Nonlinear Interpolation on Manifold of Reduced Order Models in Magnetodynamic Problems

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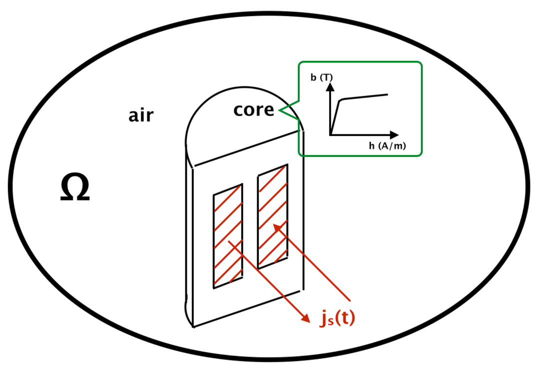
Introduction & Goal

The aim of this research is to determine efficiently the solution of a magnetodynamic problem for an unknown input parameter set based on pre-computed information. This information comes from pre-solved problems for different input parameter sets. Therefore, we have to:

- Construct reduced order magnetodynamic models (ROM).
- Reuse information from different pre-computed solutions.

This method is demonstrated on a nonlinear inductor-core system where the input frequency is a parameter.



Formulations

The magnetodynamic problem is ruled by (1) and can be split into linear and nonlinear parts as in (2).

$$\mathbf{curl}\,(\nu\,\mathbf{curl}\,a(t)) + \sigma\partial_t a(t) = \mathbf{j_s}(t) \quad (1)$$

$$\Leftrightarrow \underbrace{\mathbf{curl}\,(\nu_0\,\mathbf{curl}\,a(t)) + \sigma\partial_t a(t) - \mathbf{j_s}(t)}_{\text{Linear part}} = \underbrace{-\mathbf{curl}\,(\tilde{\nu}\,\mathbf{curl}\,a(t))}_{\text{Nonlinear part}} \quad (2)$$

POD [1, 2, 3]

Proper Orthogonal Decomposition (POD) is very efficient to obtain reduced order magnetodynamic models in the linear case, or in the nonlinear case when the input parameters do not vary. The reduced and full vector of unknowns, resp. x_r and x, are linked with the reduction basis Ψ by

$$x = \Psi x_r$$

where $x \in \mathbb{R}^{n\times 1}$, $x_r \in \mathbb{R}^{r\times 1}$ and $\Psi \in \mathbb{R}^{n\times r}$. We expect $r << n$ to reduce the number of unknowns.

The basis is obtained by a applying a thin SVD on a snapshot matrix S. This matrix S gathers the solutions for all time steps:

$$S = \left[x_1, x_2, \cdots, x_T\right] \in \mathbb{R}^{n\times T}$$

where T is the number of time steps. The reduced basis Ψ is obtained by

$$[U, S, V] = \text{svd}(S),$$
$$\rightarrow \Psi = U.$$

Since the number of time steps is smaller than the size of the initial unknown vector, the matrix U doesn't have to be truncated as it is usually done. Here $r = T$.

In our case, $n = 553$ and $r = 20$.

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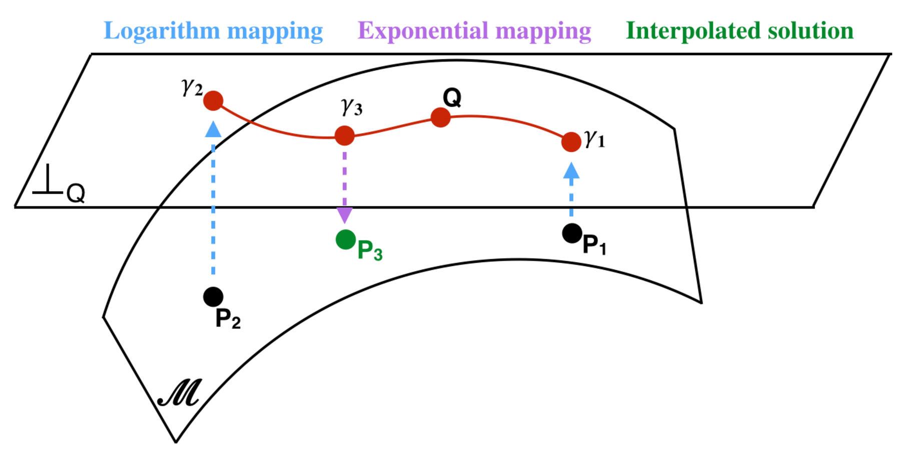
Interpolation on Manifolds [4]

For a new input parameter set, the corresponding reduced basis Ψ^* is unknown. Thanks to their orthogonality property, these reduction bases Ψ lie on the orthogonal manifold where interpolation can be performed. To this end, the reduction bases are mapped to the tangent space at point Q using the logarithm mapping:

$$\gamma = \text{Log}_Q(P) = \text{LOGM}(Q^T P).$$

These projections lie on a plane space and can be interpolated using a traditional technique (e.g. Lagrange inteprolation) to obtain the projection of Ψ^*. This interpolated projection is mapped back on the orthogonal manifold using the exponential mapping:

$$P = \text{Exp}_Q(\gamma) = Q\,\text{EXPM}(\gamma).$$



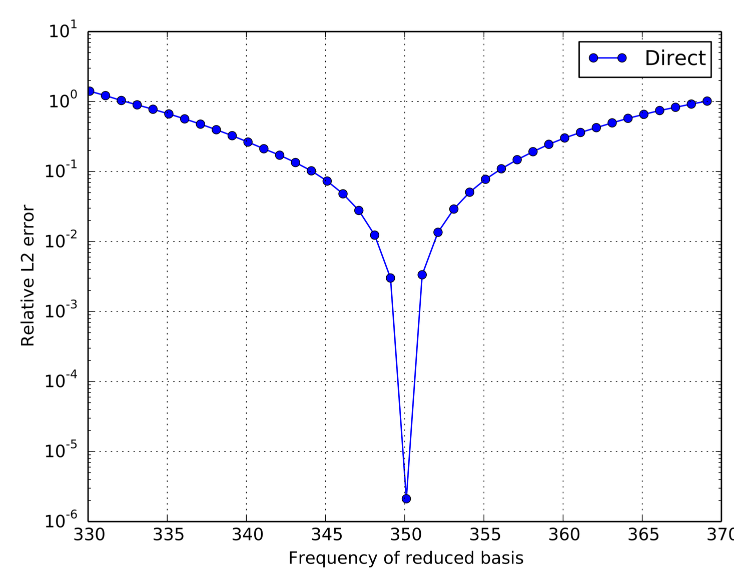
In our case, P_3 is the unknown reduced basis Ψ^*.

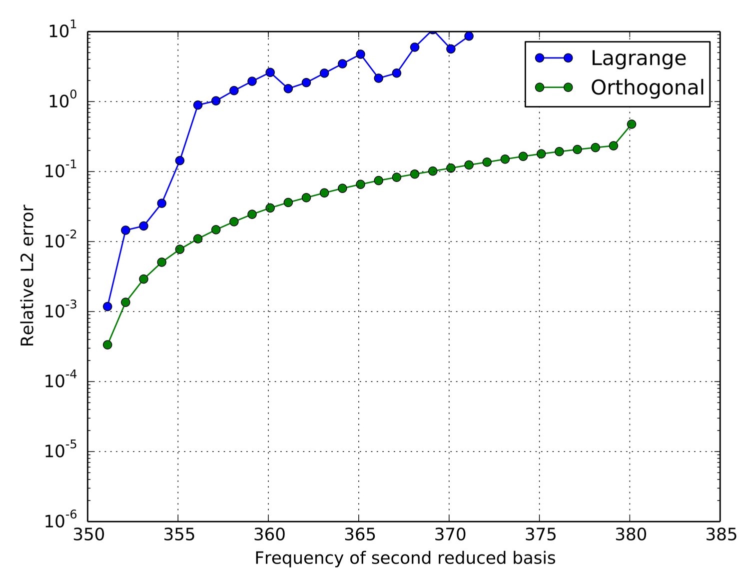
Results

The input excitation of the nonlinear inductor-core system is given by $\mathbf{j_s}(t) = J\sin(\omega t)$. Due to dynamical effects, the solution spatially changes with the input frequency. Here are some results for frequency 350Hz.

The ROM is directly constructed with a reduced basis Ψ without interpolation. The L2 error between the ROM and the full model is plotted below where the reduction basis varies from 330 Hz to 370 Hz.

The ROM is constructed with an interpolated reduced basis. The interpolation uses two basis around 350Hz. The first reduced basis Ψ_1 is fixed from solution at 320Hz. The second one Ψ_2 is varying from 351Hz to 380Hz.





The proposed method gives better results than the direct approach and a classical Lagrange interpolation.

[1] Schilders, *Model order reduction: theory, research aspects and applications.* Springer-Verlag, 2008.

[2] T. Henneron and S. Clénet, "Model Order Reduction of Non-Linear Magnetostatic Problems Based on POD and DEI Methods," *Magnetics, IEEE Transactions on*, 2014.

[3] J. Aguado and F. Chinesta, "DEIM-based PGD for parametric nonlinear model order reduction," in *6th International Conference on Adaptative Modeling and Simulation*, 2013.

[4] D. Amsallem, "Interpolation on manifolds of CFD-based fluid and finite element-based structural reduced-order models for on-line aeroelastic predictions," 2010.