Stellar acoustic radii and ages from seismic inversion techniques

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Abstract

Context: Determining stellar characteristics such as the radius, mass or age is crucial for the study of stellar evolution, exoplanetary systems or the characterisation of stellar populations in the Galaxy. Asteroseismology is currently the most promising tool to accurately determine these characteristics. However, a key question is how to reduce the model dependence of asteroseismic methods.

Method: We extend the SOLA inversion technique to new global characteristics in addition to the mean density (see Reese et al. 2012). We apply our methodology to the acoustic radius and an age indicator based on the sound speed derivative. The results from SOLA inversions are compared with estimates based on the small and large frequency separations for several test cases, including differing mixing-lengths, and the presence or absence of non-adiabatic effects or turbulent pressure.

Results: We show that SOLA inversions yield accurate results in all test cases, unlike the other techniques which are more sensitive to surface effects. We observe that the acoustic radius and mean density inversions are more robust than the age indicator inversions, which are limited to relatively young stars with radiative cores.

Theoretical approach

The inversion procedure relies on the variational principle and the frequency-structure relation for adiabatic stellar oscillations. The fundamental equations used to carry out the inversions are the following:

$$\frac{\delta\nu_{n,\ell}}{\nu_{n,\ell}} = \int_0^1 K^{n,\ell}_{s_1,s_2}\frac{\delta s_1}{s_1}dx + \int_0^1 K^{n,\ell}_{s_2,s_1}\frac{\delta s_2}{s_2}dx + \frac{\mathcal{G}(\nu)}{Q_{n,\ell}}$$

$$\text{with: } \frac{\delta s}{s} = \frac{s_{obs} - s_{ref}}{s_{ref}}$$

where s_1, s_2 are structural variables such as Γ_1, c^2, ρ_0, ... $K^{n,l}_{s_i,s_j}$ the structural kernels, $\mathcal{G}$ an ad-hoc surface correction and $Q^{n,l}$ a normalisation factor.

The relative perturbation of a global characteristic A (e.g. $\bar{\rho}, \tau$, ...) can be related to structural variables:

$$\frac{\delta A_{obs}}{A} = \int_0^1 \mathcal{T}(x)\frac{\delta s_1}{s_1}dx + \int_0^1 \mathcal{T}_{cross}(x)\frac{\delta s_2}{s_2}dx$$

$\mathcal{T}$ and $\mathcal{T}_{cross}$ being the target functions. In the SOLA method, we minimise the following cost function to find the optimal frequency combination to reproduce $\frac{\delta A_{obs}}{A}$:

$$\mathcal{J}_A = \int_0^1 \left[K_{avg}(x) - \mathcal{T}(x)\right]^2 dx + \beta\int_0^1 \left[K_{cross}(x) - \mathcal{T}_{cross}\right]^2 dx + \tan(\theta)\sum_i (c_i\sigma_i)^2 + \lambda\sum_i [c_i - f]$$

Application to the acoustic radius, mean density and age indicator

We apply inversion techniques to the acoustic radius, τ, and the age indicator, t, which are defined as follows:

$$\tau = \int_0^R \frac{dr}{c} \qquad t = \int_0^R \frac{1}{r}\frac{dc}{dr}dr$$

SOLA inversions are compared with other techniques based on asteroseismic indices. Indeed, τ and t are related to the large and small frequency separations as follows:

$$\tau \simeq \frac{1}{2\Delta\nu} \qquad t \simeq \frac{-4\pi^2\nu\tilde{\delta}\nu}{(4\ell+6)\Delta\nu}$$

where ℓ is the degree of the mode and $\tilde{\delta}\nu$ the small frequency separation. We illustrate various kernels from SOLA inversions and from estimates based on asteroseismic indices in the following figures:

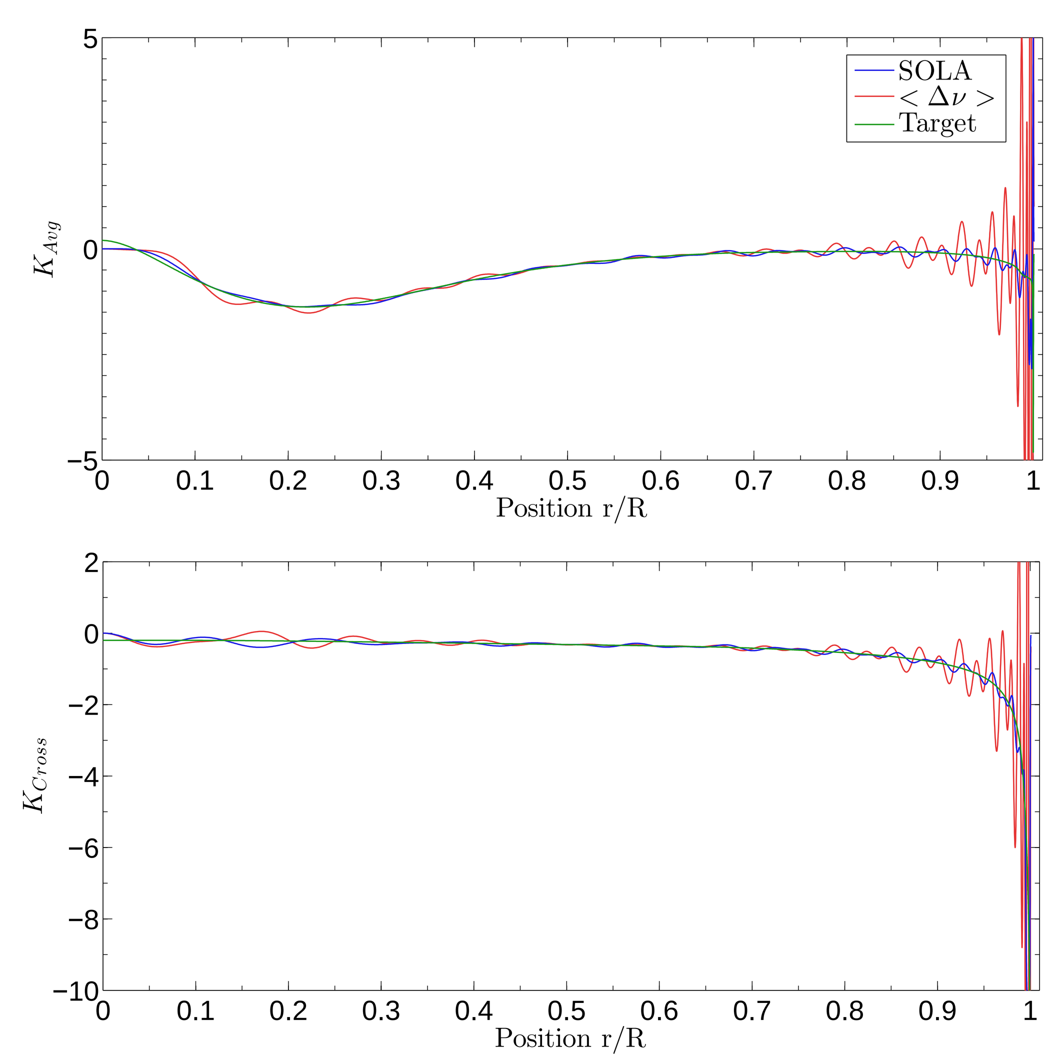

Fig. 1: Kernels for τ inversions using ρ_0, Γ_1 as s_1, s_2.

Reliability: The quality of the inversion depends on how well the averaging and cross-term kernels fit their respective targets.

Results for a grid of models

Method: we use 93 main sequence and pre-main sequence models (Marques et al. 2008) to determine the values of the τ and t indicators in the target (M = 0.9 $M_\odot$, age = 1.492 Gyr, R = 0.821 $R_\odot$ which includes surface effects). We used 33 frequencies with $\ell = 0-2$ and $n = 15-25$. The results are presented in figures 1 and 2. The vertical lines give the position of the best model from the grid in terms of $\tilde{\delta}\nu$ and $\Delta\nu$. Each point gives the result for a given reference model of the grid.

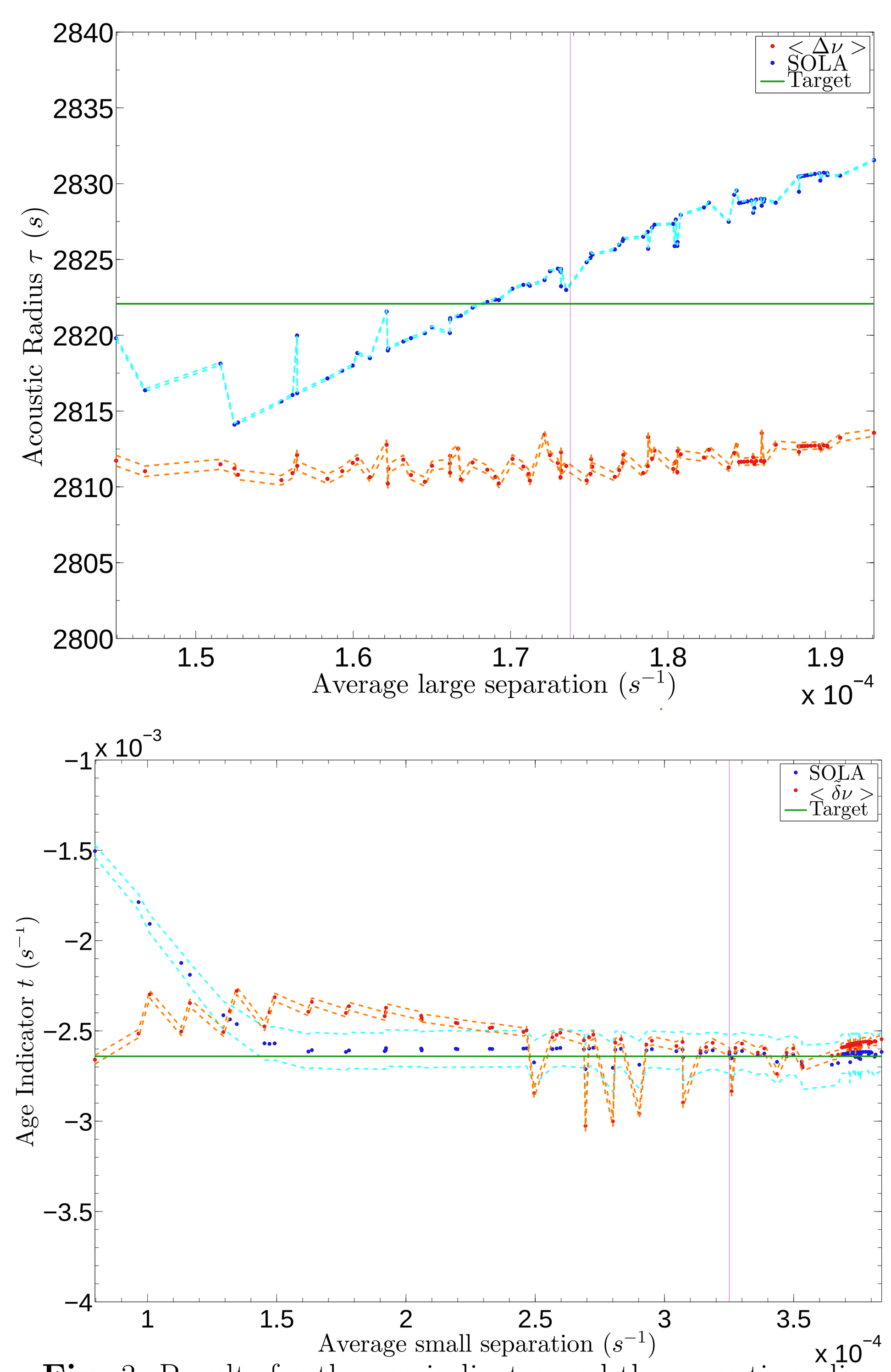

Fig. 3: Results for the age indicator and the acoustic radius.

Results: SOLA inversions yield better results when surface effects are present. Estimates based on seismic indices sometimes benefit from fortuitous error compensation, thereby leading to accurate results.

Results using forward modelling

Analysis: the results for t show that we need a *criterion* for selecting a reference model. Indeed, the inversion equations rely on the variational principle, which assumes that *the reference model is close to the star*. Therefore, the reference model needs to be chosen carefully, so we choose forward modelling. With the help of the OSM software, we fit $\langle\Delta\nu\rangle$ and $\tilde{\delta}\nu(\nu)$ using the same modes as in the previous tests.

Method: 3 targets are used, with masses ranging from 0.95 $M_\odot$ to 1.05 $M_\odot$ and ages ranging from 1.5 Gyr to 6 Gyr. They differ from the reference models by their mixing-length parameter (Model$_{\alpha_{conv}}$) and the presence or absence of turbulent pressure (Model$_{turb}$) or non-adiabatic effects in their frequencies (Model$_{nad_{1,2}}$). Results for the τ and t inversions are plotted in the figures below.

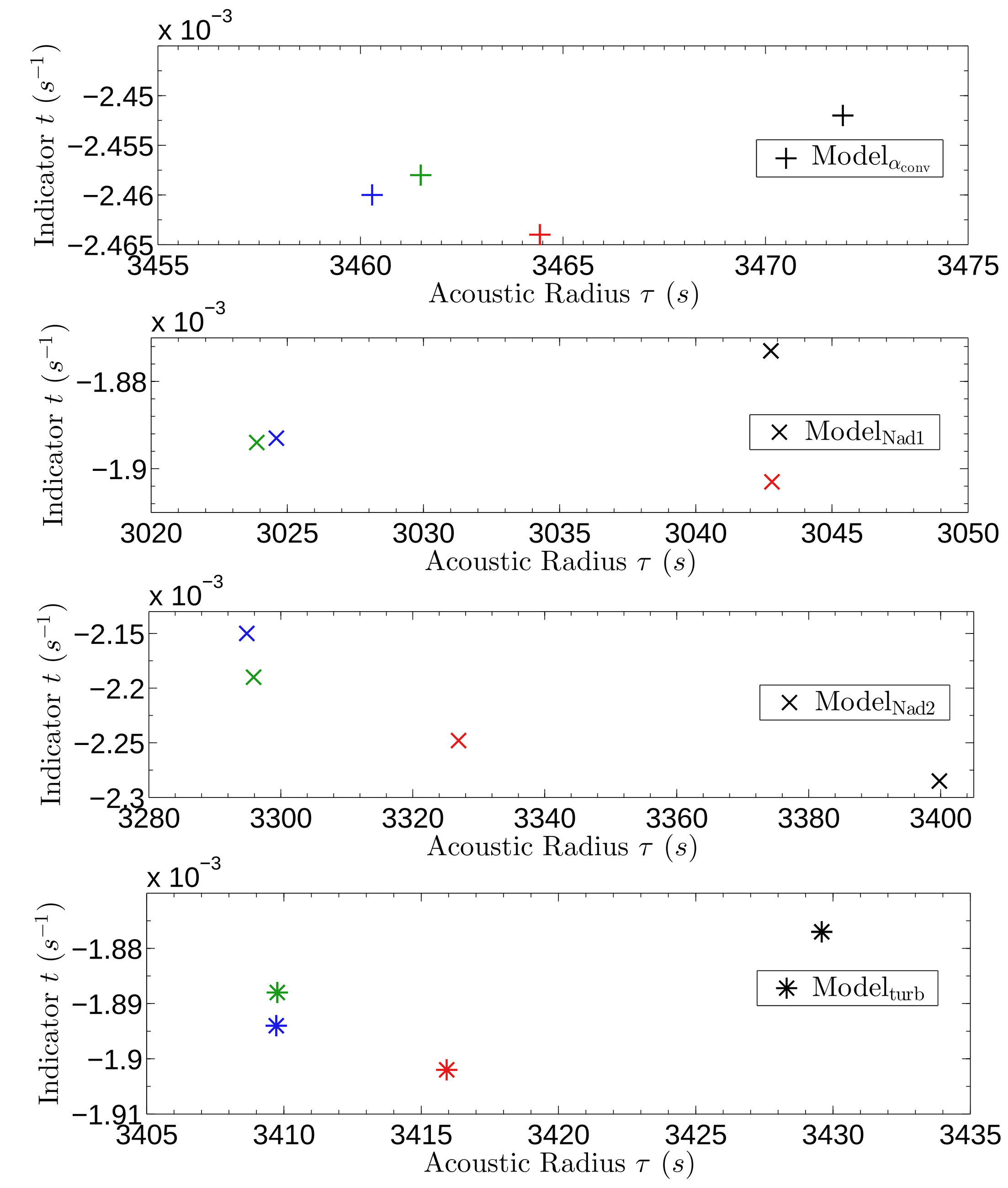

Fig. 5: Results for τ and t for each of the test cases. The results from forward modelling are in black, the estimates based on seismic indices in red, the results from SOLA inversions in blue and the true values for the targets in green.

Results and conclusions

The combination **forward modelling + SOLA inversions** can be used to accurately determine the indicators t, τ and $\bar{\rho}$ (not presented here) in observed stars. SOLA inversions always improves the accuracy of the determination of the indicator.

The SOLA approach is able to overcome limitations of forward modelling, is more accurate than estimates based on asteroseismic indices and allows us to choose the structural characteristics we wish to determine.

Prospects and conclusions

Conlusion: we set the basis of a *new framework* to determine stellar global characteristics using less model-dependent inversion techniques. We show that SOLA methods can handle the physical complexity of observed stars and overcome the limitations of forward modelling.

What's next? Provide a set of global characteristics allowing us to determine more accurately the age of stars and their structural properties.

How? The figure on the right shows evolutionary tracks of models with different masses in a t vs τ diagram. The sharp transition at higher masses result from the apparition of the convective core during the evolution of the star. This demonstrates the great sensitivity of global structural characteristics and thus their diagnostic potential.

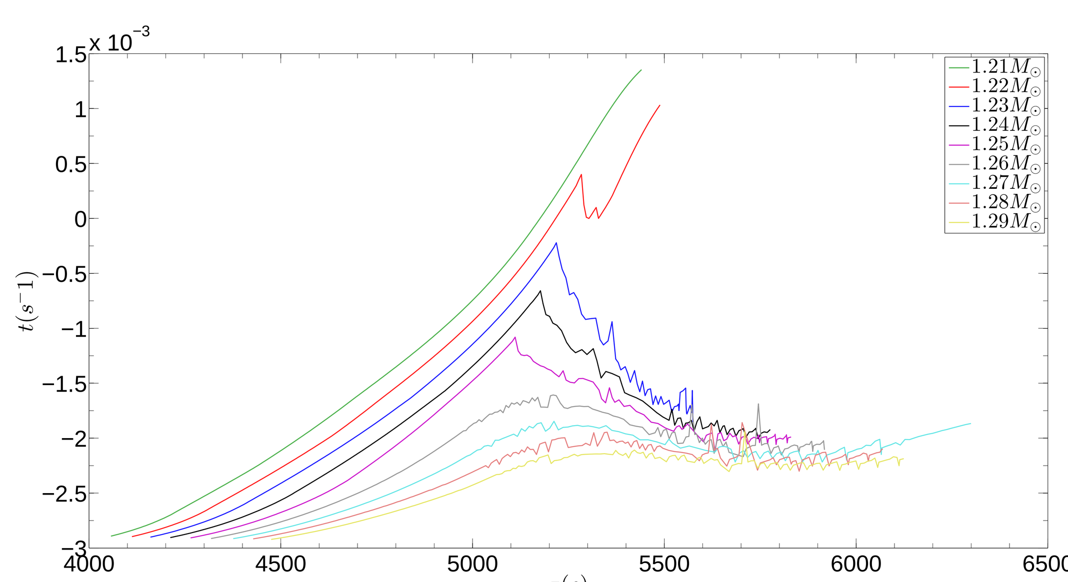

Fig. 4: Evolutionary tracks for massive stars in t vs τ plot.

References

- Dupret, 2001, A&A, 366, 166
- Marques et al., 2008, Ap&SS, 316, 173
- Reese et al., 2012, A&A, 539, A63