Identification of Nonlinear Systems Using the Restoring Force Surface Method

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ABSTRACT
The objective of this paper is to demonstrate the performance of the restoring force surface method in the identification of nonlinear systems. The experimental application of the method has been studied and numerical results on a single degree-of-freedom system are described

INTRODUCTION

The identification of the dynamic characteristics of linear systems is now widely used but interest in nonlinear systems is increasing. Identification of nonlinear systems ranges from methods which simply detect the presence or type of a nonlinearity to those which seek to quantify the dynamic behavior through a mathematical model. In this latter category is the nonparametric identification scheme called the restoring force surface method.

THEORETICAL BACKGROUND

The restoring force surface method is based on Newton's second law :

$$m\,\ddot{x}(t) + f(x(t),\dot{x}(t)) = p(t) \qquad (1)$$

where $p(t)$ is the external load and $f(x,\dot{x})$ is the restoring force, i.e. a nonlinear function of the displacement and velocity. The time histories of the displacement and its derivatives, and of the applied force are assumed to be measured. In practice, the data must be sampled simultaneously at regular intervals. From equation (1), it is possible to find the restoring force defined as $f_i = p_i - m\,\ddot{x}_i$ where subscript i refers to the i[th]-sampled value. Thus, for each sampling instant a triplet $(x_i, \dot{x}_i, f_i)$ is found, i.e. the value of the restoring force is known for each point in the phase plane $(x_i, \dot{x}_i)$.

It is important to describe the system by a mathematical model. The usual way is to fit to the data a model of the form :

$$f(x,\dot{x}) = \sum_{i=0}^{m}\sum_{j=0}^{n} \alpha_{ij}\; x^i\; \dot{x}^j \qquad (2)$$

Least-squares parameter estimation can be used to obtain the values of the coefficients α_{ij}. To have a measure of the error between of the measured x_i and predicted $\hat{x}_i$ values, the Mean-Square Error (MSE) is defined as :

$$MSE(x) = \frac{100}{N\,\sigma_x^2}\;\sum_i (x_i - \hat{x}_i)^2 \qquad (3)$$

where N is the total number of samples and σ_x^2 is the variance of the measured input. Experience shows that an MSE of less than 5% indicates good agreement while a value of less than 1% reflects an excellent fit. To have some means of determining which terms are significant and which terms can be safely discarded, the significance factor [1] is used. Roughly speaking, it represents the percentage contributed to the model variance by the term.

DATA PROCESSING

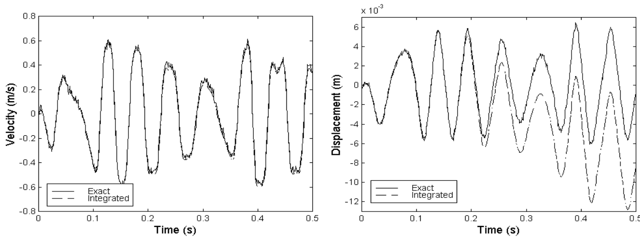
The method requires to measure displacement, velocity, acceleration and force time histories at each degree of freedom. With the aim of reducing the number of acquisition channels, numerical integration and/or differentiation procedures may be adopted. Since numerical differentiation is a notoriously difficult procedure [2], the practical solution is to measure $\ddot{x}(t)$ and numerically integrate to find $\dot{x}(t)$ and $x(t)$.

For example the nonlinear system

$$\ddot{x} + 20\dot{x} + 10000x + 5.10^8 x^3 = p(t) \qquad (4)$$

was simulated where $p(t)$ was a white noise sequence band-limited into the 10-20Hz range. White Gaussian noise was added to $p(t)$ and $\ddot{x}(t)$ in such a way that the noise contributed to 5% of the signal RMS value. The sampling interval was set to 0.001 sec. The acceleration data were then integrated twice using the trapezium rule. To fix the arbitrary constants introduced by the integration, the mean of the velocity was removed and a linear drift component was removed from the displacement signal. The resulting velocity and displacement are shown in Figure 1. For the sake of clarity, only the beginning of the signals is plotted.

The velocity is excellent and in fact is always estimated well. However, a large low frequency component has been introduced into the displacement signal due to the fact that the integration process basically acts as an amplifier of the low frequency and means components.



(a) (b)

Figure 1 : Comparison of the exact/integrated velocity (a) and displacement (b)

To improve the displacement one can high-pass filter the data using a Butterworth filter. Figure 2 shows the displacement signal filtered by a high pass filter with cut off at 2 Hz. This time, the low frequency problems are absent.

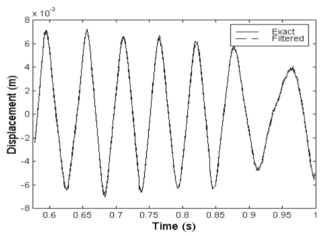


Figure 2 : Comparison of the exact/integrated and filtered displacement

PARAMETER ESTIMATION

At this point, the mass of the system was assumed to be known, the input force $p(t)$ and the acceleration $\ddot{x}(t)$ were measured, and the acceleration was integrated twice to obtain $x(t)$ and $\dot{x}(t)$. Thus, the value of the restoring force can be computed at each sampling instant and the restoring force surface is plotted (Figure 3) using the procedure of Crawley and O'Donnell [3]. The surface is nonlinear and it can be seen that the cubic term is significant.

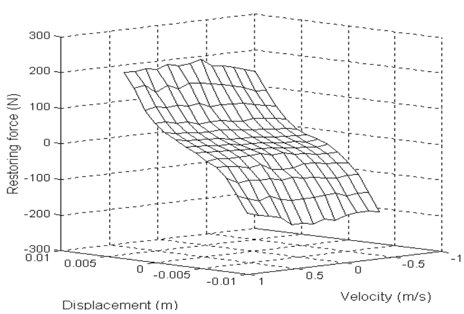


Figure 3 : Restoring force surface

As the type of the nonlinearity is not known a priori, it is better to assume a nonlinear force of the form given by equation (2). The cross terms of the model are usually small and are very often neglected. One problem that arises is that it may be difficult to choose the right model order. Looking at the evolution of the MSE as a function of indices m and n (Figure 4) demonstrates that the choice of a model order higher than 3 has no more influence on the MSE.

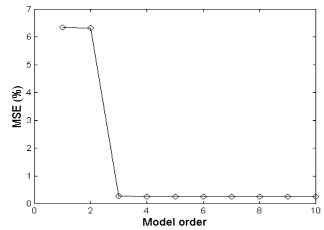


Figure 4 : MSE vs. model order

The identified coefficients for order 3 (MSE=0.27) are shown in Table 1 and confirm that the identification procedure gives excellent results.

	α_{10}	α_{01}	α_{20}	α_{02}	α_{30}	α_{03}
Exact	10000	20	0	0	$5\ 10^{8}$	0
Ident.	10076	16.42	-48732	1.17	$5\ 10^{8}$	10.42
Sign. Factor	**15.71**	**0.51**	$4\ 10^{-4}$	$2\ 10^{-4}$	**38.42**	$2\ 10^{-5}$

Table 1 : Identification results

Three significant terms are identified : a linear stiffness (α_{10}), a linear damping (α_{01}) and a cubic stiffness (α_{30}) whose values are very close to the exact values. The three other terms can be discarded since their significance factors are of slight importance.

CONCLUSION

It has been shown that the restoring force surface method allows to perform a reliable identification of nonlinear systems. Moreover, an important advantage of the method is that it does not require an a priori estimate of the system. Further work on more complex systems is foreseen.

REFERENCES

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