ASTEROSEISMIC INVERSIONS IN THE CONTEXT OF PLATO

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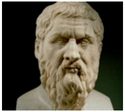
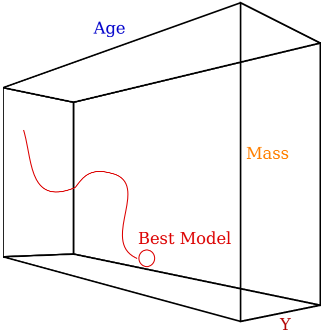


Definition:

Using seismic (and non-seismic) constraints to determine the optimal set of parameters describing the **theoretical model** of a **real star**.

Typically: Mass, age, Y, Z, α_{MLT}, α_{ov}, ... with a given physics (opacities, convection treatment, extra-mixing, ...)

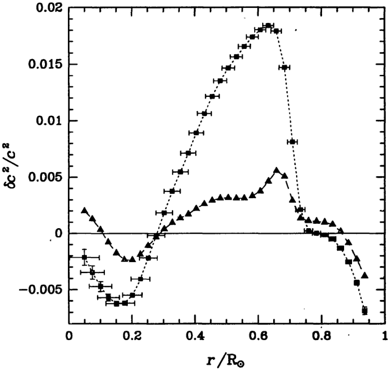


- **Limitations?**
- **Physically representative?**

Bias on determinations of fundamental parameters (Mass, Age, Radius)

Improving the use of seismic information:

1. By using **seismic indicators** (glitches, fine analysis of frequency combinations,...).
2. By relating seismic information to structural corrections using a **less model-dependent approach**.



Basu et al. 1996

- Not subject to the limitations of forward modelling
- Sufficient number of frequencies required

Inversions of integrated quantities

Using the SOLA method (Pijpers & Thompson 1994), we determine integrated quantities.

Illustration for the mean density (Reese et al. 2012)**:**

$$\bar{\rho} = \int_0^R 4\pi \frac{\rho}{R^3} r^2 \, dr \tag{1}$$

$$\frac{\delta\nu_i}{\nu_i} = \int_0^R K^i_{\rho,\Gamma_1} \frac{\delta\rho}{\rho} dr + \int_0^R K^i_{\Gamma_1,\rho} \frac{\delta\Gamma_1}{\Gamma_1} dr \tag{2}$$

$$\frac{\delta\bar{\rho}_{inv}}{\bar{\rho}} = \sum_i c_i \frac{\delta\nu_i}{\nu_i} \approx \int_0^R 4\pi \frac{\rho}{R^3 \bar{\rho}} r^2 \frac{\delta\rho}{\rho} dr \tag{3}$$

Provided accurate results up to 0.5% and a way to assess its errors.

General approach (1)

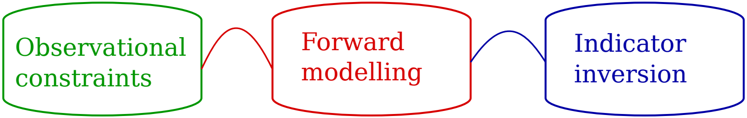
But we are not limited in the choice of the integrated quantity:

$$A = \int_0^R f(s_1, s_2, r)\, dr$$

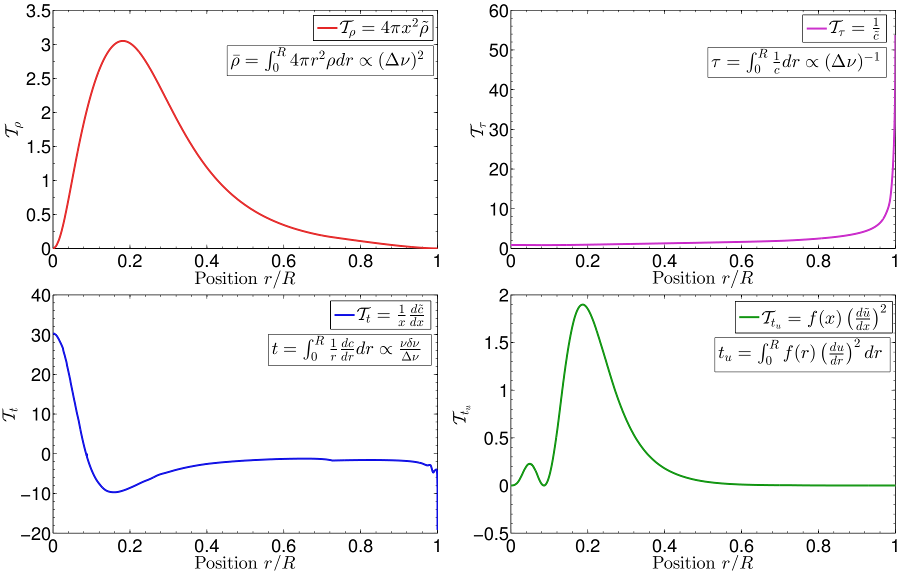
$$\frac{\delta A}{A} = \int_0^R \mathcal{T}_A \frac{\delta s_1}{s_1}\, dr + \int_0^R \mathcal{T}_{A,\mathrm{cross}} \frac{\delta s_2}{s_2}\, dr$$

With s_1,s_2 being ρ, c^2, Γ_1, $u = \frac{P}{\rho}$, Y, ... defining custom-made targets functions $\mathcal{T}_A$ and $\mathcal{T}_{A,\mathrm{cross}}$.

Combining approaches:



General approach (2): new indicators

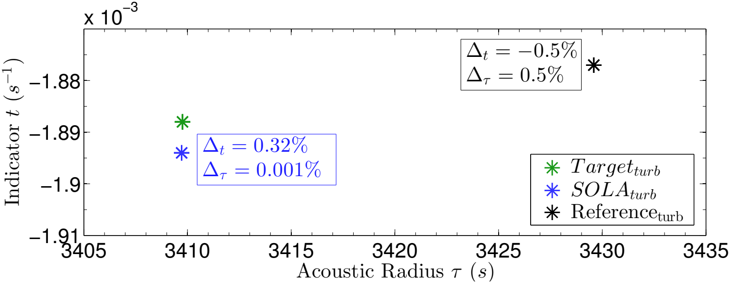


PROOF OF CONCEPT

Test strategy (see Buldgen et al. 2014, arXiv:1411.2416)

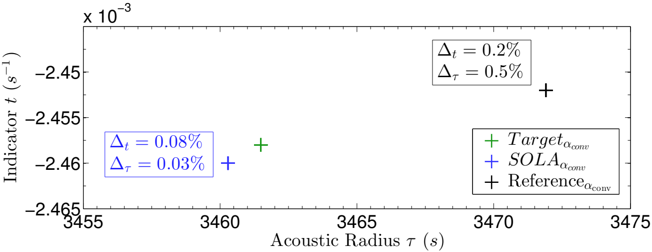
- Build target including complex physics
- Seismic modelling using simple physics
- Carry out **inversions for indicators**

1.0 $M_\odot$, 4.0 Gyr, with turbulent pressure ($< \Delta\nu > < \delta\nu >$ with $Mass$, Age)

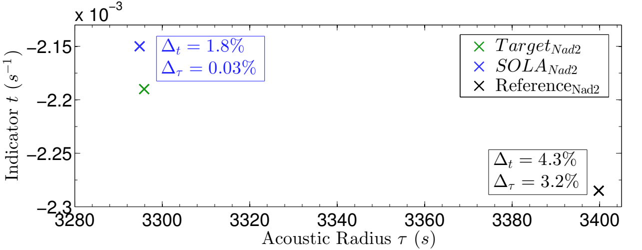


RESULTS WITH MULTIPLE INDICATORS (1)

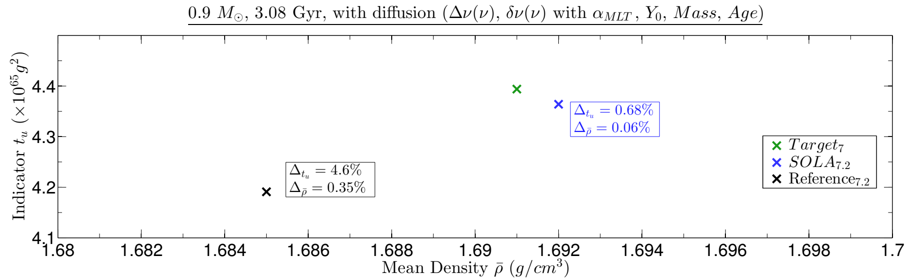
$1.05\ M_\odot$, 1.5 Gyr, $\alpha_{\rm MLT} = 1.7$ ($< \Delta\nu > < \delta\nu >$ with *Mass*, *Age* and $\alpha_{\rm MLT} = 1.522$)

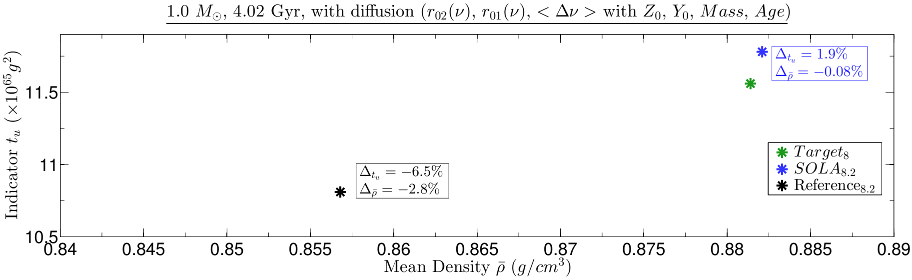


$1.0\ M_\odot$, 3.0 Gyr, non-adiabatic frequencies ($< \Delta\nu > < \delta\nu >$ with *Mass*, *Age*)



RESULTS WITH MULTIPLE INDICATORS (2)





Conclusions and upcoming work

Objectives:

- Offer **new constraints** on stellar models and **overcome the current accuracy limitations** in ages, mass and radii determinations.

What we have:

1. accurate (usually $< 1\%$) determinations of τ, t, t_u and $\bar{\rho}$
2. method tested for various sets of modes (50 freq, $\ell = 0, 1, 2, (3)$)
3. method tested for various constraints on forward modelling

Prospects:

1. new indicators (helium, surface temperature gradient,...)
2. test on new generation of models
3. apply to observations (Plato data is crucial)

Thank you for your attention!

General expression for any A:

$$\frac{\delta A}{A} = \int_0^R \mathcal{T}_A \frac{\delta s_1}{s_1} dr + \int_0^R \mathcal{T}_{A,\text{cross}} \frac{\delta s_2}{s_2} dr \tag{4}$$

Target functions:

$$\mathcal{T}_\tau = \frac{1}{2c\tau} - \frac{Gm(r)}{r^2}\rho \left[\int_0^x \frac{1}{2c\tau P} ds\right] \qquad \mathcal{T}_{\tau\text{cross}} = \frac{-1}{2c\tau}$$

$$- 4\pi G r^2 \left[\int_r^R \left(\frac{\rho}{s^2} \int_0^s \frac{1}{2c\tau P} dt\right)\right] ds$$

$$\mathcal{T}_{\bar{\rho}} = 4\pi \frac{\rho}{R^3 \bar{\rho}} r^2$$

$$\mathcal{T}_t = \frac{\frac{1}{r}\frac{dc}{dr}}{\int_0^R \frac{1}{r}\frac{dc}{dr} dr}$$

$$\mathcal{T}_{t_u} = \frac{-2u_0}{t_u} \frac{d}{dr}\left(f(r)\frac{du}{dr}\right) \tag{5}$$

Obtaining new kernels: From Masters et al. 1979:

$$\int_0^R K^{n,l}_{s_1,s_2}\frac{\delta s_1}{s_1}dr+\int_0^R K^{n,l}_{s_2,s_1}\frac{\delta s_2}{s_2}dr=\int_0^R K^{n,l}_{s_3,s_4}\frac{\delta s_3}{s_3}dr+\int_0^R K^{n,l}_{s_4,s_3}\frac{\delta s_4}{s_4}dr$$

Relate (s_1, s_2) to (s_3, s_4) (e.g. (ρ, Γ_1) to (u_0, Γ_1) in the simplest cases)

$\Rightarrow$ new kernels by solving a differential equation.
Ex: (u_0, Γ_1), (u_0, Y), (c^2, Γ_1), (c_2, Y), (Γ_1, Y), (P_0, Γ_1), (P_0, Y), (N_2, c^2),...

Table : Set of modes used in the accuracy tests

	Set 1	Set 2	Set 3	Set 4
$\ell = 0$	$n = 5 - 24$	$n = 9 - 28$	$n = 5 - 27$	$n = 9 - 34$
$\ell = 1$	$n = 5 - 24$	$n = 9 - 28$	$n = 5 - 27$	$n = 9 - 34$
$\ell = 2$	$n = 5 - 24$	$n = 9 - 28$	$n = 5 - 27$	$n = 9 - 34$
	Set 5	Set 6	Set 7	Cyg 16A
$\ell = 0$	$n = 11 - 24$	$n = 11 - 26$	$n = 9 - 23$	$n = 12 - 27$
$\ell = 1$	$n = 11 - 24$	$n = 11 - 26$	$n = 9 - 23$	$n = 11 - 27$
$\ell = 2$	$n = 11 - 24$	$n = 11 - 26$	$n = 9 - 23$	$n = 11 - 24$
$\ell = 3$	$n = 9 - 20$	$n = 12 - 22$	$n = 15 - 24$	$n = 15 - 21$
	Cyg 16B	Set 33		
$\ell = 0$	$n = 13 - 26$	$15 - 25$		
$\ell = 1$	$n = 13 - 26$	$15 - 25$		
$\ell = 2$	$n = 12 - 25$	$15 - 25$		
$\ell = 3$	$n = 17 - 24$	$-$		

Target properties:

- Total number: 57
- Mass range: 0.9 – 1.1 $M_\odot$
- Age range: 1.1 – 8.1 Gyr
- Z_0 range: 0.01 – 0.02
- X_0 range: 0.73 – 0.68
- No convective cores!
- Including various mismatches: Y_0, Z_0, diffusion, heavy elements mixing.

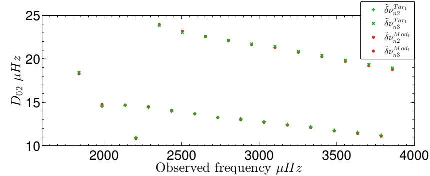
Observational constraints:

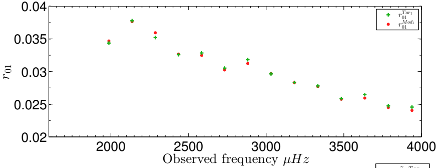
$\Delta\nu(\nu)$, $\delta\nu(\nu)$, $r_{02}(\nu)$, $r_{01}(\nu)$, $< \Delta\nu >$, $< \delta\nu >$.

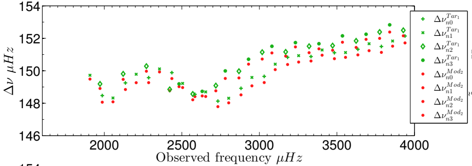
Free parameters:

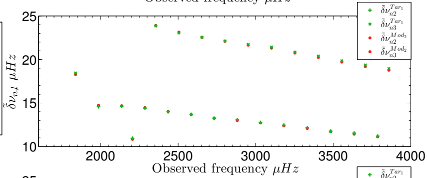
α_{MLT}, Z_0, Y_0, Age, Mass.

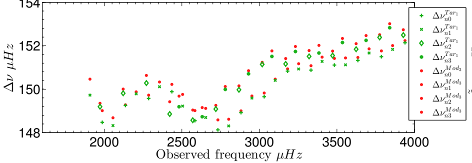
APPENDICES

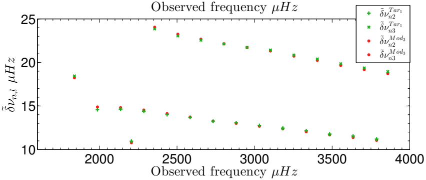

D_{02} μHz
25
20
15
10
2000
2500
3000
3500
4000
Observed frequency μHz
$\tilde{\delta\nu}^{Tar_1}_{n2}$
$\tilde{\delta\nu}^{Tar_1}_{n3}$
$\tilde{\delta\nu}^{Mod_1}_{n2}$
$\tilde{\delta\nu}^{Mod_1}_{n3}$


r_{01}
0.04
0.035
0.03
0.025
0.02
2000
2500
3000
3500
4000
Observed frequency μHz
$r^{Tar_1}_{01}$
$r^{Mod_1}_{01}$

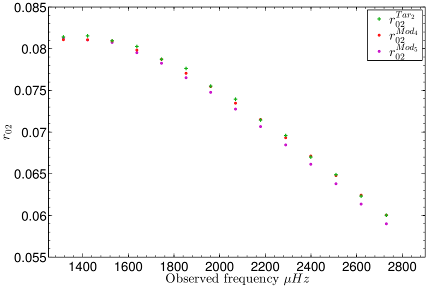

$\Delta\nu$ μHz
154
152
150
148
146
2000
2500
3000
3500
4000
Observed frequency μHz
$\Delta\nu^{Tar_1}_{n0}$
$\Delta\nu^{Tar_1}_{n1}$
$\Delta\nu^{Tar_1}_{n2}$
$\Delta\nu^{Tar_1}_{n3}$
$\Delta\nu^{Mod_2}_{n0}$
$\Delta\nu^{Mod_2}_{n1}$
$\Delta\nu^{Mod_2}_{n2}$
$\Delta\nu^{Mod_2}_{n3}$


$\tilde{\delta\nu}_{n,l}$ μHz
25
20
15
10
2000
2500
3000
3500
4000
Observed frequency μHz
$\tilde{\delta\nu}^{Tar_1}_{n2}$
$\tilde{\delta\nu}^{Tar_1}_{n3}$
$\tilde{\delta\nu}^{Mod_2}_{n2}$
$\tilde{\delta\nu}^{Mod_2}_{n3}$

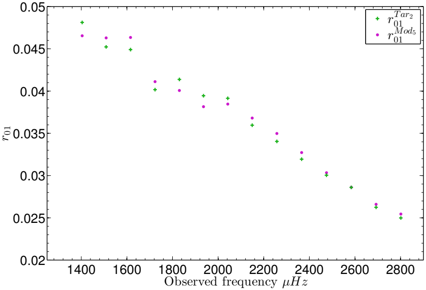

$\Delta\nu$ μHz
154
152
150
148
2000
2500
3000
3500
4000
Observed frequency μHz
$\Delta\nu^{Tar_1}_{n0}$
$\Delta\nu^{Tar_1}_{n1}$
$\Delta\nu^{Tar_1}_{n2}$
$\Delta\nu^{Tar_1}_{n3}$
$\Delta\nu^{Mod_3}_{n0}$
$\Delta\nu^{Mod_3}_{n1}$
$\Delta\nu^{Mod_3}_{n2}$
$\Delta\nu^{Mod_3}_{n3}$


$\tilde{\delta\nu}_{n,l}$ μHz
25
20
15
10
2000
2500
3000
3500
4000
Observed frequency μHz
$\tilde{\delta\nu}^{Tar_1}_{n2}$
$\tilde{\delta\nu}^{Tar_1}_{n3}$
$\tilde{\delta\nu}^{Mod_3}_{n2}$
$\tilde{\delta\nu}^{Mod_3}_{n3}$

Appendices



0.085
0.08
0.075
0.07
0.065
0.06
0.055
r_{02}
1400
1600
1800
2000
2200
2400
2600
2800
Observed frequency μHz
$r_{02}^{Tar_2}$
$r_{02}^{Mod_4}$
$r_{02}^{Mod_5}$



0.05
0.045
0.04
0.035
0.03
0.025
0.02
r_{01}
1400
1600
1800
2000
2200
2400
2600
2800
Observed frequency μHz
$r_{01}^{Tar_2}$
$r_{01}^{Mod_5}$

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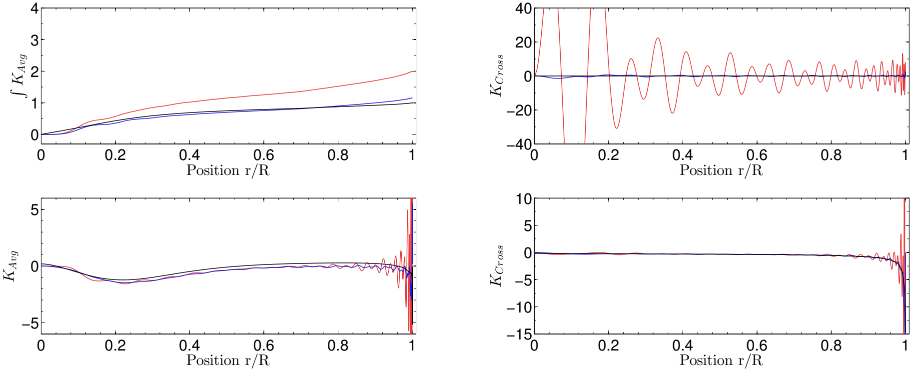
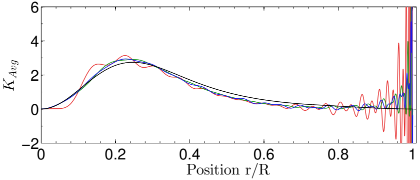
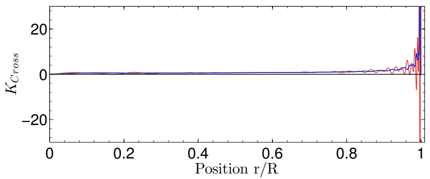


∫ K_Avg
K_Cross
K_Avg
K_Cross
Position r/R
Position r/R
Position r/R
Position r/R

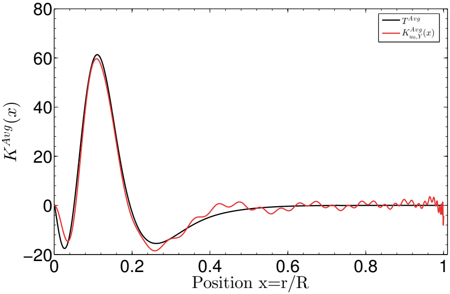
APPENDICES



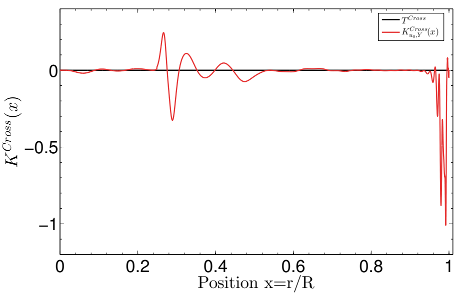
K_{Avg}
Position r/R



K_{Cross}
Position r/R



T^{Avg}
$K^{Avg}_{u_0,Y}(x)$
$K^{Avg}(x)$
Position x=r/R



T^{Cross}
$K^{Cross}_{u_0,Y}(x)$
$K^{Cross}(x)$
Position x=r/R

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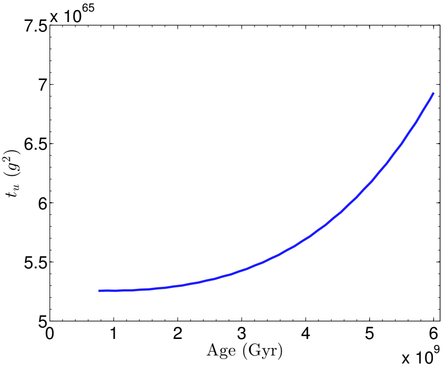
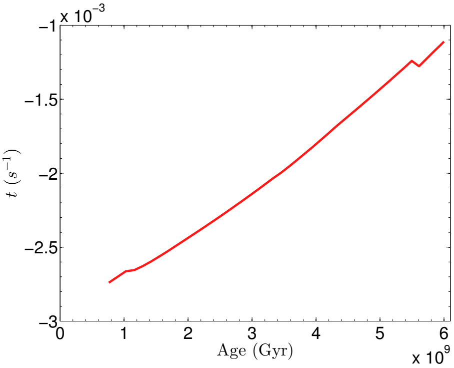


x 10^65
7.5
7
6.5
6
5.5
5
0
1
2
3
4
5
6
Age (Gyr)
x 10^9
t_u (g^2)



x 10^−3
−1
−1.5
−2
−2.5
−3
0
1
2
3
4
5
6
Age (Gyr)
x 10^9
t (s^{-1})