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Optimization methods for advanced design of aircraft panels: a comparison

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Abstract. Advanced nonlinear analyses developed for estimating structural responses for recent applications for the aerospace industry lead to expensive computational times. However optimization procedures are necessary to quickly provide optimal designs. Several possible optimization methods are available in the literature, based on either local or global approximations, which may or may not include sensitivities (gradient computations), and which may or may not be able to resort to parallelism facilities. In this paper Sequential Convex Programming (SCP), Derivative Free Optimization techniques (DFO), Surrogate Based Optimization (SBO) and Genetic Algorithm (GA) approaches are compared in the design of stiffened aircraft panels with respect to local and global instabilities (buckling and collapse). The computations are carried out with software developed for the European aeronautical industry. The specificities of each optimization method, the results obtained, computational time considerations and their adequacy to the studied problems are discussed.

1. Introduction

Since the pioneering work of Schmit [1,2] in the early seventies, in which the linear behavior of truss structures made of isotropic material under static loading were considered, the complexity of structural optimization problems has increased significantly in two aspects.

The first concerns the selection of the design variables and, as a consequence, the possibility to investigate not only the optimal size [3] and shape [4,5] but also the optimal topology [6] and material properties [7, 8], as depicted in Figure 1.

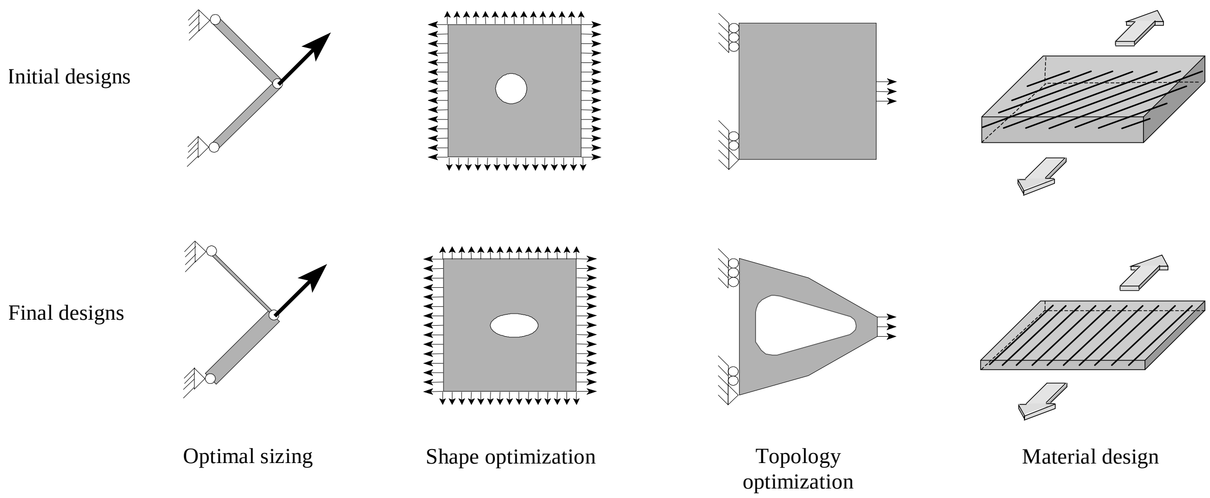

Figure 1. Structural optimization problems.

Secondly, nonlinearities have been taken into account in the formulation of the optimization problem. Those nonlinear effects can appear at the material [9] and/or at the geometrical levels [10] where so-called post-buckling and collapse scenarios are studied. Including such nonlinearities makes the problem much more intricate since both the optimization task and the structural analysis are nonlinear. Computing the sensitivities for the gradient-based optimization methods is more complex. For an overview of the developments carried out in the field, see [11,12].

In this paper we consider the optimization of stiffened panels used in aircraft constructions. Linear and nonlinear analyses based on the finite element method are carried out with *SAMCEF* [13], in order to estimate the buckling loads and the collapse (ultimate) load. These structural analyses are described briefly since they produce the functions appearing in the formulation of the optimization problem. The *BOSS quattro* task manager and optimization tool box is then briefly presented, together with the available optimization methods. These methods are then applied to two industrial test cases on stiffened panels and their efficiency in solving the problems is compared.

2. Functions entering the optimization problem

The aircraft panels considered in this paper include a flat skin and one stiffener. Two functions associated with their structural instability (buckling) and collapse (ultimate failure) are defined; both essential in the design of aircraft thin walled panels subjected to compression [14]. These two functions are implicit in the design variables. It is the case that they can not be expressed analytically and can only be evaluated with the finite element approach [15].

2.1 Buckling load factors λ_j

In the finite element formulation, the buckling loads are the eigenvalues λ of the following problem

$$(\mathbf{K} - \lambda_j \mathbf{S})\mathbf{\Phi}_j = 0 \qquad j = 1,2,... \tag{1}$$

where **K** is the stiffness matrix, **S** is the geometric stiffness matrix, and Φ are the eigen-modes (nodal displacements). The j[th] buckling load λ_j is the factor by which the applied load must be multiplied for the structure to become unstable with respect to the corresponding eigen-mode Φ_j. In an optimal design, the buckling loads should be larger than or equal to a prescribed value (say 0.8 or 1.2), meaning that the structure will buckle for a controlled (even desired) proportion of the applied load. Buckling is illustrated in Figure 2. This type of function is difficult to deal with, since buckling can be local and mode crossing can occur depending on the design variable values.

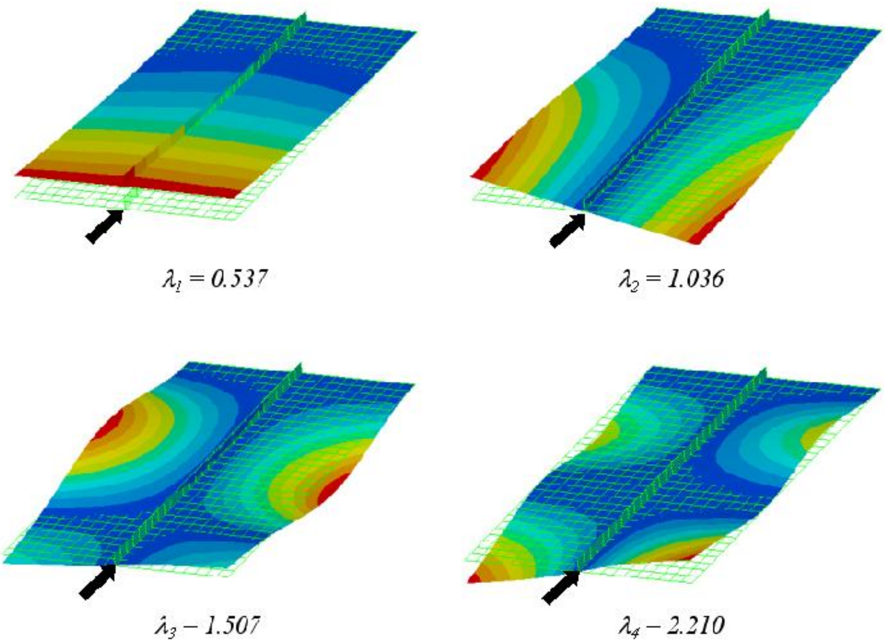

Figure 2. A stiffened panel buckling under a compressive load.
First four modes (Φ_j, j=1,…,4) are shown together with the associated buckling load factors.

2.2 Collapse load factor $\lambda_{collapse}$

Even when a stiffened panel buckles, it still can sustain a higher proportion of the applied load. This is observed experimentally [16] and can be modeled by means of the finite element approach (Figure

3). To compute the *collapse load*, which is the ultimate load that the structure can support, the analysis method must deal with geometric non-linearities. In this case, one is looking for successive equilibrium states for increasing values of the applied load. As can be seen in Figure 3, a maximum load can be estimated, corresponding to the collapse of the structure where large transversal displacements take place. After this peak, equilibrium is attained only if the load decreases, which results in an unstable configuration. To follow this unstable equilibrium path, Riks' so-called *continuation method* is used [17]. In an optimal design, the load factor $\lambda_{collapse}$ at collapse should be larger than or equal to unity, meaning that the structure can sustain its in-service loading.

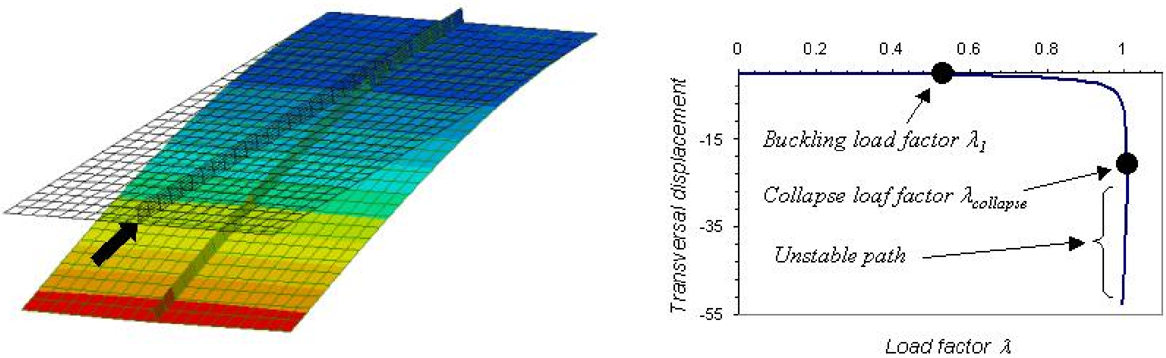

Figure 3. Collapse analysis in a stiffened panel and equilibrium path

The "linearized" buckling analysis based on the eigen-problem (1) is only an approximation of reality. Nonlinear analysis is more accurate since all nonlinear effects are taken into account, so allowing less conservative, lighter optimal designs. The main drawback of a non-linear analysis is that it is more expensive in terms of CPU.

2.3 Sensitivity analysis issues

To use gradient-based optimization methods, the first-order derivatives of the structural responses must be available. The sensitivity analyses for buckling and collapse are presented and discussed in [18,19] but for sake of brevity and clarity the details are not reported here. It is however important to note that the sensitivity of non-linear structural functions, such as the collapse load, is difficult to determine theoretically and to implement in a finite element code.

3. Optimization methods compared on aircraft panel optimization

3.1 The BOSS quattro optimization toolbox

The computational framework chosen for defining then running the optimization process is *BOSS quattro*, an open application manager for parametric design and optimization [20]. *BOSS quattro* allows a complete integration of the finite element software (e.g. *SAMCEF*) mentioned above for linear and nonlinear finite element analyses. As an application manager, it deals with the iterative design loop, alternating the structural analyses (including the sensitivity analysis, when needed) and the call to the optimizers. Several optimization methods are available in *BOSS quattro*: SQP, Augmented Lagrangian Method, specific approximation methods dedicated to structural optimization, derivative-free algorithms such as Genetic Algorithms, Surrogate-Based methods, and the more classical response surface capabilities. Since *BOSS quattro* is an open architecture, external optimizers can also be linked to it.

Three specific classes of optimization methods available in *BOSS quattro* are described in this section: Sequential Convex Programming (SCP), Surrogate-Based Optimization (SBO) and Derivative-Free Optimization (DFO). These methods are used to solve problem (2) below, where **x** is the set of design variables, w denotes the objective function to be minimized, and λ_j (j=1,…,m) are the constraints of the problem:

$$\min w(\mathbf{x})$$

$$a_j(\mathbf{x}) \geq a_j^{\min}\ ,\ j=1,\ldots,m \qquad (2)$$
$$\underline{x_i} \leq x_i \leq \overline{x_i}\ ,\ i=1,\ldots,n$$

In these three methods, a solution of the problem (2) is obtained by solving approximated problems (3) successively, where the ~ symbol denotes an approximation of the corresponding function, and *k* is the iteration index.

$$\min \tilde{w}(\mathbf{x})$$
$$\tilde{a}_j^{(k)}(\mathbf{x}) \geq a_j^{\min} \qquad j=1,\ldots,m \qquad (3)$$
$$\underline{x}_i^{(k)} \leq x_i \leq \overline{x}_i^{(k)} \qquad i=1,\ldots,n$$

SCP and SBO methods are original implementations [21,22], DFO is an external solver that has been especially linked to *BOSS quattro* for the purpose of a comparison of optimization methods. A standard genetic algorithm (GA) is also used, but not described here. For more details, see [23].

3.2 Sequential Convex Programming (SCP)
The solution of the initial implicit optimization problem (2) is replaced with the solution of a sequence of approximated sub-problems (3) which are explicit in terms of the design variables. The explicit and convex optimization problem (3) is solved by dedicated methods of mathematical programming, as described in [24], based on quadratic approximations solved by a dual approach. Building an approximated problem requires structural and sensitivity analyses (obtained from the finite element method). Solving the related explicit problem no longer necessitates a finite element analysis since the problem is now explicit in terms of the design variables. Each approximation is based on a particular first-order Taylor series expansion. The SCP method used in this paper generates mixed monotonous/non monotonous approximations, depending on the change of sign of the derivatives of the structural responses at two successive iterations [25]. The solution procedure for an unconstrained problem is illustrated in Figure 4. This method generates successive local approximations, and is therefore more likely to become trapped in a (possible) local optimum. It has proven to be reliable in many structural optimization problems [18,21,22] and usually provides a solution in few iterations (i.e. few structural and sensitivity analyses), irrespective of the size of the problem.

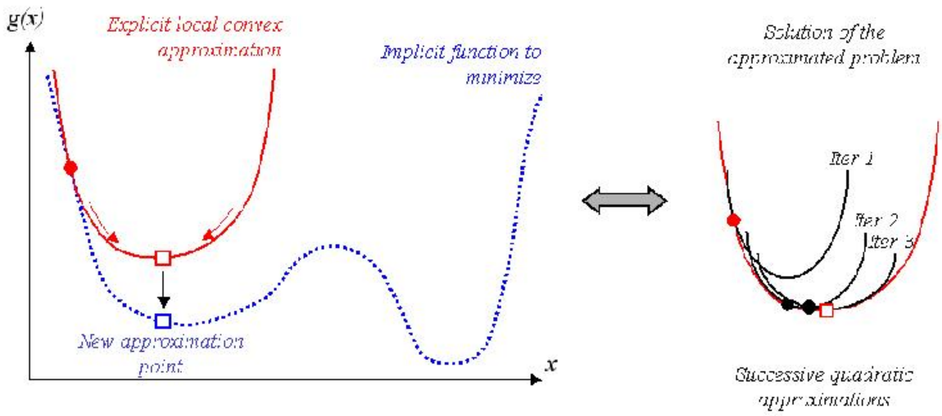

Figure 4. Solution procedure for the Sequential Convex Programming approach.

3.3 Derivative-Free Optimization (DFO)
Derivative-free optimization (see [26] for a recent monograph) aims at solving non linear optimization problems based on the function values only. The reason for not using derivatives is that the derivatives of some functions are impossible or very difficult to compute. Moreover their analytical

expression may be unknown, which can occur when the values of such a function correspond to the output of some "black box" software, measurements or experiments for instance.

DFO methods are based on the following principle: rather than approximating the missing derivatives of a function, which often proves to be expensive (through e.g. finite difference schemes), the function itself is approximated on the basis of its known values. An improvement in the objective function is then derived from this model.

This concept can be advantageously combined with a trust-region approach (see [27]), as shown in a series of papers (e.g. [28,29,30]): available function values are used to build a polynomial model interpolating the function at those points where it is known, and the model is then minimized within a trust region, yielding a new – potentially good – point. To check this possible improvement and compute trust-region ratios, the function is evaluated at the new point – thus possibly enlarging the interpolation set – and the whole process may be repeated until convergence is achieved (as represented in Figure 5).

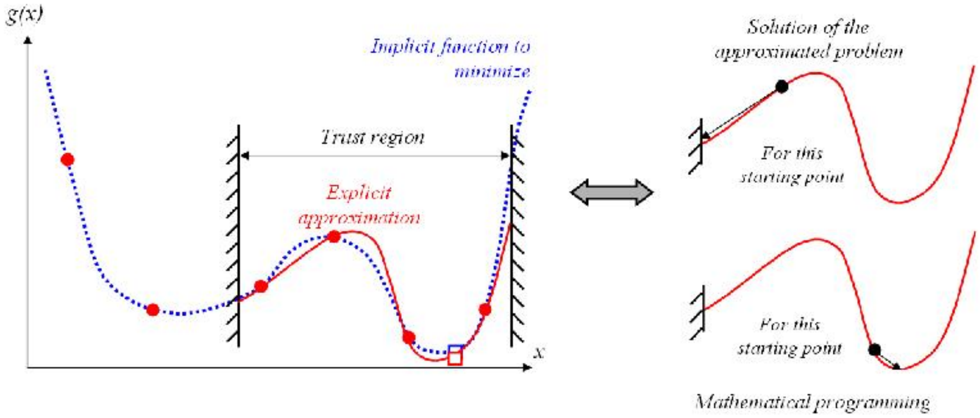

Figure 5. Solution procedure for the Derivative-Free Optimization approach.

Other components of the method include multivariate interpolation polynomials, namely (quadratic) Newton fundamental polynomials (see e.g. [31]), and a suitable strategy for adequately managing the geometry of the interpolation sets. Note also that the procedure may be generalized for handling constrained problems.

The results presented in Section 5 below include those obtained with DFO, an open source Fortran 77 package written by Scheinberg at IBM and available from COIN-OR [32]. The open nature of *BOSS quattro* as a task manager made it possible to build the necessary procedures for calling DFO as optimizer.

3.4 Surrogate-Based Optimization (SBO)

The third method tested is also the most recent one in *BOSS quattro*. A *surrogate model* – also known as *response surface* or *metamodel* – is essentially a low-definition function that approximates another function. It is expected to be a simpler representation of the original function, less accurate but much cheaper to evaluate. Classical and popular surrogate models are polynomial response surfaces, Kriging, support vector machines and artificial neural networks.

A simple surrogate-based method (called Basic SBO in the sequel) involves firstly the construction of the surrogate – based on available function values – then its direct optimization using a suitable algorithm. A genetic algorithm is often chosen given the variable nature of possible surrogates and the lack of direct derivatives.

The main drawback of such an approach is obviously the fact that the model remains unchanged in the course of the optimization process, hence the idea of an adaptive scheme, where an initial set of function values is first evaluated then used to compute a first surrogate model. This provides responses for an optimizer which in turn produces an optimal solution, at least from the point of view of the surrogate. The difference with the basic approach is that the original function is then evaluated at the corresponding point to check the accuracy of the surrogate model at the optimum. The output

from this validation step and some possible convergence criteria thereby provide the information necessary to determine whether to stop the whole process or to compute further function values to improve the surrogate. In the latter case, a further loop including the surrogate optimization and the optimum validation is performed. The SBO method we use for the comparisons of Section 4 below (see [21] for more details) features an additional "trust-region mechanism to ensure that the inner optimizer does not generate points outside the region where the surrogate is valid.
This is summarized on Figure 7 below.

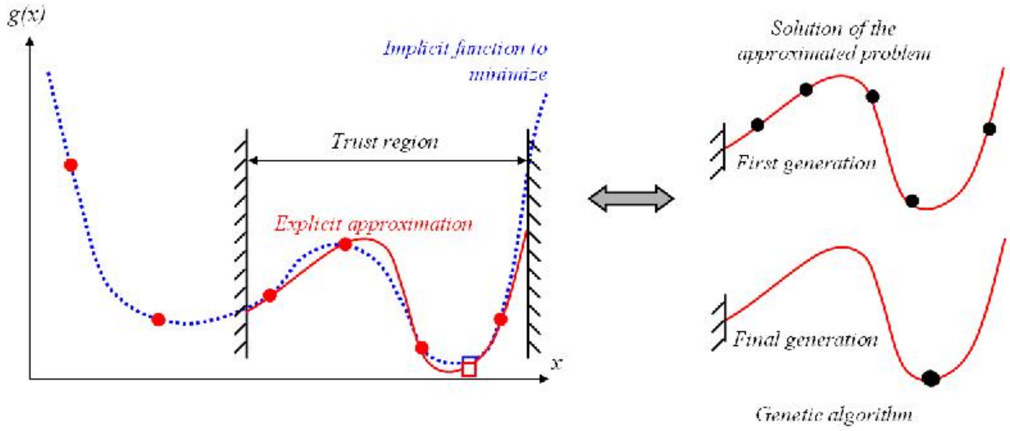

Figure 7. Solution procedure for the SBO approach.

An additional comment may be required at this point. While from a pure lexical point of view the SBO method may be considered as a *derivative-free method*, it remains a slightly different approach from the methods of the DFO family discussed above. The major difference between those two classes of algorithms is the fact that in practice SBO may use any type of surrogate, without specific requirements on possible properties (like differentiability), since a genetic algorithm will solve the subproblem.

4 Industrial application: optimal design of stiffened panels

Two applications are considered. The optimization problem to be solved is given in (2), while the structural functions entering the problem are explained in Section 2. For both applications, the objective function is related to the weight, which is to be minimized. Both problems include sizing and shape design variables.

The following five approaches are tested and compared:

- Sequential Convex Programming, SCP (see section 3.1);
- Derivative Free Optimization, DFO (see section 3.2);
- Surrogate-Based Optimization, SBO (section 3.3);
- the basic implementation of SBO (***Basic SBO)***, which amounts to a single iteration involving just one construction of the surrogate then its use within a genetic algorithm.
- a standard Genetic Algorithm (GA), with direct calls to simulation tools, without surrogate.

Some details about the parameters and setup may help understanding the results below:

- For the sole gradient-based method of the panel, namely ***SCP***, results are provided for derivatives computed with a semi-analytical approach (***SCP-sa***) and through a finite difference scheme (***SCP-fd***). Of course this last solution does not take advantage of the fact that sensitivities may be available directly from the finite-element simulation modules but this manner of proceeding allows a fair comparison with the other approaches, which do not require gradients of the original functions.
- ***DFO*** is run with the default values for parameters. No initial point is provided, so the first task performed by DFO is to generate two points, being the minimum required for computing a model. Since the Hessian matrix of the quadratic polynomial model is symmetric, a

complete model will be obtained after at least (n+1)(n+2)/2 evaluations (where n is the number of design variables), that is 28 and 55 evaluations respectively in the test cases studied below.

- The ***Basic SBO*** involves firstly the construction of an initial database with a central composite design of experiments (77 points for n=6 and 531 points for n=9). The buckling and collapse load factors are then approximated by neural networks (1000 iterations for training) while the other functions (section and aspect ratios) are used "as is" by a genetic algorithm (population of n x 10 individuals).
- ***SBO*** uses a Latin hypercube method to generate an initial set of points. Surrogates are neural networks (with 5000 iterations for training) and each iteration allows an enrichment of the database with up to 5 points evaluated in parallel.
- ***GA*** is used with a population including n x 10 individuals.

4.1 Z stiffened metallic panel

In this first problem, we want to find the best design of the metallic stiffened panel represented in Figure 8. Design variables are the lengths and thicknesses of the stiffener profile (*aft, wt, fft and ffw, sh*) and the thickness of the skin panel (*st*).

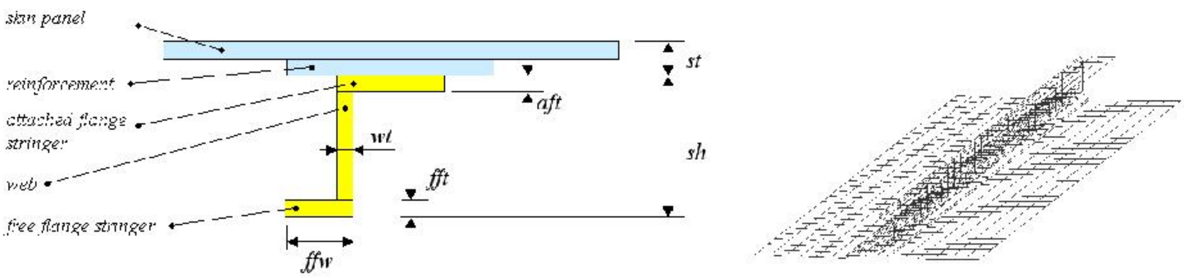

Figure 8. Stiffened metallic panel and associated design variables.

The objective function is the area of the profile of Figure 8 (skin panel and stiffener). The panel is subjected to axial and compression forces. The buckling and collapse loads must be larger than 1. The first 40 buckling loads are considered in the problem. Besides constraints on buckling and on collapse, some aspect ratios are also considered; the related constraints are expressed as follows:

a. Attached flange (AFR) : $3 < \dfrac{26.8 + wt}{aft} < 20$

b. Web (WR) : $3 < \dfrac{sh - aft - fft}{wt} < 20$

c. Free flange (FFR) : $3 < \dfrac{ffw}{fft} < 10$

d. Attached flange and skin thicknesses (AFSR) : $1.3 < \dfrac{aft}{st}$

As described earlier in Section 2, the buckling and collapse loads functions are naturally the most difficult functions to handle here since they are the output of finite-element analyses. The complete set of results is displayed in Table 1. The solution is presumed to be obtained when, for a feasible design, the relative variation of the design variables or the objective function is first lower than 1%.

	SCP-sa	SCP-fd	DFO	GA	Basic SBO	SBO
Iterations	20	29	132	33	-	39
Function evaluations	20	197	216	1955	77+1	255
Variables at optimum						
sh	47.754	46.299869	39.57937	45.8235	43.8235	46.7532
aft	3.009	3.097095	3.66015	3.63294	2.80471	3.80582
fft	2	2.096575	3.6894	2.32157	3.95294	3.51732
ffw	16.182	19.15786	11.44599	17.3882	14.7765	10.8911
wt	2.137	2.056794	3.91017	2.01412	1.90118	2.04861
st	2.07	2.059153	2.274	2.02667	1.6	1.98933
Functions at optimum						
Section (weight)	550.89	552.063541	655.1053	556.791	471.09729	553.761
$\lambda_j \geq 1$	1	1.000192	1.53838	1.03851	1.52798	1.015
$\lambda_{collapse} \geq 1$	0.999	1.013024	0.99999	1.02629	0.51826	1.00039
AFR	8.951	9.317372	8.39039	7.93135	10.2332	3.09642
WR	19.999	19.985567	8.24255	19.7947	19.49623	19.21312
FFR	8.091	9.13769	3.10239	7.48985	3.7381	3.12393
AFSR	1.453	1.504062	1.60956	1.79257	1.75294	1.91312

Table 1: Numerical results for the metallic stiffened panel optimization.

A first comment concerns the quality of the solutions: the basic SBO method leads unsurprisingly to the worst solution (the collapse reserve factor is violated by 50%), which simply confirms the need for an adaptive scheme as previously discussed. While SCP, the genetic algorithm and SBO provide very similar solutions, DFO cannot decrease the section below 655.10537, which remains high in comparison with other results. This might suggest that it stalled at a local solution, as indicated by the values of the variables at optimum, which are significantly different to those produced by the other approaches. The convergence history for SCP-sa and SCP-fd is different and can be explained by some approximations made in the semi-analytical sensitivity of the buckling loads [18]. Using SCP-sa is of course more advantageous in a CPU point of view, but requires the computation of the semi-analytical derivatives. When finite differences are used, parallel computations are performed to decrease the computational time.

Let us now compare the computational costs of each method. SCP is the cheapest method, followed by SBO, while a standard genetic algorithm is about ten times more expensive. (Note that for the latter method, the optimal solution was actually found after 1800 function evaluations). In terms of pure performance, SCP may launch all runs with perturbed values of variables for finite differences in parallel (which means 6 runs in this case) while the chosen parameters for SBO allow at most 5 runs in parallel (for database enrichment).

4.2 Z-stiffened composite panel

The stiffened panel considered in this problem is illustrated in Figure 9. It includes a metallic stiffener and a composite skin, linked with specific rivets elements. The structure is subjected to axial and shear forces. The design variables are the lengths and thicknesses of the stiffener profile (*aft*, *wt*, *fft*, *ffw* and *sh*). The skin is made of a [90/45/-45/0] laminate. The ply thicknesses *thick1*, *thick2*, *thick3* and *thick4*, related to plies oriented at 0, 45, 90 and –45° respectively, are also variable. The problem therefore includes 9 design variables.

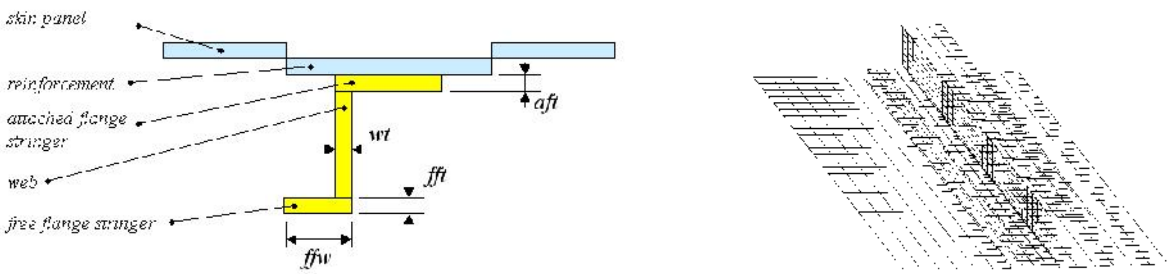

Figure 9. Stiffened composite panel and associated design variables.

Problem (2) is solved, where *w* is the structural weight. The buckling and collapse loads must be larger than 1 and 1.2, respectively. The first 12 buckling loads are considered in the problem. The solution can be assumed when, for a feasible design, the relative variation of the design variables or the objective function first becomes lower than 0.1%.

	SCP-sa	**SCP-fd**	**DFO**	**GA**	**Basic SBO**	**SBO**
Iterations	22	14	225	30	-	28
Function evaluations	22	131	406	2611	531+1	254
Variables at optimum						
sh	35.6450	40.8036	36.1301	36.3292	35.2473	38.8058
aft	1.6000	1.6000	1.6006	1.7296	2.1365	1.6197
fft	4.0000	2.0000	2.0000	3.0201	3.1173	2.1725
ffw	9.1998	14.5648	20.1031	11.2978	12.0844	13.3344
wt	1.6000	1.6000	1.7047	1.8156	1.6018	1.6015
thick1 (0 deg.)	0.3895	0.2086	0.4443	0.3413	0.4754	0.4060
thick2 (45 deg.)	0.3634	0.2000	0.1999	0.2346	0.2524	0.2000
thick3 (90 deg.)	0.2042	0.2000	0.2000	0.2329	0.3540	0.2018
thick4 (-45 deg.)	0.2013	0.5934	0.9790	0.6975	1.0000	0.6120
Functions at optimum						
Mass	0.3613	0.3334	0.3627	0.3652	0.4042	0.3413
$\lambda_j \geq 1$	0.9999	1.0130	1.7712	1.6242	1.7018	1.5182
$\lambda_{collapse} \geq 1.2$	1.1999	1.1949	1.1999	1.2524	1.3469	1.1992

Table 2: Numerical results for the composite stiffened panel optimization.

As for the first test case, the basic SBO produces the worst solution overall. However it is important to notice a much better behavior of the method (no constraint violation at solution). This is probably due to the fact that the central composite design required 531 function evaluations (since there are 9 variables) and thereby produced more accurate response surfaces. Both runs of SCP yield the best solutions, with both reserve factors active at optimal solution. The other methods (DFO, GA and SBO) found local solutions and they did not manage to secure the activation of the buckling reserve factor. Note that DFO performs better than in the previous test case and yields a mass close to that obtained by SCP-sa.

In terms of CPU costs, the trend observed in the previous section is confirmed, with SCP and SBO requiring fewer function evaluations than the other methods. DFO confirms its better performance in the second test case by reducing the gap with SBO, while the standard GA remains much more expensive. The major difference between SCP-sa and the other methods (SCP-fd, DFO, GA and SBO) is that the latter require more than one function evaluation per iteration, which can be parallelized.

5 Conclusions

This paper focuses on the application of various recent optimization algorithms to solve two industrial optimization problems in the framework of airplane design. These problems may be considered challenging for they involve the output of both linear and nonlinear structural analysis simulation codes in the constraints. The numerical experiments demonstrate initially that classical gradient-based methods remain competitive, even when finite differences are used. However, with the increase of the finite element model size and resulting simulation times, surrogate-based optimization methods appear promising. The challenges in that research field lies both in the use of adequate models and in the development of methods able to exploit the maximum amount of information from a limited number of simulations.

Our test cases also suggest that mixed approaches combining approximation or surrogate models (for complex functions) and the original functions (for simple analytical expressions like aspect ratios) should perform well. A further advantage of surrogate-based methods is that their exploratory nature makes them particularly well-suited to parallel implementations.

To conclude, for problems where complex non-linear analyses are considered in the design problem, SCP should be used when the derivatives are available. If the sensitivities are not computed, SBO appears to be an interesting alternative.

Finally, since the size of industrial optimization problems is tending to increase (both in terms of number of variables and number of functions), future work will also be dedicated to comparisons involving such larger problems.

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