ODYSSEUS 2015

"Multi-period vehicle assignment problem with stochastic transportation order availability"

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QuantOM

comex combinatorial optimization: metaheuristics & exact methods

"Multi-period vehicle assignment problem with stochastic transportation order availability"

THE PROBLEM

Vehicle assignment problem

To maximize profit : select orders to be transported by trucks (FTL-PDP) References : W.B Powell

Multi-period

Confirmed and projected orders provided over some periods
Repetitive decision process period per period over an horizon

Stochastic transportation order availability

Projected transportation order realize or vanish

Find profitable paths over time for a fleet of vehicles

Multi-period and stochastic data ! Is it worthwhile ?

Outlines

- Multi-period information and decision framework
- The Deterministic Vehicle Assignment Problem
- An example
- The Stochastic version
- Bounds : a-priori and a-posteriori information
- Algorithms : single or multiple scenarios approaches
- Simulation
- Instances and Results
- Robustness Analysis
- Conclusions

Multi-period : Rolling horizon

Decision : in t and $t = 1, 2, .., T - H$ => **Policy** $\neq$ Solution

t	t+1,...,t+RH	t+RH+1,...,t+H	t+H+1,...,T
Deterministic		**Stochastic**	**Tail**

Parts : decision period, deterministic, stochastic horizons

Case study (*Period* $=$ *day*) **:**

1. Rolling horizon $H = 4P$
2. Deterministic $RH = 1P$, Stochastic $3P$

Dynamism of the system :

1. Solve, get decisions and apply actions in t (info out)
2. Roll-over 1 period, updates (info in) $t + 1 \rightarrow t'$
   2.1 stochastic gets deterministic $t + RH + 1 \rightarrow t' + RH$
   2.2 new stochastic info in $t + H + 1 \rightarrow t' + H$
3. Go to 1 with $t \rightarrow t + 1 = t'$

Deterministic Vehicle Assignment Problem

- Set of **Cities** $C_1, ..., C_N$ and transportation times $TT_{(C_1,\ C_2)}$
- Set of **Periods** $1, ..., T$
- Set of **Orders** $j \in J$ $(DepC_j,\ ArrC_j,\ DepP_j,\ ArrP_j,\ Gain_j)$
- Set of **Trucks** $i \in I$ $(LocC_i,\ Un/Loaded_i$ value 0 or $j)$

Actions : Carry $Order_j$, **Wait in** $LocC_i$, **Unladen to** $DepC_j$

Objective : profitable paths i.e. maximize (Gains-Costs)
subject to :
Max 1 Order per Truck, max 1 Truck per Order
Flow conservation constraints

Network flow structure : Polynomially Solvable

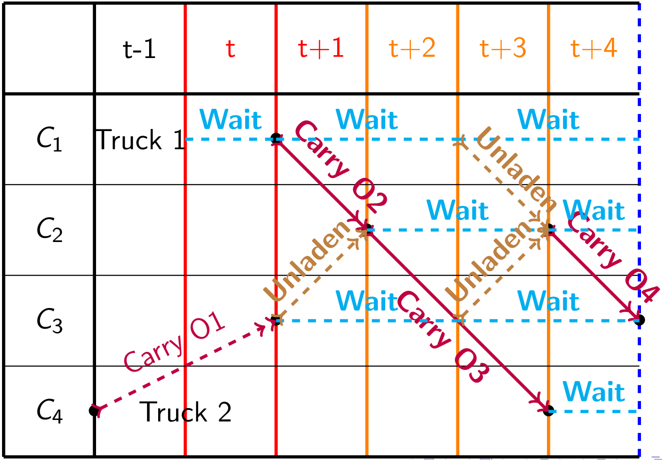
Remarks :

- Full-Truck-Load (FTL), no preemption
- Unloading at the end of $t = ArrP_j$ if $un/loaded_i = j$
- $ArrP_j = DepP_j + TT_{(DepC_j,\ ArrC_j)}$

Deterministic Vehicle Assignment Problem

Decisions for a truck i

- **Carry Oj** if $LocC_i = DepC_j$ (Gain)
- **Wait** in $LocC_i$ (Cost)
- Move **Unladen** from $LocC_i$ to $DepC_j$ (Cost)



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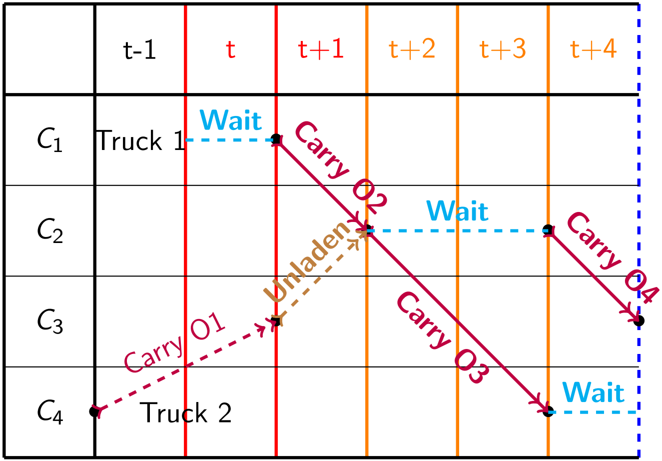
Deterministic Vehicle Assignment Problem

An optimal solution for a period t



Multi-period policy : cumulated value of actions in t

Stochastic Vehicle Assignment Problem

Stochastic framework : **Transportation Order Availability**
If $DepP_j \in t + RH + 1, .., t + H$, (*) the availability of order j is represented by a discrete distribution law :

$$P(q_j = x) = \begin{cases} p_j & if \;\; x = 1 \\ 1 - p_j & if \;\; x = 0 \end{cases} \quad (1)$$

Projection q_j materializes (=1) or not (=0)
when $t + RH + 1 \rightarrow t' + RH$

Scenario : specific outcome of $q_j, \; \forall j \in J$ if (*).

Deterministic equivalent : **scenario tree**, e.g. $(2^{30} = 1.10^9)$

Simulation : Stochastic becomes approximate Deterministic

=> 1 scenario technique

=> Multiple scenarios : Separately or Simultaneously

Specific Scenarios => Bounds

Optimal policy for the stochastic problem : E^*

Bounds from deterministic scenarios :

1. Myopic or a-priori policy over RH : O^*_{RH}
2. Oracle or a-posteriori policy over H : O^*_H
3. Oracle or a-posteriori solution over T : O^*_T

Expected Value Scenario => Expected Value 'Solution' EVS

Maximization : $O^*_T \geq O^*_H \geq E^* \geq EVS \geq O^*_{RH}$

VPI : Value of the Perfect Information O^*_T - $E^* \geq 0$

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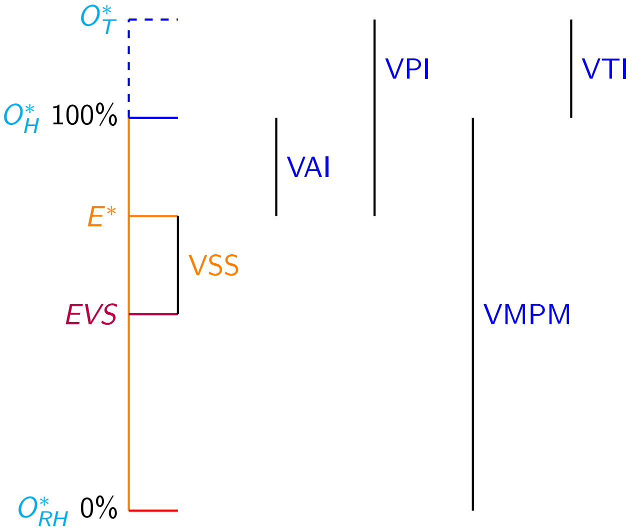
A picture : maximization



Problem : Found E^* the optimal policy

Algorithms : single or multiple scenarios

=> **Aim** : Approximations of E^*

Single scenarios (Mean, Modal, "Optimist", Dedicated)

Multiple Scenarios Approaches (MSA)

- **Consensus (Cs) :**
  1. Solve N scenarios
  2. Create a new solution with frequent actions in t
- **Restricted Expectation (RE) :** Solve N scenarios i & j and cross-evaluate action i in t over scenarios j
  1. Insert actions of solution i in t in scenario j (over $t + RH + 1, .., t + H$)
  2. Scenarios $i \neq j$ $(i, j \in N) \Rightarrow N - 1$ Solutions (i in j)
  3. Cumulated value of N Solutions (i + i in j)
  4. Select the best action i in t
- **Subtree :** Solve ST scenarios and add non-anticipativity constraints (Linear Relaxation = Optimal in practice)

Statistical validation and Biases

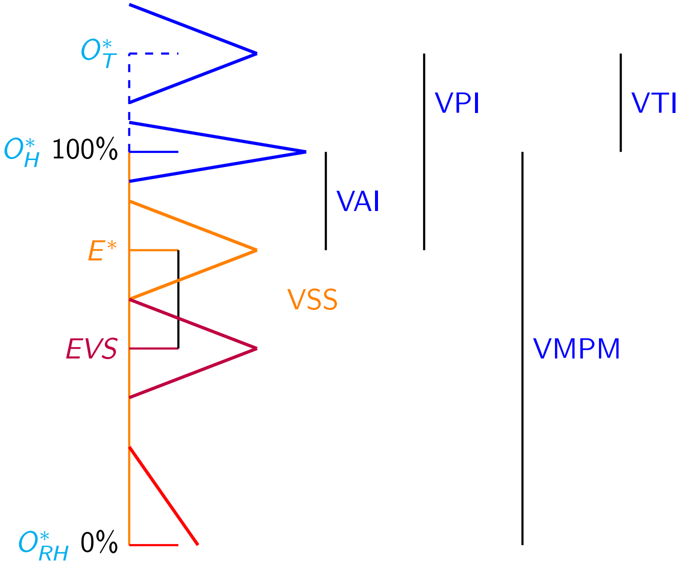
Statistical validation :
Can not solve and can not test all scenarios
How to compare Policy 1 with Policy 2 values ? (μ_1, μ_2)
=> restricted set of test scenarios

Outclassment = significant statistical difference of means

"Paired sample comparison"
Hypothesis : $\mu_1 \neq \mu_2$, $\mu_1 > \mu_2$?
Solve 30 scenarios by instance
Normality check, confidence level, Z-test
"With a 95% confidence level, one can claim that Policy 1 outclasses, overperforms, is (not) equal to Policy 2"

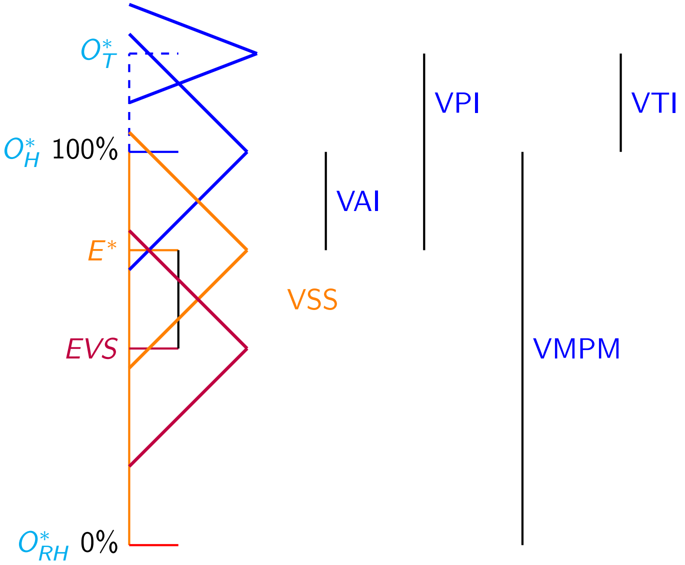
Warm-up and End of horizon biases :
Warm-up : remove H periods
End of horizon : T long, unit per period

A picture : no outclassment



Every additional information or model is valuable

A picture : outclassment (Rem : Overlap*)



Additional information might not be statistically relevant

Instances and Results

10 Trucks, 10-**15**-**20**-25 Cities, **150**-200 Orders, 20P ($RH = 1$, $H = 4$) stochasticity linked to city sizes, Subtree (30 scenarios)

Info		LB	EVS			UB
Inst./Alg.	O^*_T	O^*_{RH}	EVS	Cs	ST_{30}	O^*_H
5-15-25 A	222.0	0	73.6	**80.0**	79.2	100
6-15-25 A	156.1	0	78.6	**90.8**	89.7	100
7-15-25 A	171.0	0	57.2	68.0	**70.7**	100
8-15-25 A	187.3	0	**54.3**	13.8	53.4	100
5-15-25 B	153.1	0	57.7	61.2	**81.6**	100
6-15-25 B	165.7	0	55.8	42.8	**60.3**	100
7-15-25 B	194.7	0	56.5	60.4	**61.0**	100
8-15-25 B	201.4	0	86.7	60.8	**100.0**	100
5-15-25 C	192.4	0	64.1	53.8	**78.8**	100
6-15-25 C	125.9	0	62.7	78.3	**88.0**	100
7-15-25 C	179.2	0	63.9	49.6	**70.4**	100
8-15-25 C	192.0	0	47.0	20.0	**63.5**	100
5-20-25 A	195.1	0	63.9	45.2	**65.9**	100
6-20-25 A	153.8	0	52.1	54.4	**74.3**	100
7-20-25 A	253.9	0	38.6	32.1	**44.5**	100
8-20-25 A	225.7	0	7.3	-36.5	**21.9**	100
5-20-25 B	141.9	0	62.9	33.2	**68.4**	100
6-20-25 B	147.4	0	62.7	53.4	**74.2**	100
7-20-25 B	176.7	0	52.1	52.7	**66.1**	100
8-20-25 B	165.1	0	49.8	25.6	**54.2**	100
5-20-25 C	171.7	0	51.4	61.2	**67.7**	100
6-20-25 C	215.3	0	39.1	23.6	**56.1**	100
7-20-25 C	142.9	0	53.6	54.0	**61.3**	100
8-20-25 C	150.3	0	67.3	41.7	**71.3**	100
Average	**178.4**	0	**56.6**	46.7	**67.6**	100

Results analysis

Neither the graphs, nor the laws influence the results
The VPI, VTI are high on average 110.8%, 78.4%

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Results analysis

ST_{30} best values (average 2/3 of the gap) except twice for Cs and once for EVS. ST_{30} never under-performs

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Results analysis

EVS performs "well", only 11% behind ST_{30}

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Robustness analysis

Robustness : forecast availabilities based on a probability p in algorithm ST^p compared with real availabilities p'

Table: Robustness of distribution law parameter

Forecast	**EVS**	Low	Medium	High
Alg.	EVS_{50}	ST^{30}	ST^{50}	ST^{70}
Reality Low 20%	23.8	**55.0**	48.1	20.1
Reality High 80%	60.4	67.0	84.9	**87.6**
Alg.	EVS_{50}	ST^{20}	ST^{50}	ST^{80}
Reality Medium 50%	36.4	31.9	**55.1**	30.2

Conclusions and perspectives

Conclusions :

1. Importance of stochastic multi-period models
2. VPI, VMPM, VSS are relevant information values
3. ST is the best algo and others under-perform
4. ST^{50} (calibrated with a 50% availability) is robust
5. ST solvable by a LP solver
6. e.g Independent of graph shape, size or distribution laws

Perspectives :

1. Repositioning strategy
2. Investigate the VTI
3. Compare with ADP