bind the followers.

There may be multiple optimal solutions for the lower level problem for a given set of values for the higher level decision variables. In that sense, the follower's behavior determines two possible approaches for the leader's decision. The first and most commonly chosen is an optimistic approach that assumes the follower's cooperation and subsequent choice of the most profitable solution to the leader. Whereas, the second is a pessimistic approach that assumes the follower's aggressive behavior, leading the leader to bound the damage resulting from the follower's most undesirable reaction.

The bilevel optimization problem was proven to be strongly NP-hard. Later, the results were strengthened to prove that the mere check for local or strict optimality in the linear case is an NP-hard problem as well. This intrinsic difficulty imposes a necessity for most classical solution methods to assume certain convenient properties for the functions, such as: smoothness or convexity, in order to be able to efficiently handle the problem. For a comprehensive account on the bilevel optimization paradigm, including the mathematical properties, optimality discussions and solution methods, we refer to Dempe (2002) and Colson et al. (2007). A rigorous bibliography review, containing more than one hundred references, is available as well through Vincente and Calamai (1994).

A particular strength of the above mathematical framework is the fact that it takes into account the strategic behavior of the target customers through an individual optimization problem, providing the expected demands with a degree of flexibility and realism that is not affordable through classic demand functions. The special structure of the problem fits several real-world application domains having an embedded decision hierarchy within their definition. As a preface to introduce its potential application to intermodal pricing, we now emphasize on the successful application of the bilevel optimization concept in the general transportation domain, as well as the relevant field of (network-based) pricing.

3.1. Applications in transportation

In most transportation related decisions, we may observe two autonomous, and possibly conflicting, levels of decision makers: on the upper level, the public sector or an authority seeks to improve a transportation network performance and meet certain global goals; while on the lower level, the network users make their personal travel choices in their best interest. Based on the surveys in Migdalas (1995) and Colson et al. (2007), the typical problem classes of bilevel optimization in the domain of transportation are: network design; signal setting; origin-destination (O-D) matrix estimation; hazardous materials management.

3.1.1. Applications on inter-/multimodal networks

The literature examples in the relevant context of inter-/multimodal transport are of a narrow perspective and not significantly numerous. In Clegg et al. (2001), the problem of optimizing urban multimodal transportation networks is addressed in a bilevel manner. The aim is to decide on control values, in terms of signal green-times and prices, while the travelers' (rational) route and mode choices, translated into the estimated traffic flows, are at equilibrium. Yamada et al. (2009) propose a bilevel optimization model for strategic multimodal network planning. On the upper level, a discrete network design problem is considered to select a suitable subset of actions from a number of possible ones. The lower level incorporates a *multimodal multiclass user equilibrium traffic assignment* in order to capture the decisions' influence on the traffic and freight flows. A heuristic approach, based on genetic local search, is applied to solve the problem. Zhang et al. (2013) present an optimization model for terminal networks, based on bilevel programming, and taking into account environmental costs and economies of scale. The authors conduct a search among candidate policy packages of terminal configurations and CO_2 emission prices by applying a genetic algorithm, while performing a multi-commodity flow assignment over a multimodal network to derive the travelers' decisions. In the context of energy saving, Du et al. (2014) exploit analytical methods, with the aim to bridge the gap between the designed policy instruments and their corresponding consumption output in the transportation sector. Within a bilevel optimization framework, energy consumption is minimized on the upper level over a multimodal transportation network, subject to the traffic demand distribution, resulting from a travelers' utility maximization problem on the lower level.

3.2. Pricing problems

Pricing problems consist of the leader's problem of setting taxes or prices for a set of offered services, while accounting for the followers' choice, having the freedom to settle for the taxed or the untaxed services, with the aim of minimizing their own operational costs. A thorough review of this class of bilevel problems is recently conducted by Labbé and Violin (2013). The authors start by presenting a general, as well as a linear version of the price setting problem. A graphical interpretation is used, afterwards, to illustrate the conditions under which a bounded solution is guaranteed; namely, that the follower's feasible set is both non-empty and bounded, and that there exists at least a feasible solution for the follower using only untaxed services. The latter condition is necessary to prevent the leader of setting an infinite tax to their owned services.

The scope of the problem analysis in the survey is further moved to its network-based framework, known as the network pricing problem (NPP). The leader became an authority owning a subset of arcs and aiming to maximize his/her revenue through a toll assignment scheme. The followers, in turn, became the users traveling the network in a cost minimization fashion, from their own perspective. The rest of the arcs, assumed to be owned by other network agents (or not), are subjected to fixed costs which are known a priori. Let K be a set of commodities, where each commodity k is associated with an origin o^k, a destination d^k and a demand η^k. And let A_1 and A_2 denote the set of toll arcs and toll free arcs respectively, c_a a fixed travel cost on each arc $a \in A_1 \cup A_2$, and T_a a toll on each toll arc $a \in A_1$. If the flow is given by variables x_a^k on toll arcs, and y_a^k on toll free arcs, a formulation of this problem is thus given by (Labbé and Violin, 2013):

$$\max_{T \geq 0} \quad \sum_{a \in A_1} T_a \sum_{k \in K} \eta^k x_a^k, \tag{5}$$

$$s.t. \quad (x,y) \in \arg\min_{x,y} \quad \sum_{k \in K} \left(\sum_{a \in A_1} (c_a + T_a) x_a^k + \sum_{a \in A_2} c_a y_a^k \right), \tag{6}$$

$$s.t. \quad \sum_{a \in i^+} (x_a^k + y_a^k) - \sum_{a \in i^-} (x_a^k + y_a^k) = b_i^k \qquad \forall k \in K, \forall i \in N, \tag{7}$$

$$x_a^k, y_a^k \geq 0 \qquad \forall k \in K, \forall a \in A_1 \cup A_2, \tag{8}$$

where i^- and i^+ stand for the set of arc with i as head or tail respectively, and b_a^k evaluates to -1 if i is the origin of commodity k, 1 if it is the destination, and 0 otherwise.

When negative tolls are allowed, the model is said to deal with subsidies. In the same context, two main categories of the problem are introduced: *arc pricing* in contrast to *path pricing*. For the former, the previous formulation typically holds, where the leader-owned arcs are not restricted to assume similar tolls values. In the latter category, however, tolls are associated to paths that could be even priced differently for each commodity. The special case of a polynomial number of paths is also defined; namely, the highway system. Furthermore, a clear mapping scheme of the special context of product pricing is established with respect to the arc pricing context, in terms of each pair of corresponding problem elements. The particular strength of bilevel optimization is evident within this context, where the concept of reservation prices becomes embedded in the definition of the network itself in the form of the allowed toll windows. The authors refer to Labbé et al. (98) for a two-phase procedure to convert the arc pricing problem into a single-level mixed integer problem (MIP).

Graph processing techniques have been studied in the literature to reduce the practical size of the original network. By examining the structural properties associated to the shortest path selected by each commodity, a *shortest path graph* can be constructed, where further arcs can be eliminated using path dominance reasoning. More details on the shortest path graph construction procedure are furnished by Bouhtou et al. (2007) and Van Hoesel (2008).

There remains a considerable room for experimentation in the context of the NPP. Van Hoesel (2008) proposes three possible avenues for extension: incorporating capacities on the arcs, considering other pricing mechanisms and integrating the problem of the network design. Labbé and Violin (2013), for their part, highlight a number of open issues, such as integrating real-life features into bilevel models and tackling more variants of product pricing.

4. Application to intermodal pricing

Returning to the context of intermodal pricing at the door-to-door level, we observe that the definition of the problem fits both the particulars of the product pricing problem and, by consequence, the NPP. The problem can be considered from an economic perspective where an intermodal service provider corresponds to a profit maximizing leader, a set of target customer shippers to utility maximizing (or cost minimizing) followers, intermodal service tariffs to price decisions, with an obvious parallel of reservation prices and service assignment (or arc flow) variables.

As for the representation of intermodal networks, the traditional modeling concept of *virtual networks* is certainly a solid candidate. The idea, initially proposed by Sheffi (1985), is to use pseudo-links or virtual links to represent intermodal transfers among the modal networks. As different transport operations can occur on the same infrastructure, a virtual link with a specific cost is created for each particular operation. A further development of this concept is implemented in the NODUS software (Jourquin and Beuthe, 1996). The main contribution is designing a structured notation and an automatic generation of the virtual links.

Following the above methodology to build an intermodal network, a direct application of the NPP to intermodal pricing can therefore be made possible. On the upper level, there is little variance as to the profit structure of the leader's problem. The main differences would lie in the revenue and cost structure of the intermodal carriers, and the respective necessary constraints. It is possible indeed to develop the problem in other directions that have significant impact on the foreseen costs and profits. One development would be to integrate the configuration decisions of the service network. This can possibly include decisions about enabling/disabling a certain service line, the service routing and frequency, and the trains' scheduling and length for the rail part. Special interest has been given to this vein of research in intermodal transport, and how to combine it with complex schemes of flows' bundling in order to design more efficient consolidation networks able to generate economies of scale. The interested reader is referred to the tactical planning discussions in Bontekoning et al. (2004) and Caris et al. (2008). Another interesting direction would be to account for the prospective market competition. This is particularly sensible to consider with the presence of the Council Directive 91/440/EEC (1991) on the development of the Community's railways, recognizing "the need for a greater integration of the European railway system into an increasingly competitive market" and ensuring the management independence of the railway undertakings. However, a typical direction would require a special mathematical treatment that can best translate an oligopolistic market in an equilibrium form.

The lower level, however, provides interesting points of discussion. We distinguish between two imaginable cases for the followers' problem: (1) the shippers are faced with the choice of either the offered intermodal service or (manageably) other discrete alternatives; or (2) the shippers' other network-based alternatives are sufficiently numerous as to amount to a routing problem. For the first, we discuss the application of discrete choice models; while for the second, we adopt a *traffic assignment* perspective to distinguish between the viable ways to model the shippers' routing problem.

4.1. Followers' discrete choice problem

Discrete choice analysis is the methodology used to analyze and predict the individual choice behavior in economics and management, not least for transportation and travel behavior modeling. Through statistical approaches, a functional relation is established between an expected choice and the attributes of both the corresponding decision-makers and alternatives. For an in-depth account of the concept of discrete choice analysis and its mostly applied methods, we refer to Ben-Akiva and Lerman (1985).

In many practical situations, the freight shippers' alternatives are intrinsically limited with respect to an offered intermodal service, comprising in principle the choice of highways and national routes Discrete choice models provide in that case a reasonable framework to depict the shippers' choices. The problem could potentially be formulated as a utility maximization problem, in the presence of necessary constraints, and cast into a bilevel pricing problem as previously hinted to by Tsai et al. (1994), in one of the few reviewed optimization-based approaches for intermodal pricing. Likewise, it could be scaled down to a demand analysis model in a typical product pricing problem, in the trivial case of the constraints' absence.

4.2. Followers' traffic assignment problem

In the case of a larger choice pool for the followers, the application of the *virtual networks* concept reduces their presumably two fold problem of modal choice and optimal routing to the routing part. Traditionally, the lower level problem in the variants of the NPP problem is handled as a shortest path problem, where the arc capacities are assumed unconstrained and the arc travel times, reflected in their respective costs, are regarded as constants. However, in more realistic cases, where the effects of congestion are to be taken into account, there exists a mutual dependence between the arc flows and costs. In that sense, the link flows and costs are iteratively updated, moving towards a state of equilibrium. Two main modeling branches accordingly stem from this distinction: shortest path and traffic equilibrium assignment models.

4.2.1. Shortest path assignment

In a shortest path assignment, or all-or-nothing (AON) assignment, all the trips from a certain origin to a destination zone are assigned to a single shortest (minimum cost) path among all feasible ones, assuming no congestion effects, hence constant link costs, and a unified costs perception for all drivers (Ortúzar et al., 2011). A typical formulation in a multi-commodity network is given by problem (6)-(8). Although it seems unrealistic in terms of selecting only one path for each O-D pair and ignoring real-life aspects, it may be reasonable in sparse networks, where there are few alternative routes and they are very different in cost, and to provide an insight on the desired path in the absence of congestion. Models, using the AON technique, mostly make similar assumptions and run necessary experiments in order to alleviate the effect of capacity shortage on their concerned decision horizon.

4.2.2. Traffic equilibrium assignment

The issue of congestion is explicitly considered by expressing a link cost as function of its usage level, with respect to its capacity. As the flow increases towards the capacity of a certain network link, it is conceivable that the traffic conditions deteriorate and the link speed becomes lower, which implies a higher travel time and cost, diverting in turn part of the flow to alternative, now, cheaper links. A number of such iterative procedures would take place until the fluctuation in links costs and flows would eventually converge, reaching an equilibrium configuration that can be regarded either from the user or the system perspective. The first case is known as *user equilibrium* (UE). Typical models are based on the behavioral assumption in Wardrop's first principle of traffic equilibrium (Wardrop, 1952), stating that *"the journey times on all the routes used are equal, and less than those which would be experienced by a single vehicle on any unused route"*. An equilibrium is thus attained *"when no driver can unilaterally reduce his/her travel costs by shifting to another route"*. By contrast, the other case is referred to as *system optimum* (SO). At equilibrium, traffic flows are said to satisfy Wardrop's second principle, stating that *"the average journey time is minimum"*. This implies the cooperative behavior of the drivers to minimize the total system travel time.

In our discussion, we concentrate on the UE case for the reasons that: (1) flows at UE do not generally minimize the total system travel time, however they are more behaviorally realistic than those at SO equilibrium (Migdalas, 1995); and (2) the cooperation of the network users among each other and with a supposed authority defies or complicates the whole non-cooperative setting of the Stackelberg game that we stick to. We refer to Patriksson (1994) for a rigorous treatment of the network equilibrium problem, as well as to Marcotte (1986) for a thorough account on congestion issues in the frame of bilevel network design problems.

Let $G = (A, N)$ denote an underlying network, with N representing a set of nodes, A a set of links, K a set of O-D pairs. If we define for each kth pair in K, a set of simple paths between its end nodes as P_k, the travel demand as r_k, the shortest route travel time as π_k, and the travel time and flow on the pth route in P_k as c_{pk} and h_{pk} respectively. Therefore, the UE principle can be mathematically expressed through the following conditions (Migdalas, 1995):

$$h_{pk}(c_{pk} - \pi_k) = 0, \quad \forall p \in P_k, \tag{9}$$

$$c_{pk} - \pi_k \geq 0, \quad \forall p \in P_k, \tag{10}$$

$$\sum_{p \in P_k} h_{pk} = r_k, \tag{11}$$

$$h_{pk} \geq 0, \quad \forall p \in P_k, \tag{12}$$

for every O-D pair k.

According to Migdalas (1995), there exist two main modeling approaches for the UE problem: a non-linear network model and a Variational Inequality (VI) formulation. In the first approach, for every link $a \in A$, a travel cost function $s_a(x_a)$ is defined in terms of its total flow x_a, as encountered by a user traveling on link a. A convex optimization problem can thus be formulated, for which, conditions (9)-(12) hold as the first-order optimality conditions, assuming s_a to be positive, strictly monotone increasing and continuously differentiable. In the second approach, however, the general case where the link travel functions may also depend on the flow of neighboring links is considered. Assuming similar properties on the link travel and the route cost function, the equilibrium problem can be formulated as VIs, in which case, the resulting bilevel program can be regarded as an equivalent to a mathematical program with equilibrium constraint (MPEC). As shown by Colson et al. (2007), MPECs incorporate bilevel programs, whenever the lower level problem in the latter is convex and differentiable. The reverse holds as well through replacing the lower-level VI by an optimization problem.

An argument may arise, opposing formulating the followers' problem in the specific domain of the NPP as a UE problem, relying on the point that pricing decisions are generally regarded for a larger interval than that of the congestion occurrence. However, if the network in question experiences congestion in quite a regular manner within the decision horizon, for sufficiently long periods of time, taking congestion into consideration within a NPP would therefore be justifiable. Nevertheless, to our knowledge, the literature examples incorporating a network equilibrium problem in the lower-level of a NPP, in the profit maximizing sense of the problem, are nearly non-existent.

The related field of congestion pricing, however, can be referred to as broadly along the same line. The link tolls are used to achieve a set of overall management or planning goals and a certain social welfare, quite in the same flavor as in the SO principle. Several studies have explicitly treated traffic congestion problems as Stackelberg games on networks, where the users' route choice behavior is described through a network equilibrium model. This is a wide research class that requires a rigorous review on its own; however, some examples can be noted. Yan and Lam (1996) introduce a two-arc based pricing model that involves queuing delays. A brief account on the subject of congestion pricing within bilevel programs is given by Van Hoesel (2008). Recently, a multi-objective bilevel pricing model is presented by Wang et al. (2014), in the context of sustainability maximization.

5. Conclusion

A literature review has been conducted to demonstrate the scarcity, as well as the necessity, of solid optimization frameworks, targeting pricing strategies of intermodal services from the door-to-door carriers' perspective. We propose to express the problem as a sequential non-cooperative Stackelberg game between the intermodal operators and the target customer shippers, mathematically translated in bilevel optimization; a well-suited framework for hierarchical pricing schemes. This comes at a challenging time for intermodal transport to promote its market position through a more efficient maneuver of the crucial pricing questions, eventually leading to a closer figure to the target EU modal split policies.

We provide modeling insights to represent the particular aspects of an intermodal pricing problem in a similar form to that of a NPP. More specifically, a certain carrier's profit maximization problem would be considered at the upper level, while the shippers' rational reactions to the carrier's strategies would be depicted at the lower level. A special discussion is devoted to the latter sub-problem in order to cover several real-life situations.

We finally shed light on an interesting, yet poorly utilized, direction promising a more realistic representation of the shippers' behavior, hence more educated pricing strategies; namely the utilization of network equilibrium models within the followers' problem in the potential bilevel intermodal pricing problems.

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