Composites Science & Technology, 66, pp. 1303-1314, 2006.

A general and effective approach for the optimal design of fibers reinforced composite structures

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Abstract

This paper presents a general and effective procedure based on a mathematical programming approach for composite structures optimal design, under weight, stiffness and strength criteria. Effective means that the developed approach is able to find a local optimum in few iterations, even with a large number of design variables. In addition, as the formulation may include strength constraints, it is shown that not only plies thickness but also fibers orientations should be considered as design variables. Besides this, it is explained how these angular variables can improve the structural performance. The generality of the approach is related to the fact that the developed optimization technique is not only dedicated to the design of particular laminates configurations, but is also reliable for general problems including isotropic and/or anisotropic materials, as well as other structural responses. The design problem formulation relies on a direct parameterization of the laminates in terms of the physical design variables, that is orientations and thicknesses, and not on alternative ones like the lamination parameters. Numerical applications, including a non homogeneous composite membrane design problem and an industrial test case are presented.

Key words optimization, fibers orientations, strength criterion, finite element modelization

1. Introduction

As a result of their large number of definition's parameters, the design of fibers reinforced composite structures naturally calls for optimization algorithms. However, as a result of their unusual complex structural behavior, a small amount of work has been dedicated to their optimal design compared to problems involving isotropic materials. Structural responses of composites are highly non linear, non monotonous and non convex over fibers orientations. Moreover, in problems involving fibers orientations and ply thickness design variables, the structural responses present mixing non monotonous and monotonous behaviors (Figures 1 and 2). Because of the solution complexity of this type of problems, the development or the selection of reliable optimization algorithm are difficult. An alternative parameterization based on the lamination parameters can simplify the laminate's optimization problem [1,2]. However, the related formulations are restricted to global structural responses like stiffness and don't generally include local strength constraints at the ply level. Optimal design studies are limited to particular structural responses [3-5], and to particular laminates configurations as well [6-8]. It is concluded that solving general applications requires a parameterization based on thicknesses and orientations, and dedicated reliable optimization algorithms.

This paper focuses on the problem of selecting an efficient solution procedure to handle linear elastic composite materials optimization. Applications of this procedure to real-life problems will show that the realized new progress open the door to solve composite problems in a general and robust way, as it is already the case with isotropic materials. The efficiency of the optimization procedure relies on two main points. On one hand, the selection of a rigorous formulation for design problems including strength constraints. In this context, it is shown that fibers orientations should be considered as design variables. On the second hand, an efficient gradients based optimization algorithm is presented for working simultaneously with ply thickness and fiber orientation design variables. The optimization scheme developed in [9] was found to be reliable for simple laminates design [10]. It can consider simultaneously monotonous and non monotonous structural behaviors, by generating automatically the best structural approximation of a design function according to each design variable type (Figures 1 and 2). This new solution procedure has been made available in the environment of the optimization software BOSS/Quattro [11] which is linked to the SAMCEF finite elements code [12]. Results obtained on more realistic applications of complex geometries are presented and prove the efficiency of the optimization approach. A multi-objective formulation is also carried out. Solved problems include continuous design variables, that is the thicknesses and the orientations can take any value between lower and upper bounds.

The outline of the paper is as follows. Section 2 presents the key role of the fibers orientations in the optimal design of composite structures: their influence on the structural responses like stiffness and strength is illustrated, and their usefulness in a strength design problem is demonstrated. In section 3, we briefly discuss the choice of the selected laminates parameterization. In section 4, the optimization approach used in this paper is presented. It relies on the approximation concepts approach. It efficiently and simultaneously takes into account plies thickness and fibers orientations in the optimal design. Section 5 is dedicated to the numerical applications.

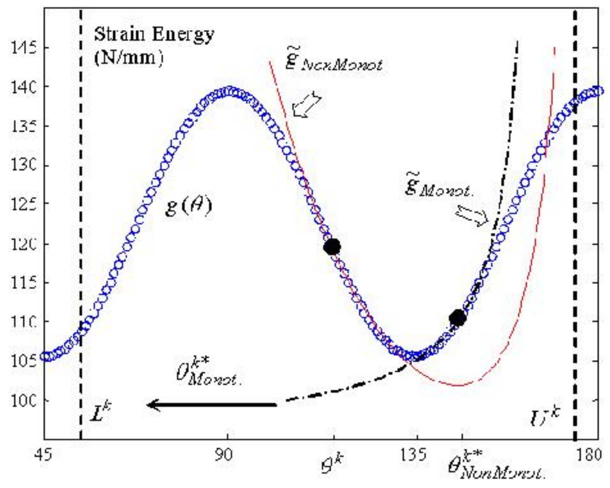

Fig. 1 Non monotonous variation of the strain energy density in a single ply laminate with respect to the fiber orientation. A non monotonous approximation of the design response is advised

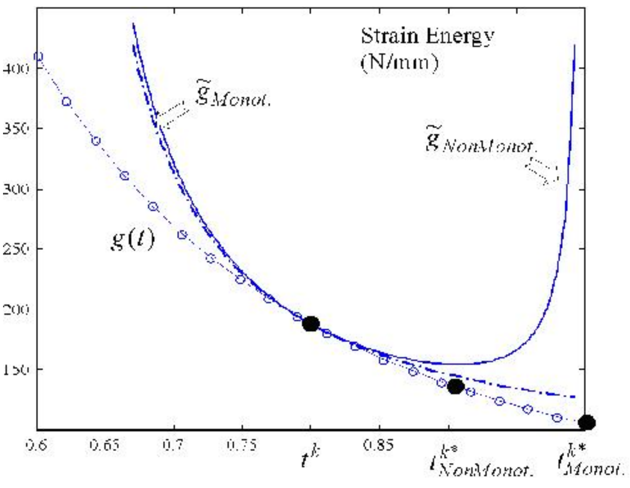

Fig. 2 Monotonous variation of the strain energy density in a single ply laminate with respect to the ply thickness. A monotonous approximation of the design response is advised

2. They key role of fibers orientations in the optimal design of laminates

In this section, it is shown that the computation of the optimal fibers orientations leads to great savings in structural performances of composite structures and can help in finding the correct optimal plies thickness in a problem including strength constraints [13].

2.1 Improvement of the structural behaviors

Figure 3 illustrates the design domain of a unidirectional subject to an in-plane shear load. The ply thickness and the fibres orientation are allowed to vary; the iso values of the strain energy density are plotted. The laminate' stiffness increases when its strain energy density decreases. Considering the initial design *(A)*, the structural stiffness may be improved by increasing the ply thickness to point *(B)*. Doing so, the design is located in a region of low structural performances characterized by a too high weight. For a constant ply thickness, the minimum of the strain energy is found for optimal values of 45 or 135°, along the direction *(AC)*. A better solution consists in changing simultaneously the fibers orientation and the ply thickness to reach the design located in *(D)*, with an optimal angle of 135° and the maximum allowable resource constraint (limited weight).

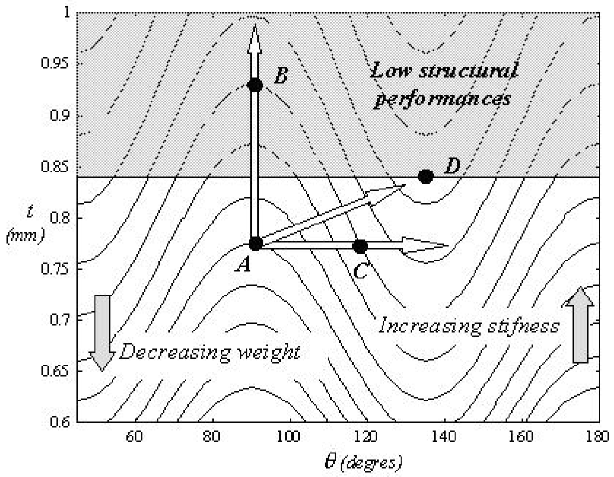

Fig. 3 Iso-values of the strain energy. Improvement of the structural performances: weight and stiffness criteria

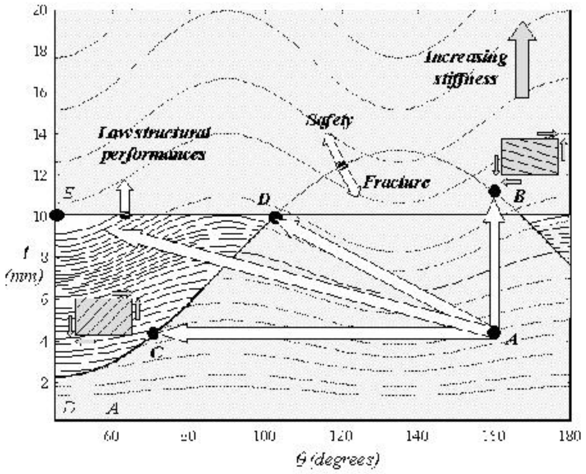

Fig. 4 Improvement of the structural performances: weight, strength and stiffness criteria

In Figure 4, the same problem is considered with a ply strength restriction. The design domain is divided in ply's failure and safety regions, according to the Tsai-Wu criterion. There also exists a region of low structural performances, again characterized by a too high ply thickness, given that a limitation on the weight is generally considered in the design problem. Iso values of the strain energy density are plotted. The structural stiffness increases in the indicated direction. Starting from the initial configuration (*A)*, a first way to design a safe laminate is to increase the ply thickness (*B*), with the risk to have a too heavy laminate. A second choice is to optimize fibers orientation at constant thickness (*C*). Finally, both ply thickness and fibers orientation can be optimized (*D*, *E*), leading to admissible solutions of higher stiffness.

2.2 Strength constraints in laminates design

The problem arising in laminates design including strength constraints was first observed by Schmit and Farschi [14]. To illustrate it, we consider a symmetric $[0/90]_S$ laminate subject to an in-plane axial load (Figure 5).

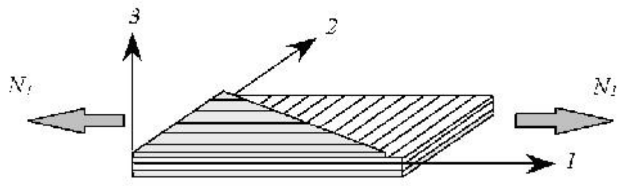

Fig. 5 Laminate under in-plane axial loading

The optimization problem (1) consists in minimizing the laminate's weight (i.e. its thickness), under the Tsai-Wu strength criterion. The plies thicknesses can vary continuously between a lower and an upper bound.

$$\begin{gathered} \min 2\times(t_{0^\circ}+t_{90^\circ}) \\ T.W.(t_i)\le 1 \qquad i=90^\circ,0^\circ \\ 0.01mm \le t_i \le 5mm \end{gathered} \tag{1}$$

The iteration history for solving problem (1) is illustrated in Figure 6. The optimization procedure used for obtaining these results is described in section 4. It is shown in Figure 6 that the optimal solution is characterized by a *90°* plies thickness equal to the lower bound *0.01mm* ('O' in Figure 6). It can be concluded that those plies tend to disappear at the solution, what is obvious according to the unidirectional load case (Figure 5). According to this observation, the problem is reformulated with only *0°* plies, that is for an initial $[0/0]_S$ laminate. As seen in Figure 6, the corresponding optimal solution is, in this case, better with regards to the total thickness of the laminate ('+' in Figure 6), although the contribution to the total thickness of the *90°* plies in the first solution was negligible.

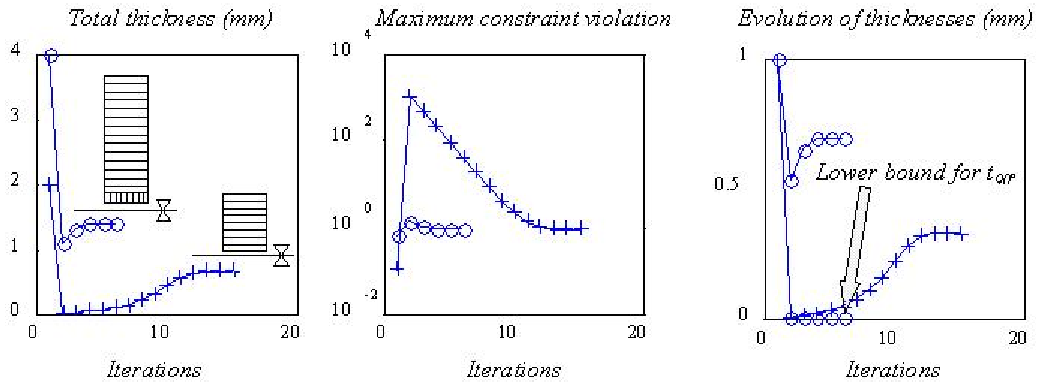

Fig. 6 Iteration history for the solution of problem (7). A strength constraint is violated when its value is larger than 1

This surprising behavior can be explained by plotting the design domain of problem (7), which is illustrated in Figure 7. The iso values of the objective function are parallel lines representing the total weight, which decreases in the indicated direction. The Tsai-Wu limiting values of each ply are also plotted. They divide the design domain into regions of safety and failure. The feasible solutions are located in the safety region. The optimal solutions for an initial $[0/90]_S$ and for an initial $[0]_S$ are the points that minimize the total thickness while satisfying the plies strength restrictions. They are noted $[0/90]_S^*$ and $[0]_S^*$ in Figure 7. Given that the true optimal solution of problem (7) is characterized by a $[0]_S$ laminate, the dashed vertical line joining the two solutions is also included in the feasible domain. Unfortunately, this part is not directly accessible by classical optimization algorithms, and so-called ε-relaxation techniques, used in topology optimization, should be used [15] to change locally the design domain and to allow the optimization procedure to reach $[0]_S^*$. It is shown here that such a mathematical manipulation doesn't have to be considered in the context of fibers reinforced composite structures design. The fibers orientations optimization can indeed help in finding the optimal solution of problem (7) corresponding to a $[0]_S$ laminate. This is illustrated in Figure 8. When fibers orientations are taken into account in the design process, the design domain changes (Figure 8), and when the *90°* plies take the optimal value of *0°*, the unreachable dashed region disappears and the true optimal solution is easily found. It is observed in Figure 8 that a great saving in structural weight can already be obtained for orientations close to their optimal values (see the '10°'s case').

With regards to those two key roles of fibers orientations, it is evident that there is a need for an effective optimization algorithm reliable for fibers and ply thickness optimization. Such an optimization method is described in section 4.

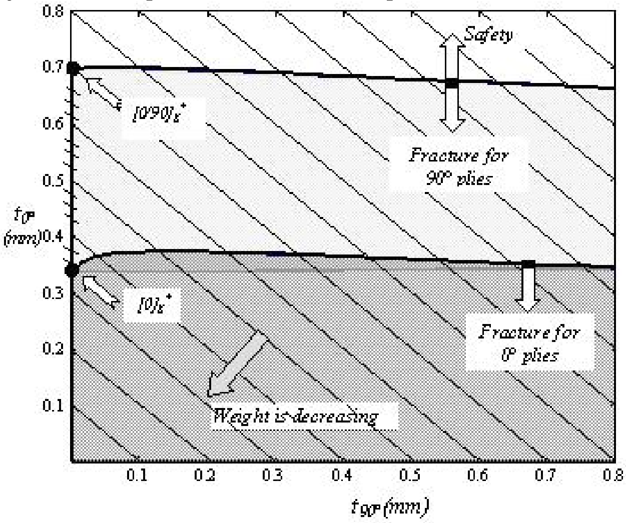

Fig. 7 Design domain of problem (1)

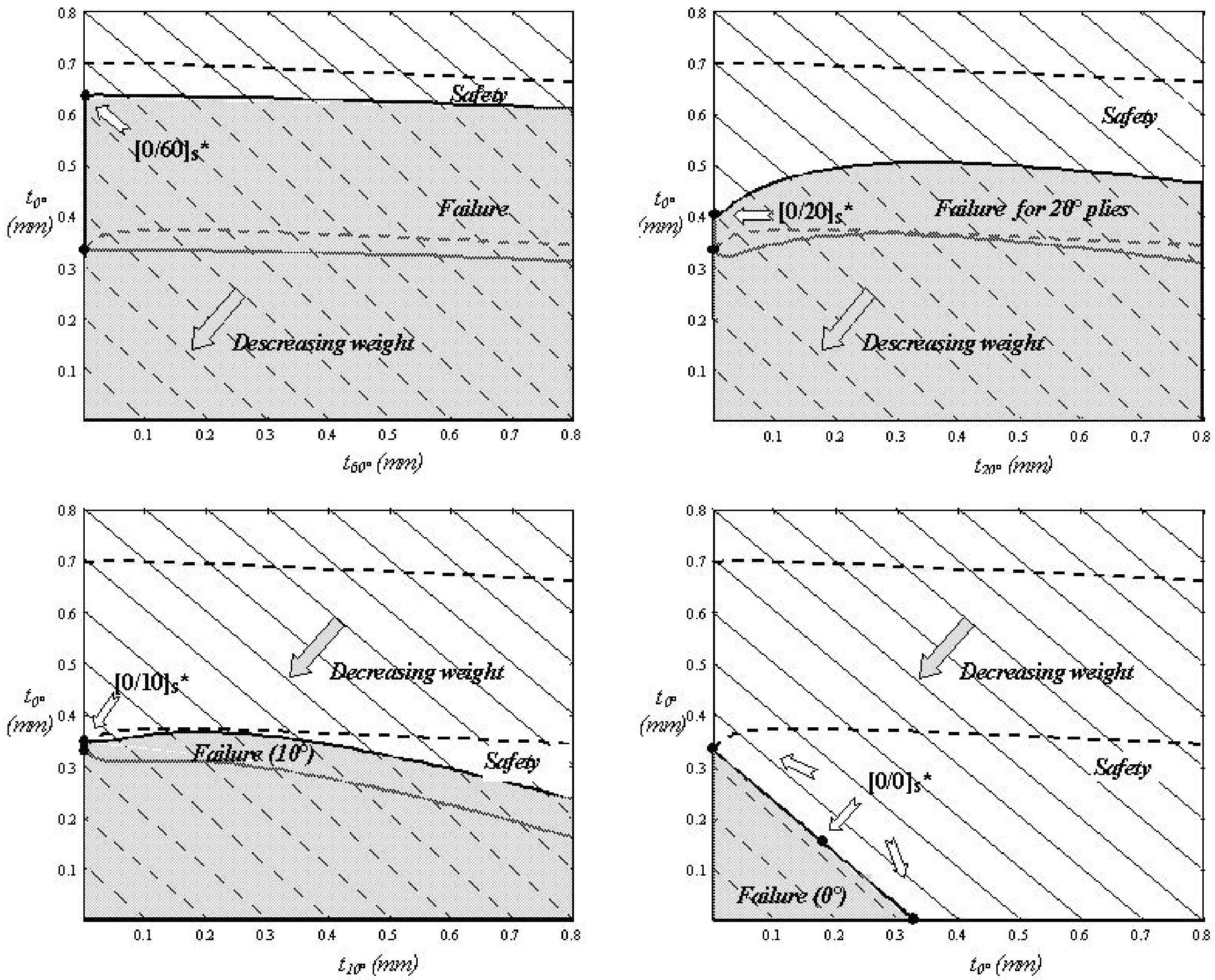

Fig 8 Design domains of design problem (1) for *[0/60]S*, *[0/20]S*, *[0/10]S* and *[0]* laminates.

3. Discussion about the parameterization and the features of the optimum

The selection of a parameterization strongly influences the generality of a design problem. As briefly explained in the introduction, at least two parameterizations can be considered for composites (see [16] for a review).

The first parameterization is defined at the ply level. It considers local design variables, that is the ply thickness t and the fibers orientation ¯ , and allows the computation of local structural responses like strengths (via a Tsai-Wu or Tsai-Hill criteria). Given that it includes physical and manufacturing variables, the results are directly exploitable by the user. However, the related optimization problem is not convex and local optima exist (Figures 1, 3 and 4). Besides this, the number of design variables increases with the number of plies.
A second parameterization consists in using global design variables called the lamination parameters. Such variables allows to characterize the global structural responses of a laminate, like stiffness, frequencies or buckling load [1-6]. However, local strengths can only be computed in very particular laminates configurations [17]. Besides this, the optimization is performed in the spaces of lamination parameters which are not yet completely known, that is only specific design problems can be solved [7,8,18]. Moreover, the solution is expressed in terms of the lamination parameters, and an inverse problem has to be solved in order to get the corresponding number of plies, thicknesses and fibers orientations [19]. The advantage of this parameterization relies on a small number of design variables and the convexity of the design space.

Since strength considerations are very important for the composite structures design, the first parameterization is selected. Plies thickness and fibers orientations are continuous design variables, that is they can take any value between a lower and an upper bound. According to the selected formulation of the problem, the design domain is not convex, and the solution will correspond to a local optimum, providing the user with a better solution or, anyway, with a feasible one. A particular mathematical programming approach called approximation concepts approach is used and is explained in the next section.

4. Optimization procedure

The approximation concepts approach is used to solve the general optimization problem (2) where $g_0(\mathbf{X})$ is the objective function and $g_j(\mathbf{X})$ are the structural restrictions. According to the generality of this approach [20,21], the structural responses can be of any type (static, dynamic, etc, including weight, stiffness, strength, frequency, etc). In this paper, however, the formulation is limited to weight, stiffness and strength criteria.

$$\begin{gathered} \min g_0(\mathbf{X}) \\ g_j(\mathbf{X}) \le g_j^{\max},\ j = 1,...,m \\ \underline{x_i} \le x_i \le \overline{x_i},\ i = 1,...,n \end{gathered} \qquad (2)$$

Here, the design variables $\mathbf{X} = \{x_i, i = 1,...,n\}$ are the plies thickness and the fibers orientations. In the approximation concepts approach, the primary optimization problem (2) is replaced at each iteration k by the solution of a sequence of explicit approximated sub-problems (3) generated through first or second order Taylor series expansion of the structural functions in terms of specific linearization variables.

$$\begin{gathered} \min \tilde{g}_0(\mathbf{X}) \\ \tilde{g}_j(\mathbf{X}) \le g_j^{\max},\ j = 1,...,m \\ \underline{x_i} \le x_i \le \overline{x_i},\ i = 1,...,n \end{gathered} \qquad (3)$$

The explicit optimization sub-problem (3) is then solved by a mathematical programming technique, based on a dual approach [20]. The generalized first order approximation scheme (4) derived in [9] is used to approximate each structural response entering the optimization problem (1). This scheme, called GBMMA-MMA, is based on the non monotonous GBMMA approximations [22] illustrated in Figure 1, and on the monotonous MMA [23] illustrated in Figure 2.

$$\begin{aligned} \tilde{g}_j(\mathbf{X}) = g_j(\mathbf{X}^k) &+ \sum_{i \in A} p_{ij}^k \left(\frac{1}{U_i^k - x_i} - \frac{1}{U_i^k - x_i^k} \right) \\ &+ \sum_{i \in A} q_{ij}^k \left(\frac{1}{x_i - L_i^k} - \frac{1}{x_i^k - L_i^k} \right) \\ &+ \sum_{+, i \in B} p_{ij}^k \left(\frac{1}{U_i^k - x_i} - \frac{1}{U_i^k - x_i^k} \right) \\ &+ \sum_{-, i \in B} q_{ij}^k \left(\frac{1}{x_i - L_i^k} - \frac{1}{x_i^k - L_i^k} \right) \end{aligned} \qquad (4)$$

In (4), U_i and L_i are two asymptotes (see Figures 1 and 2) that give the name to the method (Method of Moving

Asymptotes). Symbols $\sum(+,i)$ and $\sum(-,i)$ designate the summations over terms having positive and negative first order derivatives, respectively. *A* and *B* are the sets of design variables leading to a non monotonous and a monotonous behavior respectively, in the considered structural response. At a given stage *k* of the iterative optimization process, a monotonous, non monotonous or linear approximation is automatically selected, based on the tests (5), (6) and (7) computed for given structural response $g_j(\mathbf{X})$ and design variable x_i.

$$\frac{\partial g_j(\mathbf{X}^k)}{\partial x_i} \times \frac{\partial g_j(\mathbf{X}^{k-1})}{\partial x_i} > 0 \Rightarrow \text{MMA (monotonous)} \tag{5}$$

$$\frac{\partial g_j(\mathbf{X}^k)}{\partial x_i} \times \frac{\partial g_j(\mathbf{X}^{k-1})}{\partial x_i} < 0 \Rightarrow \text{GBMMA (non monotonous)} \tag{6}$$

$$\frac{\partial g_j(\mathbf{X}^k)}{\partial x_i} - \frac{\partial g_j(\mathbf{X}^{k-1})}{\partial x_i} = 0 \Rightarrow \text{linear expansion} \tag{7}$$

This strategy proved to be reliable simple for laminates design [10], for truss sizing and configuration [24] and for topology optimization including a large amount of design variables [25]. It is available in the BOSS/Quattro optimization toolbox, and is tested here on more realistic composite design problems, including complex geometry or a multi-objective formulation. For further details on the algorithm, see [9,24]. It should be noted that this family of approximations also includes the non monotonous GCMMA [26] that was mathematically demonstrated to be globally convergent (i.e. the algorithm will always find a local minimum, whatever may be the initial starting point). Finally, information on gradients computation can be found in [27].

5. Numerical applications

Numerical applications aim at demonstrating the efficiency of the optimization approach described in section 4 in simultaneously dealing with thicknesses and fibers orientations. Comparisons are made with other solution procedures that are well-known in structural optimization. It is shown that some optimal solutions can not be reached when only plies thicknesses are considered as design variables, justifying the use of fibers orientations as design variables, as explained in section 2. It is also underlined that when both kinds of design variables are considered in the problem, minimizing the structural weight, only function of plies thickness, is not a good strategy. In this case, the stiffness is chosen as the objective function.

5.1 Non homogeneous composite membrane optimization

This application concerns the design of a composite membrane divided in 20 regions of constant fibers orientations and made of one ply laminates. The initial configuration includes laminates with 45° or –45° layers, with equal thicknesses of 1mm. The base material is glass reinforced polyester (Tables 1 and 2) and the applied load *P* is equal to 5000N (Figure 9).

E_X	E_Y	G_{XY}	¬$_{XY}$
38050	8140	3130	0.285

Table 1 Mechanical properties of the glass reinforced polyester (in MPa)

X_T	X_C	Y_T	Y_C	S	¢:
820	491	21	107	40	1860

Table 2 Tensile, compressive and shear strengths. Values in MPa. Density ¢ in kg/m^3

20×16 quadrangular membrane multi layer finite elements are used. There are 40 design variables, that is 20 ply thicknesses and 20 fibers orientations. In the design problem (8) the compliance (i.e. the energy related to the external loads) is the objective function to be minimized (what leads to the maximization of the structural stiffness).

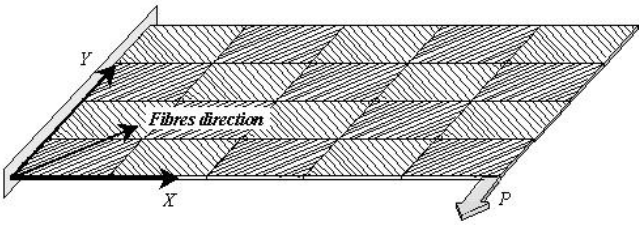

Fig. 9 Composite membrane

The strength constraint is related to the Tsai-Hill (*T.H.*) safety margin (*MS*). When this margin is positive, the applied stress can increase by a factor 1+*MS*; when *MS* is negative, the applied stress has exceeded the strength by a factor 1/(1+*MS*). The strength restriction that the optimum has to fulfill is an overall minimum safety margin of 0.15. Considering such a value allows to take into account the numerous uncertainties that surround the design of composite structures. Such a definition of the safety factor is to be used for a proportional loading. A restriction on the structural weight is also imposed. Plies thicknesses are allowed to vary continuously between 0.01 and 5 mm.

$$\begin{gathered}\min_{\underline{\ },t} Compliance \\ \min\left(\frac{1}{\sqrt{T.H.}}-1\right) \geq 0.15 \\ weight \leq 1.5kg\end{gathered} \tag{8}$$

The obtained solution is illustrated in Figures 10, 11 and 12, and the iteration history is plotted in Figure 13. Note that the initial design is not feasible. The stopping criteria is such that the relative variation of the objective function at two successive iterations be lower than 0.01%.

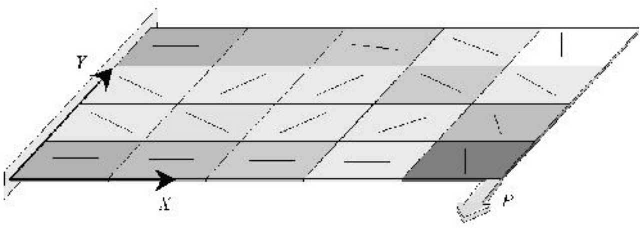

Fig. 10 Illustration of the solution

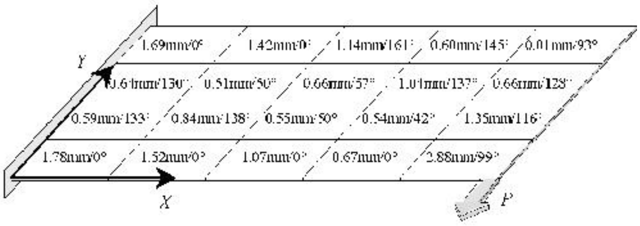

Fig. 11 Solution for the composite membrane

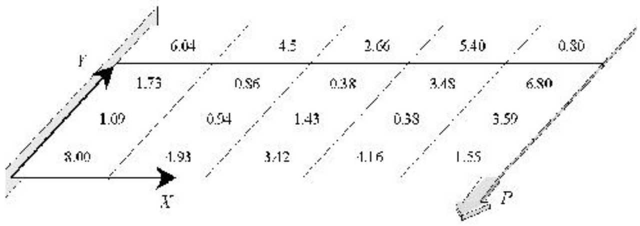

Fig. 12 Minimum Tsai-Hill safety margins in the plies.

In Figure 13, the non dimensional compliance is plotted with regards to its optimal value and the mass is normalized according to its upper allowable bound. At the iteration 13, the design is feasible and the relative variation of the objective function is of 0.3%. It is seen from Figures 12 and 13 that the overall minimum Tsai-Hill safety margin at the solution is larger than 0.15. This means that the weight still can be decreased.

If the problem is restarted with an upper bound of 1.25 kg for the structural weight, the obtained solution is quite similar to the preceding one in terms of fibers orientations. The corresponding iteration history is plotted in Figure 14. A feasible design is found after 6 iterations, while 8 iterations are needed for stabilizing the objective function value with a relative variation of 0.1%.

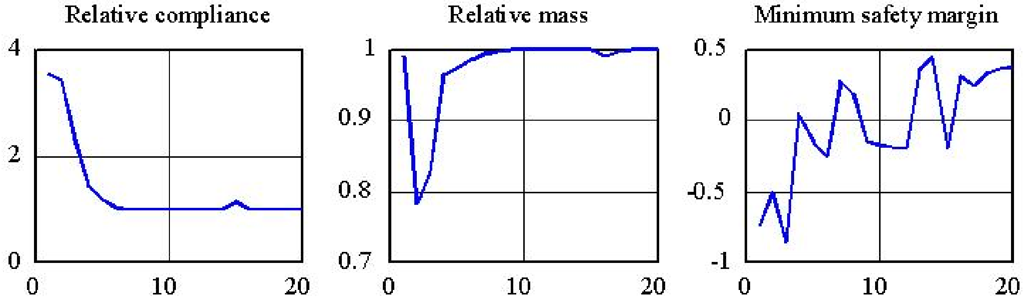

Fig. 13 Iteration history for the composite membrane optimization using GBMMA-MMA. The maximum allowable weight is of 1.5 kg

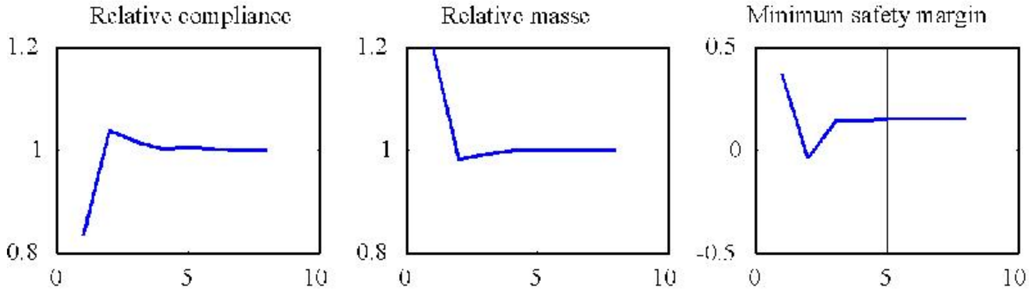

Fig. 14 Iteration history for the composite membrane optimization using GBMMA-MMA. The maximum allowable weight is of 1.25 kg

It is to be noted than when only plies thicknesses are considered in problem (8), a solution seems to be very difficult to be obtained. Although the used optimization algorithm GCMMA [26] is globally convergent, it is not able to find any optimal design in a reasonable time. After 100 iterations, the overall minimum Tsai-Hill safety margin is of -0.125 and the restriction on the mass is slightly violated. This underlines the limitation of the "only thickness" formulation and the key role of the fibers orientations in the design of composite structures, discussed in section 2. In Figure 15, the non dimensional compliance is plotted with regards to its value at iteration 100. The mass is normalized according to its upper allowable value.

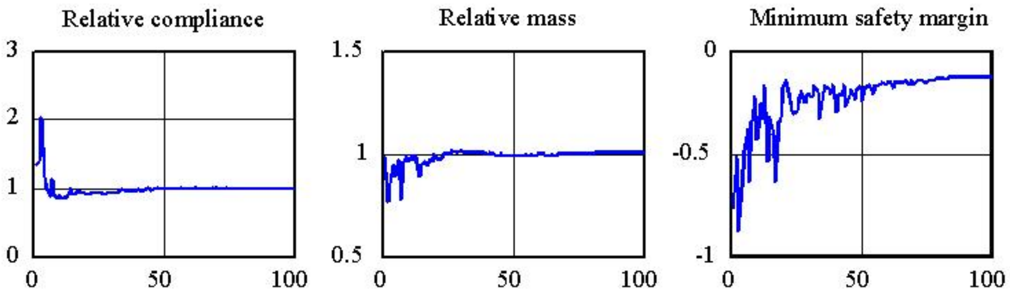

Fig. 15 Iteration history for the composite membrane optimization using GCMMA. Plies thicknesses are the only design variables

5.2 Industrial application

5.2.1 Position of the problem

The optimization procedure is applied to the design of a light rail vehicle (LRV) made of composite material (Figure 22). The sandwich panels configuration used in the LRV is symmetric and comprises skin layers of E-glass reinforced epoxy resin and the foam core is manufactured from polyurethane (Table 3).

	E_X	E_Y	G_{XY}	$\neg_{XY}$
FOAM	21.5	21.5	7.7	0.39
GLASS-EPOXY	43640	9760	3810	0.274

Table 3 Mechanical properties of the glass reinforced epoxy and the foam material (in MPa).

The initial configuration of the panel includes a 28mm thick foam core with 8×1 mm face sheets building up with 8 plies and a lay-up of *[0°/45°/-45°/90°]* $_S$ (Figure 17). The loading condition is derived from the British Railway Board standard [28]: it includes a compressive force of 1500 kN at the buffer position.

The mesh of one quarter of the wagon is presented in Figure 16. It is made of 2012 multi layer 8-noded Mindlin shell finite elements. The design domain is the dark panel also illustrated in Figure 16. It is divided in three different design zones (Figure 24). Ply thicknesses and fibers orientations are considered as design variables. Plies thicknesses are allowed to vary continuously between 0.01 and 5 mm.

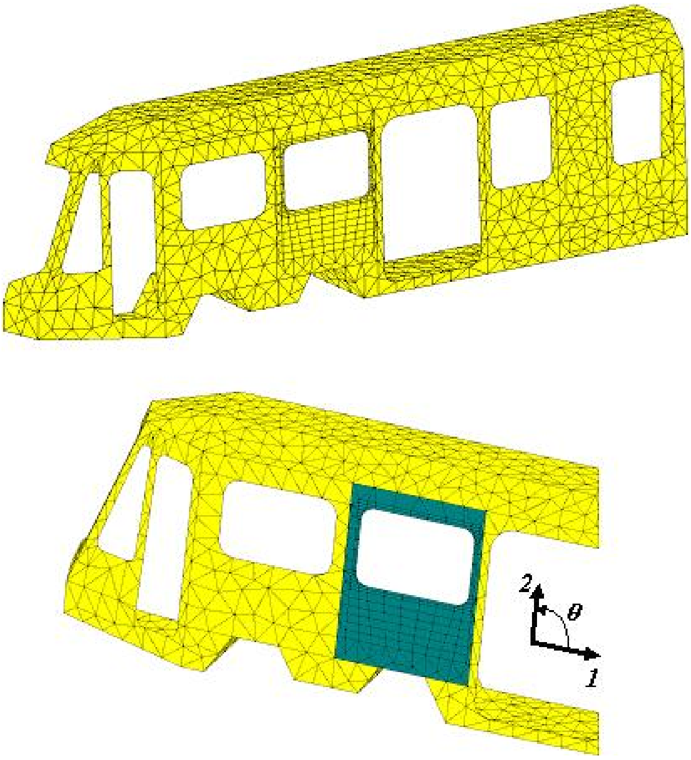

Fig. 16 Finite element model of the railway vehicle. The dark panel on the right is the part to be designed.

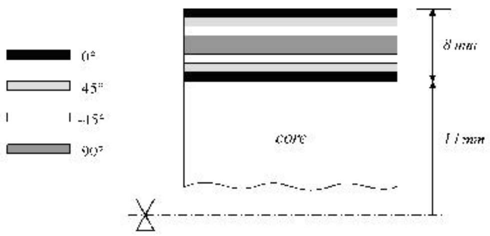

Fig. 17 Initial configuration of the sandwich panels.
Stacking sequence : $[(0/45/-45/90)_S/Z_C]_S$. Z_C is related to the core

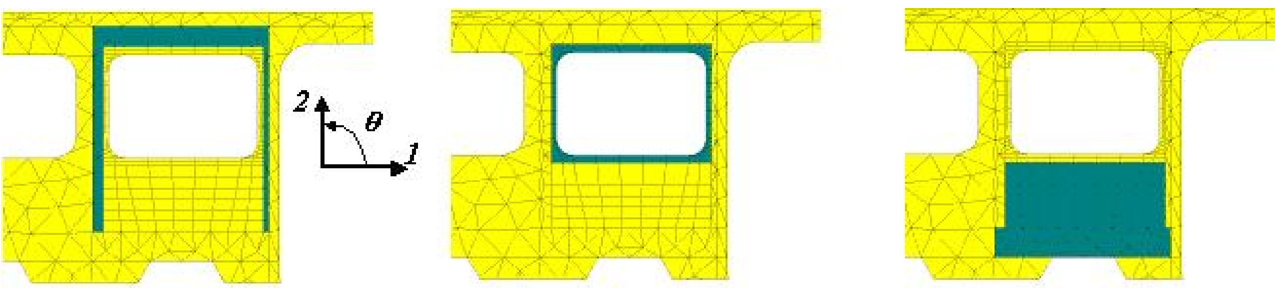

Fig. 18 The 3 design regions
Domain 1 : 74 finite elements; Domain 2 : 88 finite elements; Domain 3 : 83 finite elements

The plies thicknesses and the fibers orientations can vary in a different way in the 3 design regions, while keeping the symmetry property. The problem then includes 24 design variables (12 orientations and 12 thicknesses). One of the target is to decrease the weight. Strength constraints are also considered at the upper and lower skins of each ply via a Tsai-Hill safety margin of 0.05. 96 strength constraints are taken into account in the design problem.

5.2.2 Minimum weight design with only plies thicknesses

This first study consists in minimizing the panel weight with ply strength restrictions. The initial ply thicknesses in the 3 domains are 3.5, 8 and 3mm, respectively. The convergence history – limited to 30 iterations – is illustrated in Figure 19 for the globally convergent GCMMA method. Although the panel mass varies from 244 to 30 kg, the structural strength is not satisfied given that the minimal safety margin is much lower than 0.05. Here, both plies thickness and orientations were considered as design variables. However, as the objective function (i.e. the weight) only depends on the thicknesses, the orientations along the optimization process are not well taken into account and their variations are very small (Figure 19). The design space is then not relaxed enough during the iterative process (Figure 8), and a feasible solution can not be reached. When both kinds of design variables are considered in the problem, minimizing the structural weight, only function of plies thickness, is then not a good strategy. A function of both kinds of design variables is to be preferred [14]: in this paper, it is the structural stiffness.

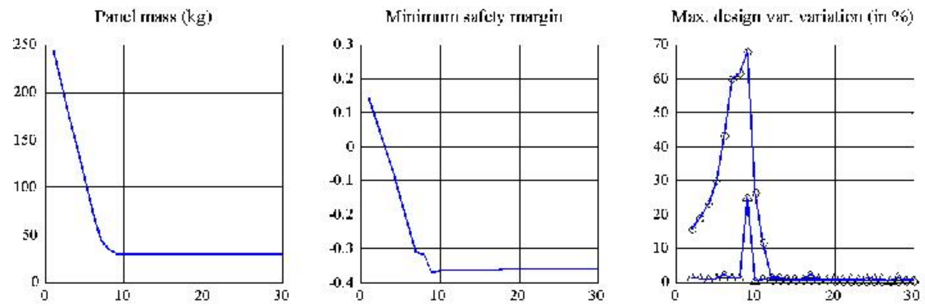

Fig. 19 Iteration history for mass minimization under strength constraints
O = thicknesses Δ orientations

5.3.3 Stiffness optimization

In this second study, the goal of the optimization is to increase the structural stiffness while maintaining the panel mass at the 30 kg obtained in the first study. For this study, the initial ply thicknesses in the 3 domains are 0.06, 0.11 and 0.06, respectively. Strength constraints are imposed on the upper and lower faces of each ply. Results are reported in Figure 20 where the number of violations at each design step is plotted with the evolutions of the compliance (values related to the optimum) and the normalized mass.

Consequently to the small initial thicknesses, the initial design is non admissible: 68 strength restrictions are violated. It is seen in Figure 20 that only 11 iterations are needed to find an admissible design. At this stage, the corresponding relative variation of the objective function is of 0.5% and the solution is supposed to be reached. The optimal stacking sequences are illustrated in Figure 21. Seeing as they are very different in the 3 design regions, a new initial parameterization is proposed for a sake of manufacturability.

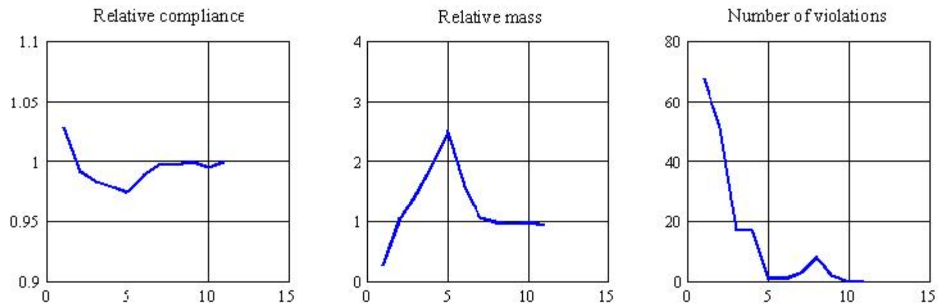

Fig. 20 Iteration history for the optimization of the LRV using GBMMA-MMA (*ICHECK = 2*).

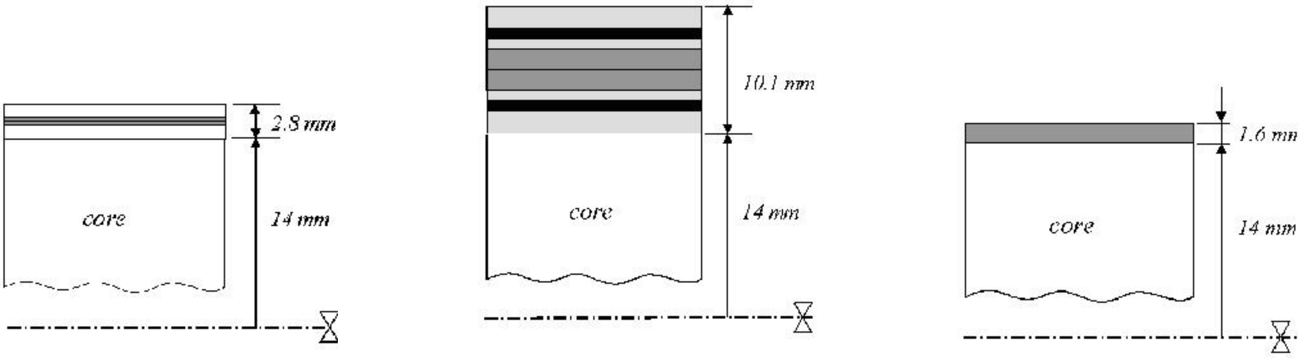

Fig. 21 Optimal stacking sequence for the problem including mass, stiffness and strengths
Domain 1 : $[(14/90)_S/Z_C]_S$; Domain 2 : $[(35/0/35/65)_S/Z_C]_S$; Domain 3 : $[90/Z_C]_S$

5.2.4 *An alternative parameterization for stiffness optimization: manufacturability considerations*

Two design regions are now considered instead of 3. Regions 1 and 3 in Figure 18 are merged. The initial symmetric stacking sequences are reported in Figure 22. The problem includes 16 design variables, and 32 strength constraints are considered. The initial mass is of 39 kg for 1mm plies thicknesses.

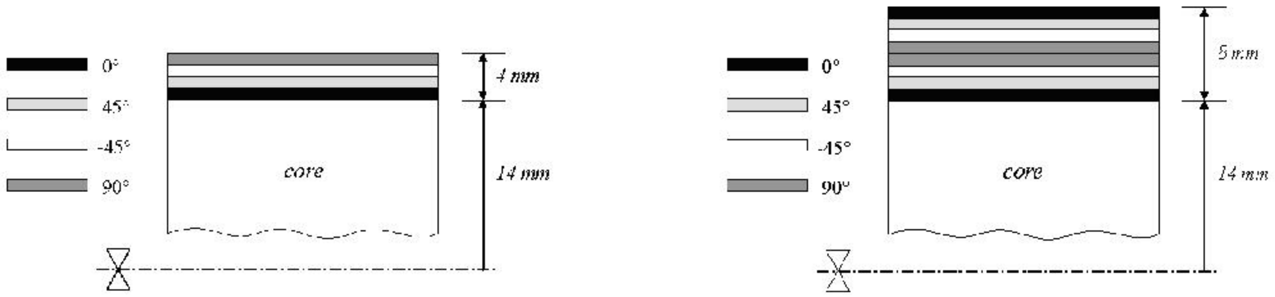

Fig. 22 Initial configuration of the sandwich panels.
Domains 1 and 3 : $[90/-45/45/0/Z_C]_S$; Domain 2 : $[(0/45/-45/90)_S/Z_C]_S$

The results for an optimal panel of 30 kg are illustrated in Figures 23 and 24. The values are normalized with respect to the optimal values. When GBMMA-MMA is used, 19 iterations are needed to reach the solution. A feasible solution is found with GCMMA in 31 iterations. When a SQP algorithm (Sequential Quadratic Programming, see [29]) is used for solving the problem, 75 iterations are not enough to reach a feasible solution. The mass restriction is slightly violated, and 8 safety margins are not satisfied (in Figure 24, only the normalized results of the 50 first iterations are reported for the SQP).

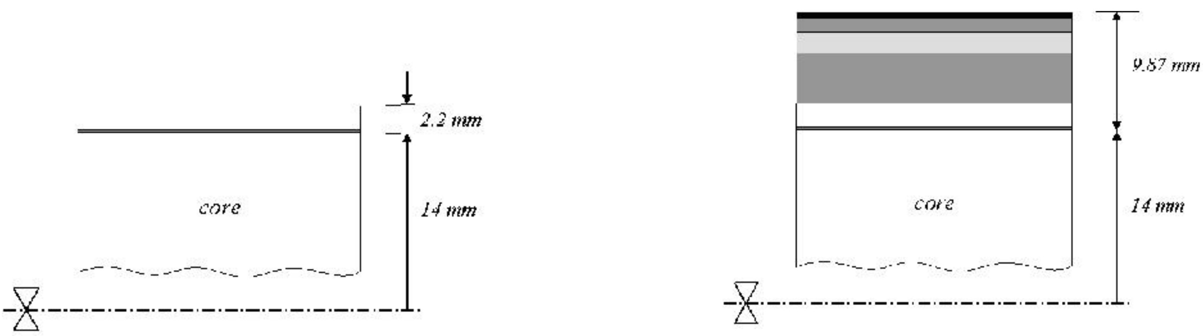

Fig. 23 Optimal stacking sequence for the problem including mass, stiffness and strengths
Domains 1 et 3 : $[11/99/Z_C]_S$; Domain 2 : $[0/93/122/93/11/99/Z_C]_S$

The resulting stacking sequence includes an identical definition in the 3 regions and a reinforcement in the part just around the window.

Composites Science & Technology, 66, pp. 1303-1314, 2006.

6. Conclusions

This paper proposed a general and efficient procedure for fibers reinforced composites optimal design. Numerical applications showed that the proposed solution strategy allows the design of composite structures under stiffness and strength criteria, over fibers orientations and plies thicknesses. Those two kinds of structural responses have to be taken into account in real-life applications, and fibers orientations must be considered for a rigorous optimization problem formulation. Besides this, great savings in structural performances can be achieved by considering such variables in the design process. Due to the lack of convexity resulting from the orientation/thickness formulation, the solution is not the global optimum. This is the price to pay for designing the structure for strength [30].

Acknowledgments Professor Claude Fleury (University of Liège, Belgium) is gratefully acknowledged for making the Conlin optimizer available. The main part of this work was performed under the contract BRPR-CT96-0332 of the European project BRITE-EURAM (COMPOPT).

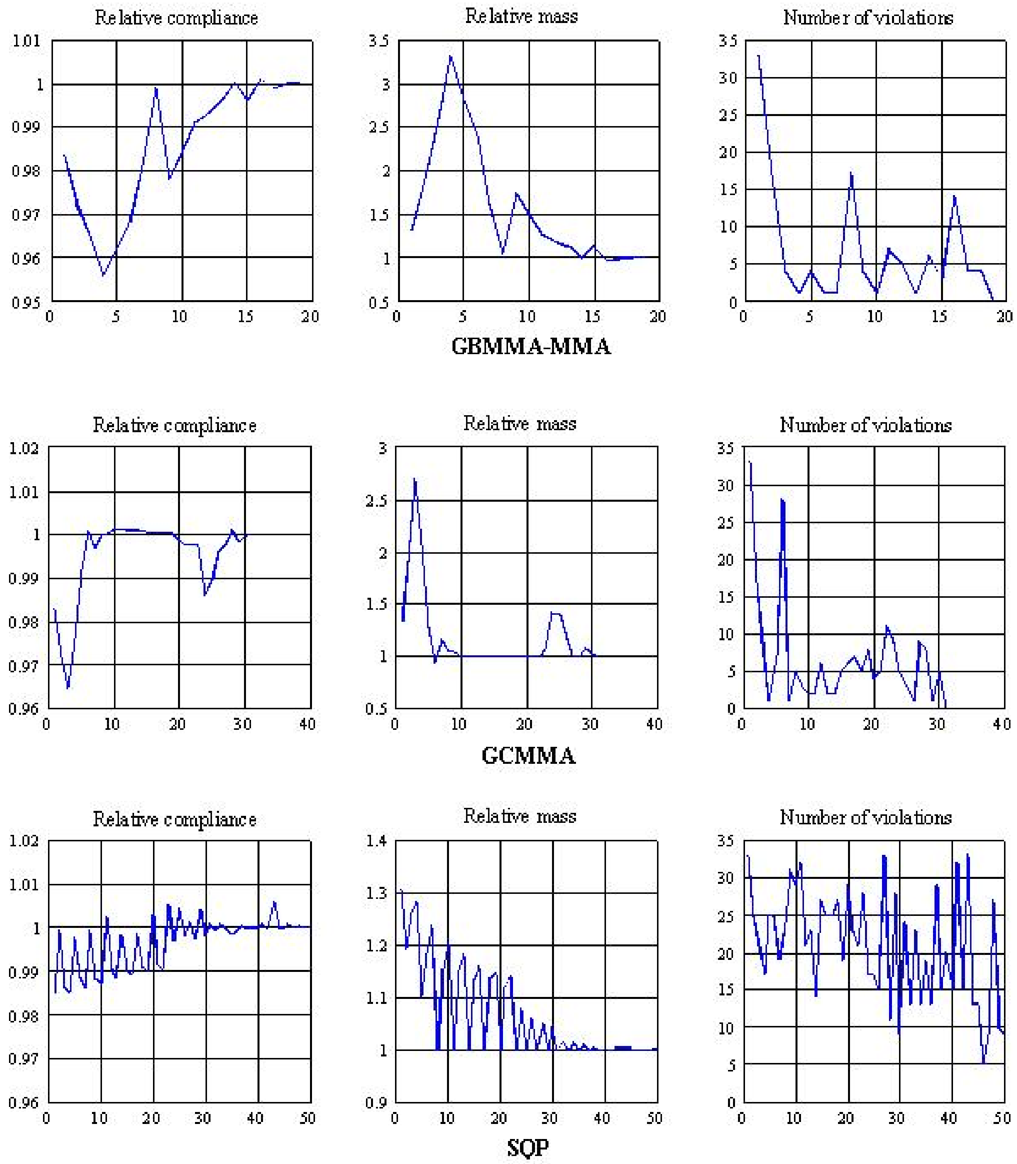

Fig. 24 Iteration history for the optimization of the LRV.
Comparison of different optimization methods. GBMMA-MMA, GCMMA and SQP

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