A stochastic multi-scale analysis for MEMS stiction failure

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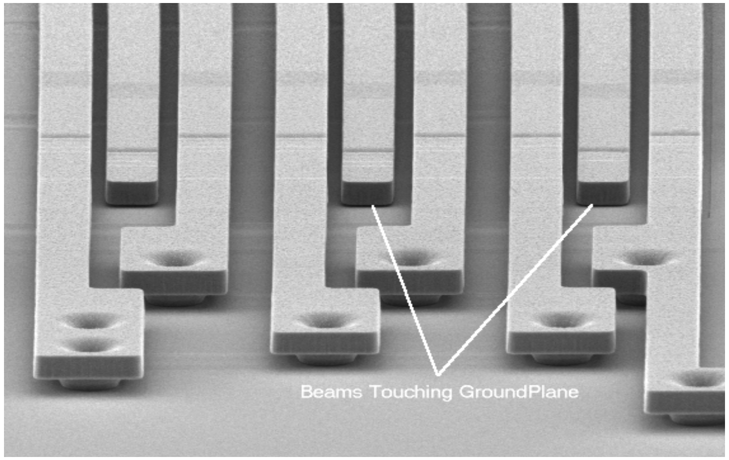
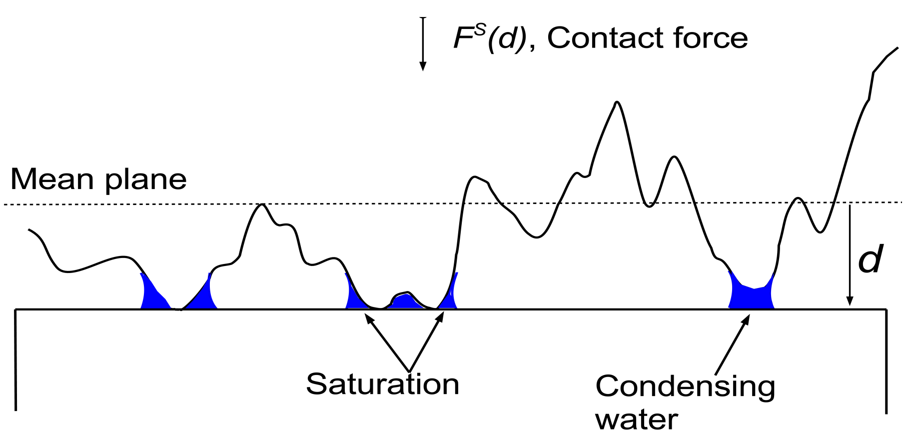
- MEMS stiction failure
  - Due to the dominance of surface adhesive forces
    - E. g., van der Waals forces and capillary forces
    - In humid condition, the capillary forces are dominant
  - Depends on the surface topologies
  - An uncertain phenomenon

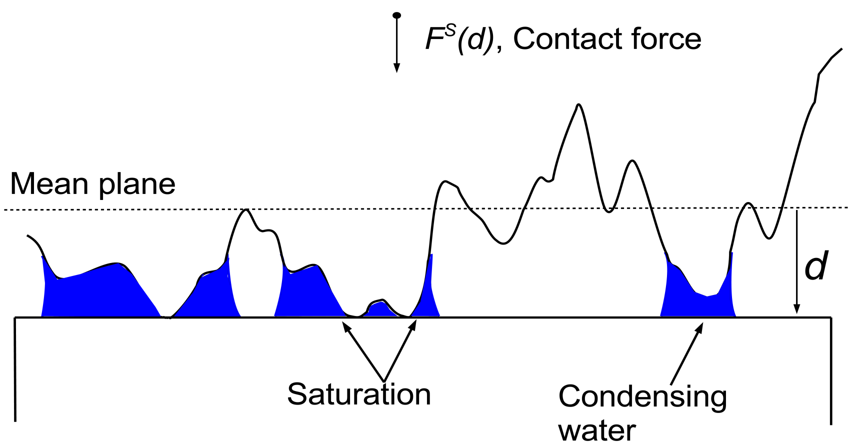


Stiction failure in a MEMS sensor

(Jeremy A.Walraven Sandia National Laboratories. Albuquerque, NM USA)



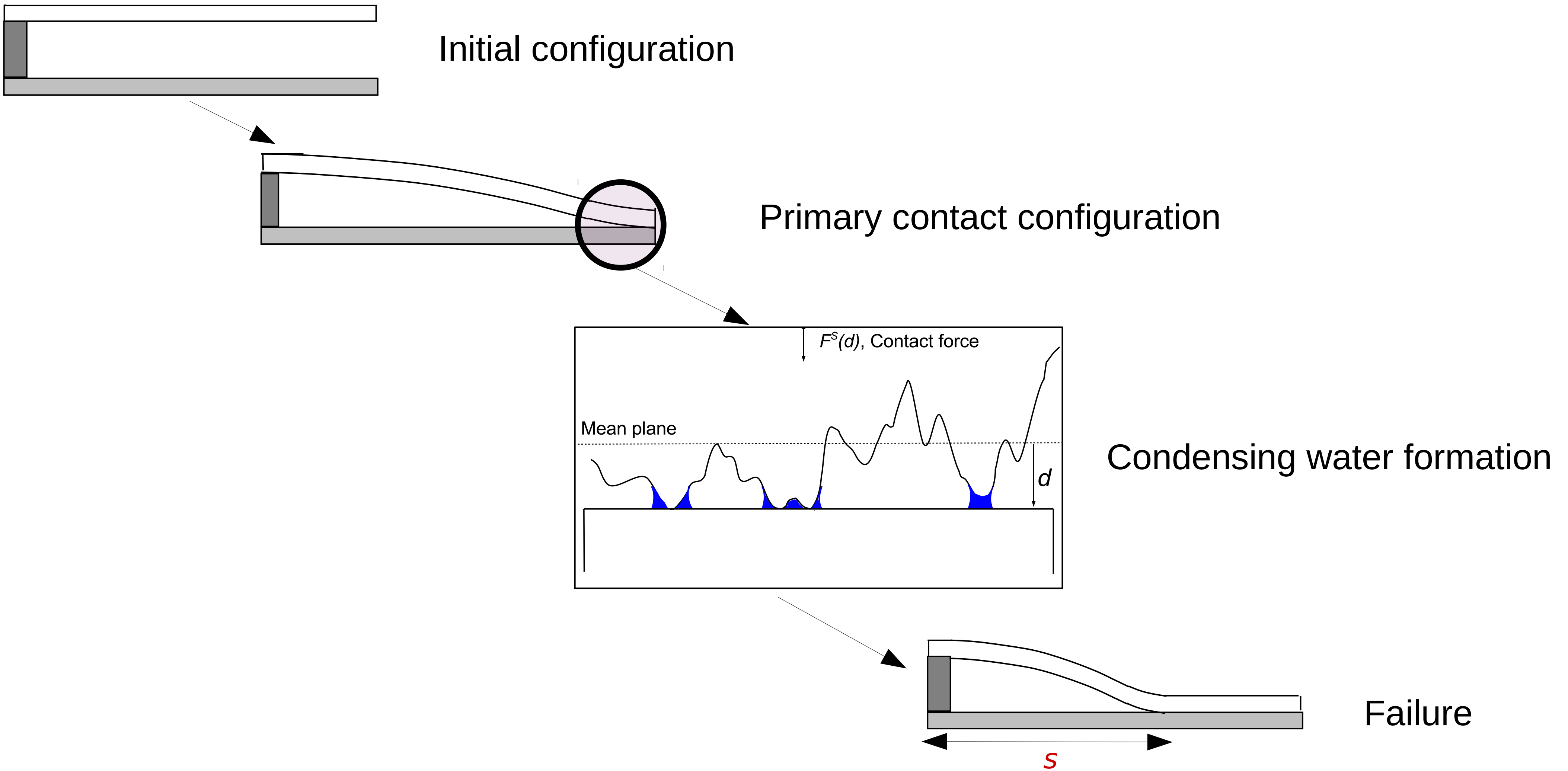
Contact zone: Low humidity levels



Contact zone: High humidity levels

Motivation

- Construct a numerical model
  - To predict the crack length s and its uncertainties from the surface topology
  - At an acceptable computational cost



- The crack length s characterizes the required energy to release the cantilever beam out of the failure configuration

Methodology

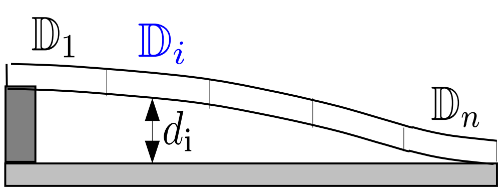
- Construct a **S**tochastic **M**ulti-scale **M**odel (**SMM**) for stiction problems
- Multi-scale component of **SMM**
  - Micro- to meso-scale model: evaluate the meso-scale contact laws from contacting topologies
  - Meso- to macro-scale model: use the meso-scale contact laws to predict the macro behaviors
- Probabilistic component of **SMM**
  - Direct method (Full Monte Carlo method)
    - Characterize the randomness of the micro-scale topology
    - Propagate the randomness through the multi-scale model
  - Indirect method through "a stochastic model of the random meso-scale contact laws" (*)
    - Implement "A stochastic model of the random meso-scale contact laws" to model the randomness of the meso-scale contact laws
      - Not a trivial task
    - Propagate the randomness of meso-scale contact laws (only) through the meso- to macro-scale model
      - Lower computational cost

(*) A Clément, C Soize, J. Yvonnet*, Uncertainty quantification in computational stochastic multi-scale analysis of nonlinear elastic materials*

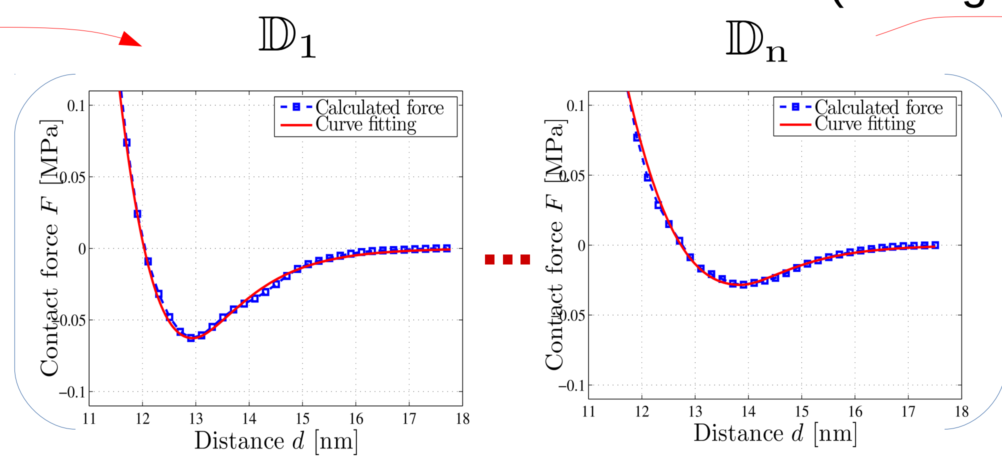
Multi-scale component of SMM

- Meso-scale contact law: force-distance function modeling the interaction of two contacting bodies
  - The bridge between micro and macro-scales
  - The key ingredient of this research

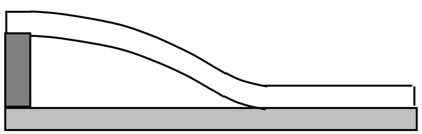
1 . Discretization



2. Contact modeling (*)



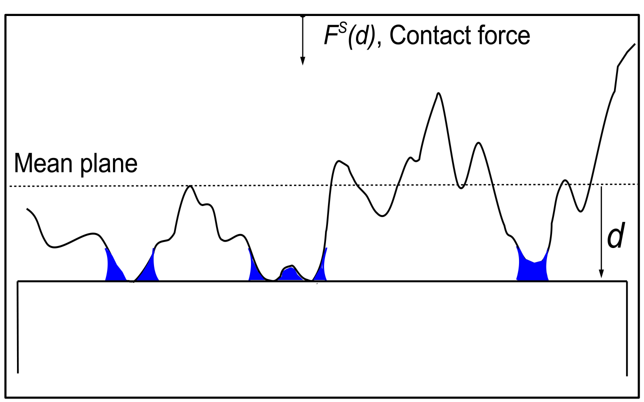
3. Finite Element model
(n integration points)



Macro-scale behavior

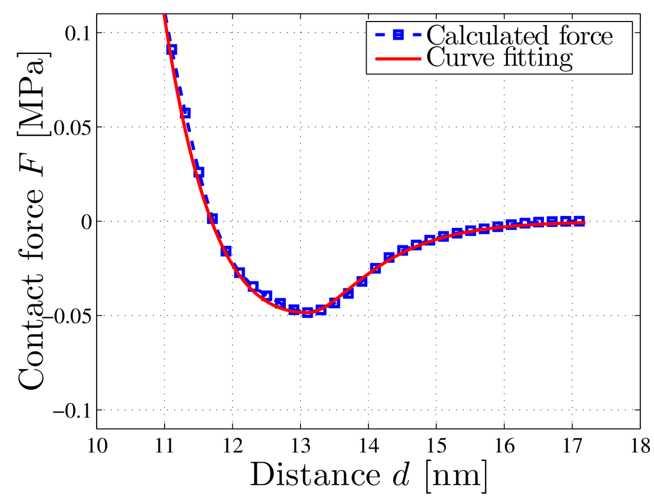
(*) Details of Contact modeling procedure

Micro-scale topologies $\mathbb{D}_i$



Analytical contact models
- Meniscus
- Laplace pressure
- Asperity contact models

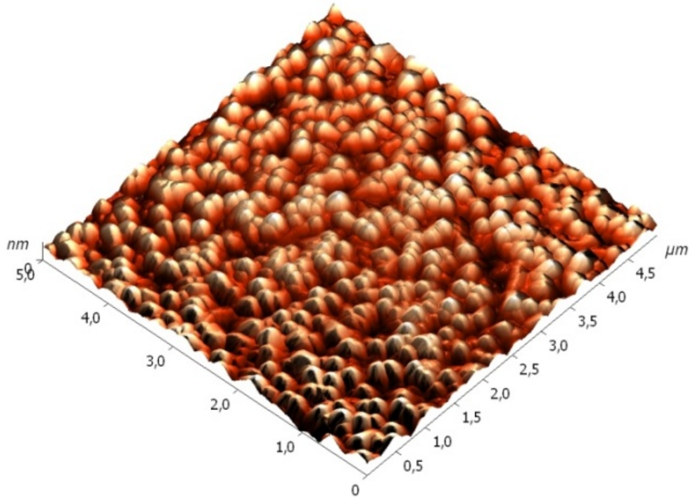
Meso-scale contact law

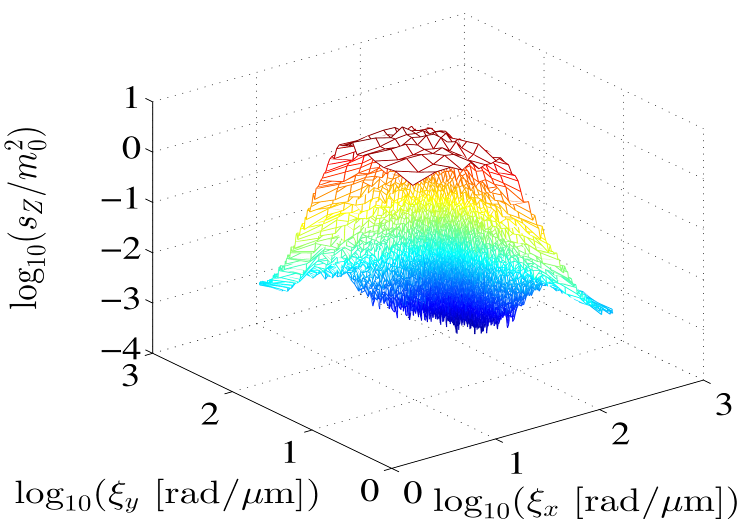


Probabilistic component of **SMM**: Direct method (Full Monte Carlo Method)

- Characterize the rough surface as a stationary Gaussian random field

Characterization:
Surface topology





Atomic Force Measurements

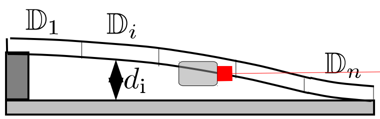
Topology Spectral Density

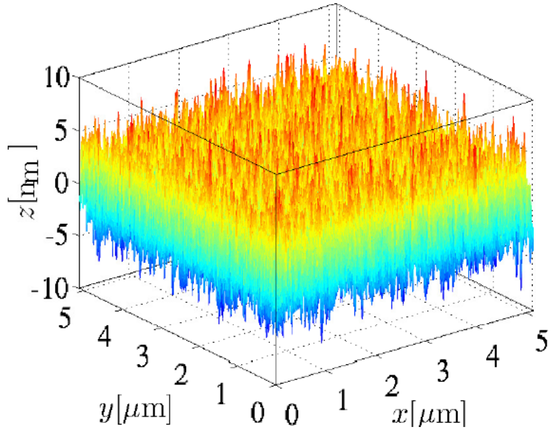
Probabilistic component of **SMM**: Direct method (Full Monte Carlo Method)

- Propagate the randomness through the multi-scale model

1. Surface generator

Zoom to the topologies (*)

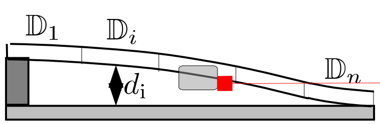


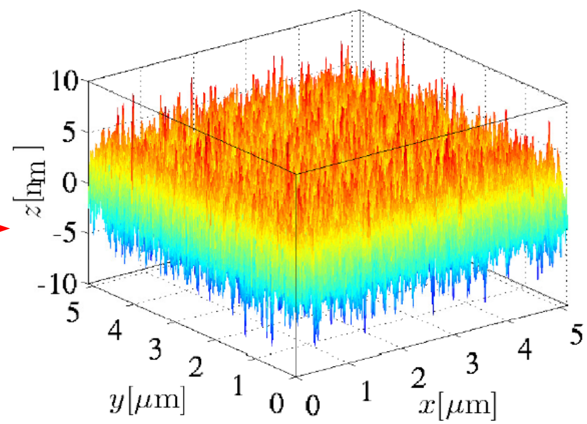


Topology Spectral Density

...

(different topologies)

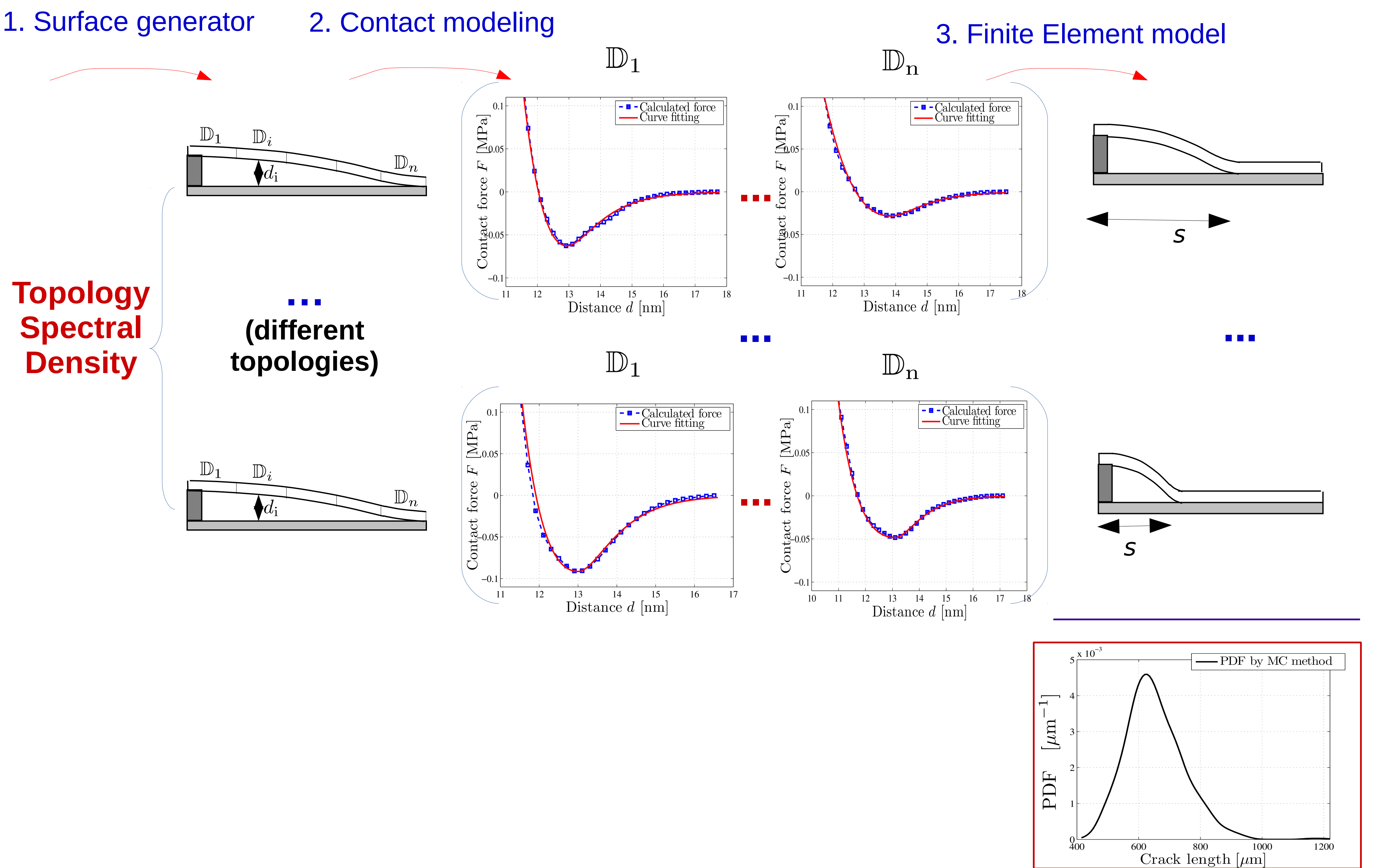




(*) The axes have different scales: the *x* and *y* axes units are µm and the *z* axis one is nm

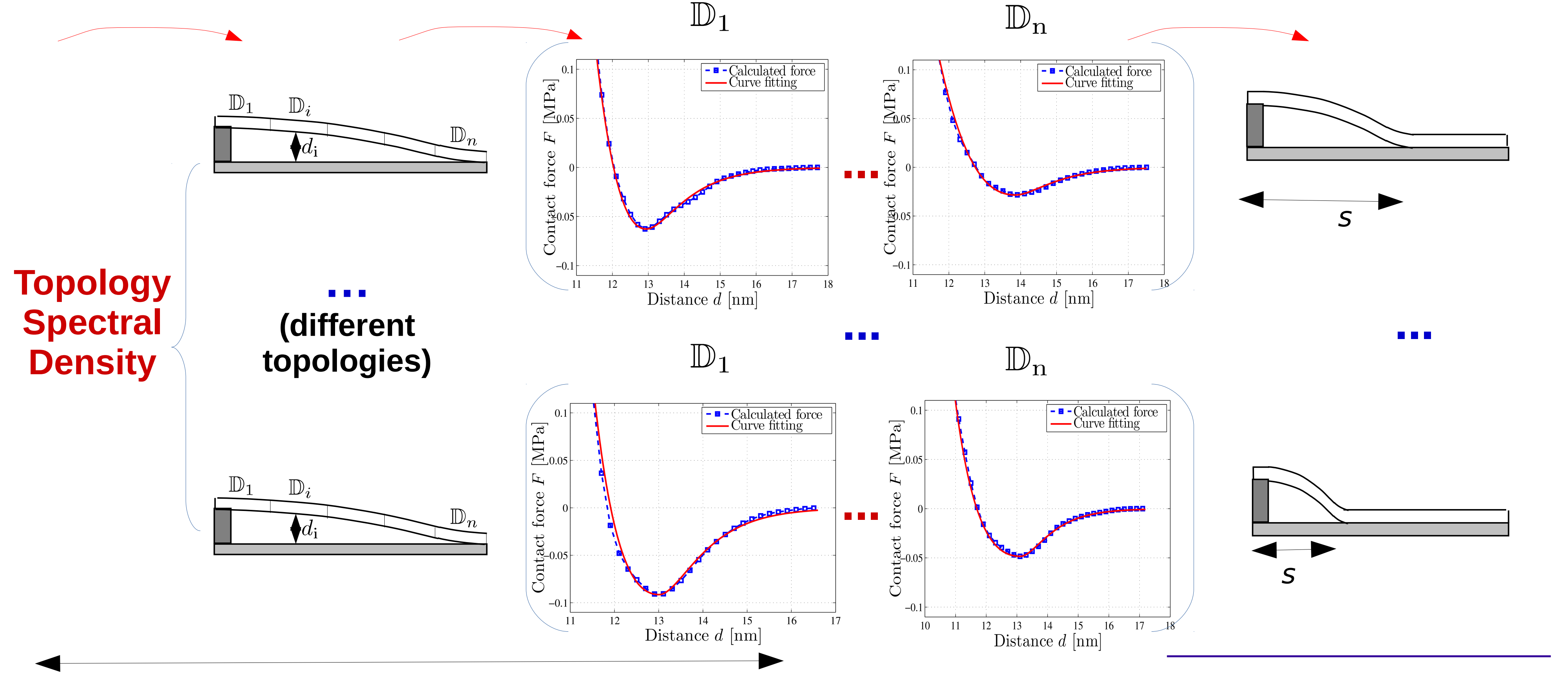
Probabilistic component of **SMM**: Direct method (Full Monte Carlo Method)

- Propagate the randomness through the multi-scale model

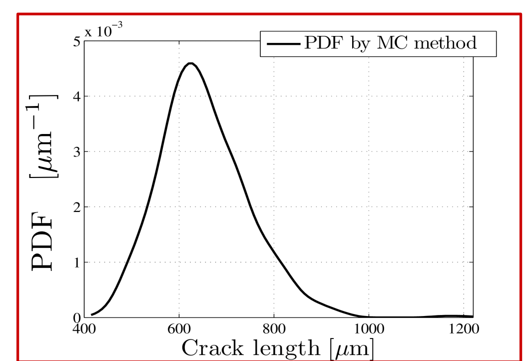


Probabilistic component of **SMM**: Direct method (Full Monte Carlo Method)

1. Surface generator 2. Contact modeling 3. Finite Element model



- A time consuming process
- Requires a big memory
- Motivation for constructing the indirect method through a stochastic model of meso-scale contact laws
  - to represent the probability distribution of the meso-scale contact laws

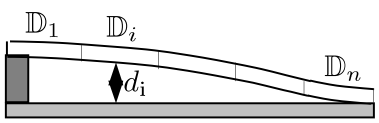


Probabilistic component of **SMM**: Direct method (Full Monte Carlo Method)

1. Surface generator

2. Contact modeling

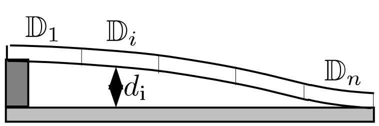
3. Finite Element model



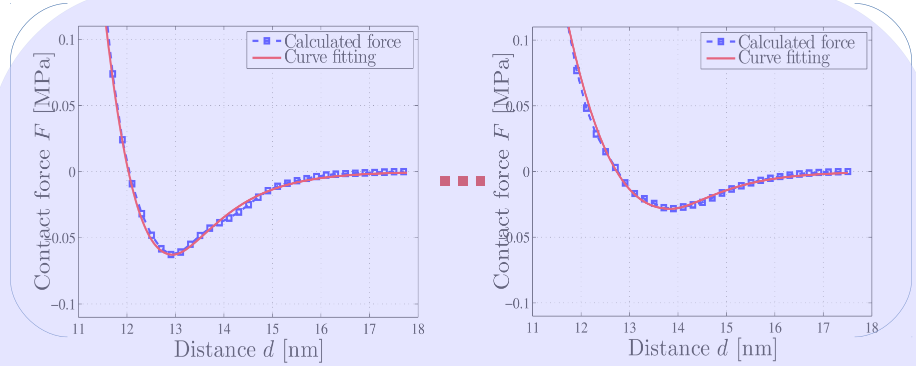
Topology Spectral Density

...

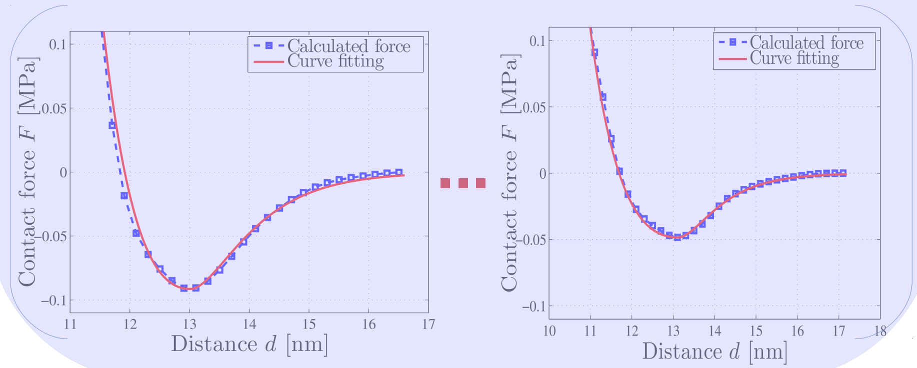
(different topologies)

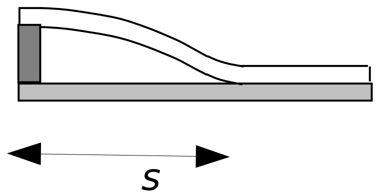


$\mathbb{D}_1$ $\mathbb{D}_n$



Reference random contact law fields





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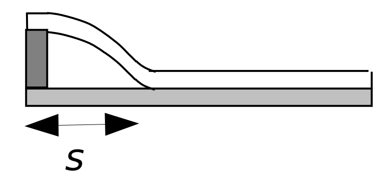
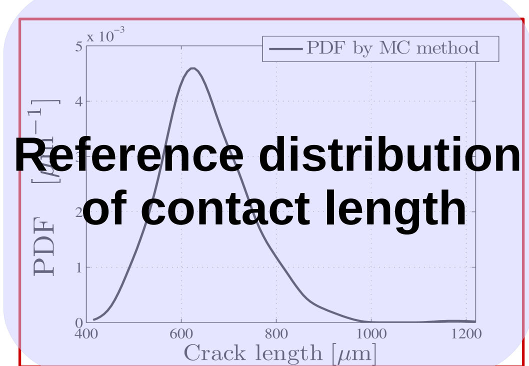




Reference distribution of contact length

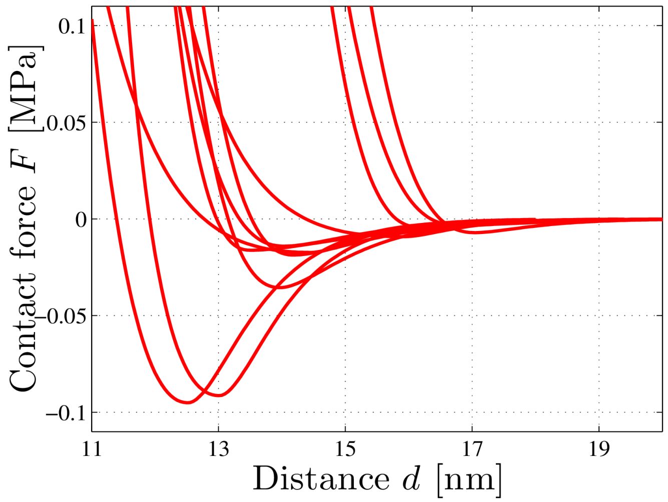
Indirect method: Stochastic model of random contact laws

- The stochastic model of random contact laws *T* represents the probability distribution of meso-scale contact laws
    - *T*: matching a random vector of a basic distribution, e. g., Gaussian one, to a random contact law

Basic vector value random variable: e.g., Gaussian one

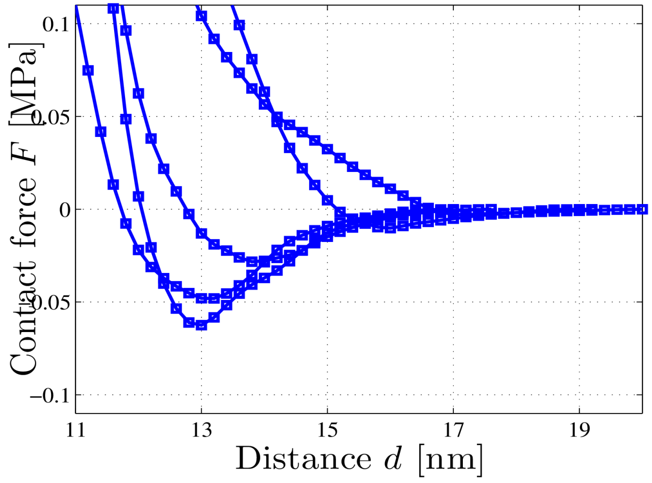
$$\hat{F}(d) = T(d, \xi)$$

Generated contact laws



Probabilistic Approximation

N observed contact laws from **SMM** using Monte Carlo method

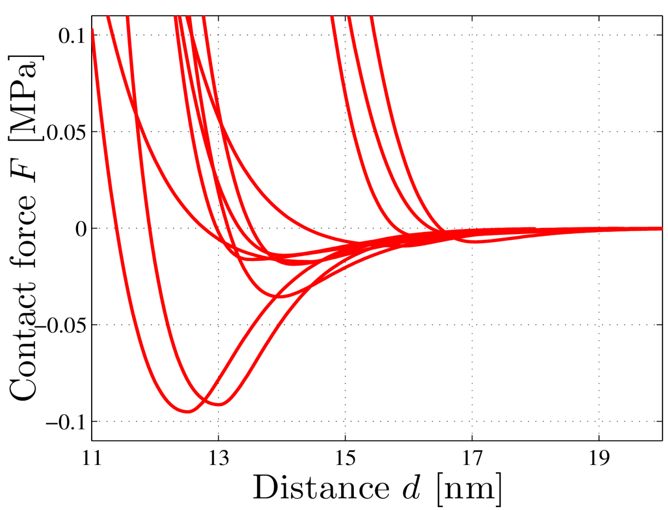


- Remark: The correlation of neighboring contact forces can be neglected

Stochastic model of random contact laws

- Reduced-order process
  - Fitting the adhesive contact laws using an analytical function (modified Morse potential) computed from the reduced parameters
  - Each contact law corresponds to a vector of reduced parameters and vice versa
- Randomness modeling process
  - Using Polynomial Chaos Expansion as the mean to represent the probability distribution of the reduced parameters

Generated contact laws



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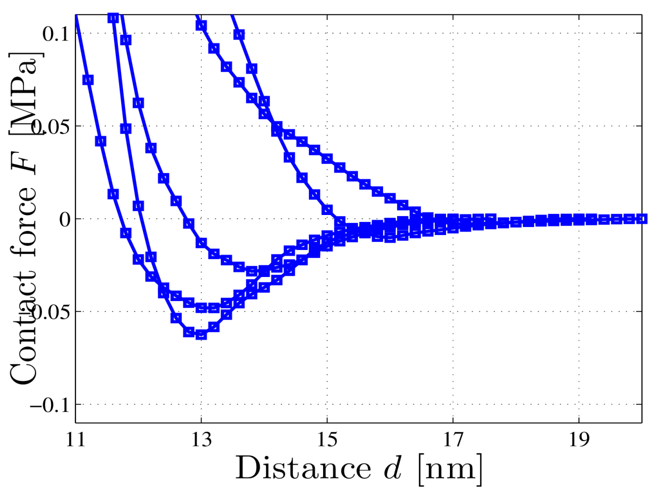
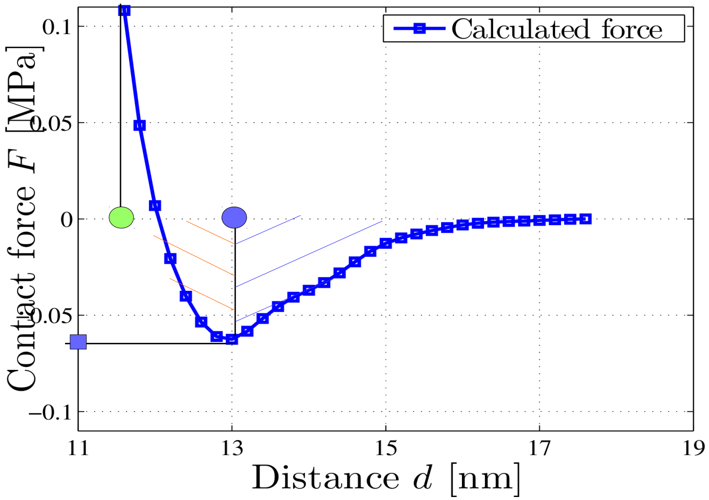


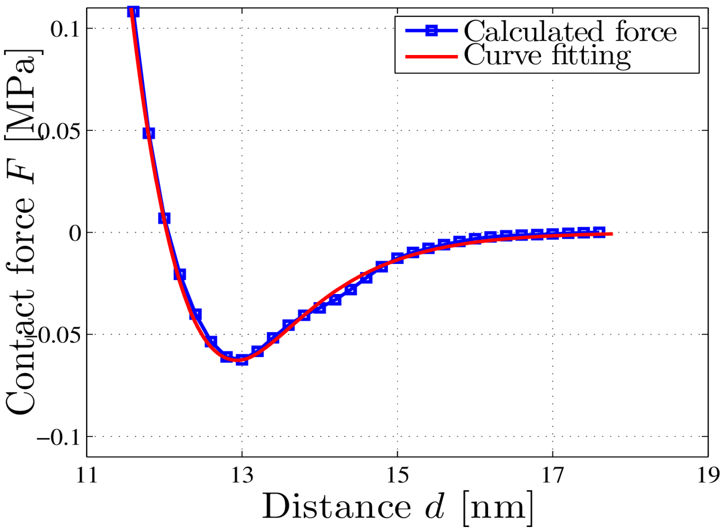
Stochastic model of random contact laws: Reduced Order process

- Reduced-order process
  - Fitting the contact laws using an analytical function (modified Morse potential)

Observed meso-scale contact laws

$\forall F$





Extracting parameters

Reconstructing

Φ

Vectors of contact law's reduced parameters

$$\lambda = \begin{pmatrix} \log(E_{\mathrm{left}}) \\ \log(F_{\max}) \\ \log(E_{\mathrm{right}}) \\ d_{\max} \\ d_{\mathrm{limit}} \end{pmatrix}$$

- The logarithm is applied to enforce the positivity of $E_{\mathrm{left}};\quad E_{\mathrm{right}};\quad F_{\max}$

- Using Hermite polynomial chaos expansion to construct the stochastic model:

$$\begin{bmatrix} \log(E_{\text{left}}) \\ \log(F_{\max}) \\ \log(E_{\text{right}}) \\ d_{\max} \\ d_{\text{limit}} \end{bmatrix} \overset{\text{d.}}{=} \sum_{k=0}^{N_{\text{p}}} \underbrace{c_k}_{\text{Coefficients Vector}} \underbrace{\psi_k}_{\text{Hermite Polynomial}} \underbrace{(\xi)}_{\text{Gaussian random vector}}$$

- The coefficients are found by solving Maximum Likelihood problem
  - Likelihood function is computed using multivariate kernel density estimation with Scott's data-based rule for the optimal bandwidth
  - The constraint of identical covariance

$$CC^{\mathrm{T}} = \mathrm{Cov}(\Lambda) \quad \text{with } C = [c_1 c_2 \ldots c_{N_{\mathrm{p}}}]$$

- The coefficient matrix can be rewritten as

$$C = [\mathrm{Cov}(\Lambda)^{-1/2}]^{\mathrm{T}} S$$

where S is defined on the Stiefel manifold $SS^T = I$

Implementation (Summary)

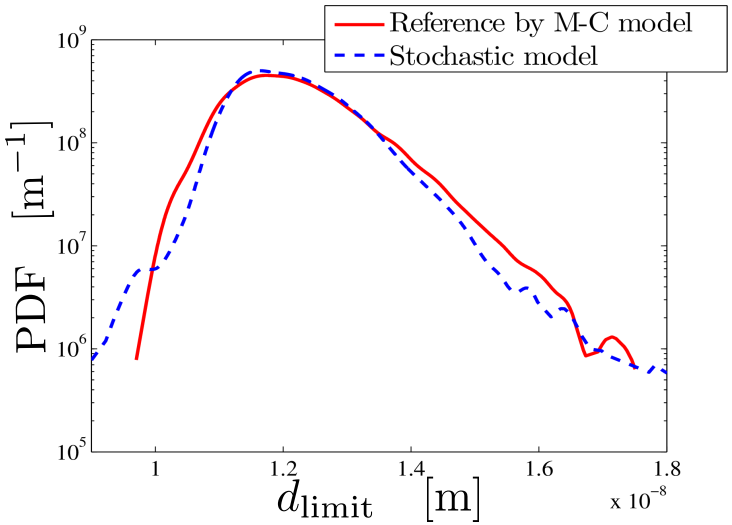
- Multi-scale component of **SMM**
  - Using analytical contact model for rough surfaces (*) to solve the Micro- to meso-scale model
  - Using Finite Element model of Euler-Bernoulli beam theory with a Newton-Raphson algorithm for dealing with the nonlinearity of contact laws to solve the meso- to macro-scale model
- Probabilistic component of **SMM**
  - Using Spectral Representation with Fast Fourier Transform implementation for the simulation of the stationary Gaussian random field of topologies (**)
  - Using gradient-free optimization in which a line-search technique and the orthogonal directions obtained by Gram–Schmidt process are applied to solve the maximum likelihood problem of PCE.

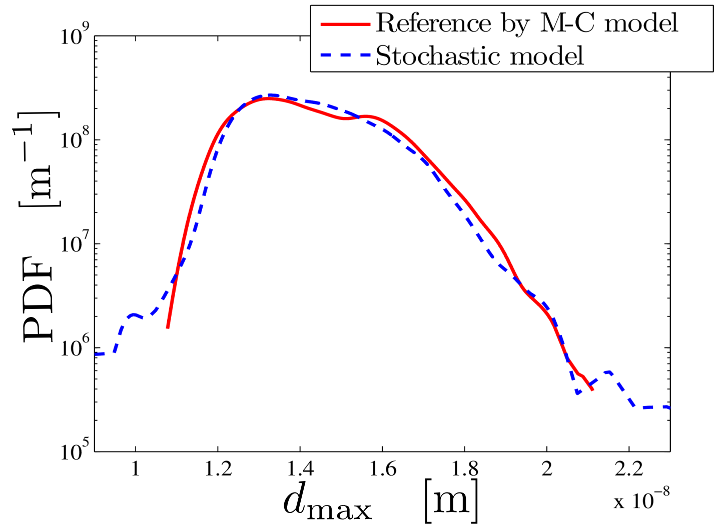
(*) TV Hoang et *al.*, *A probabilistic model for predicting the uncertainties of the humid stiction phenomenon on hard materials*

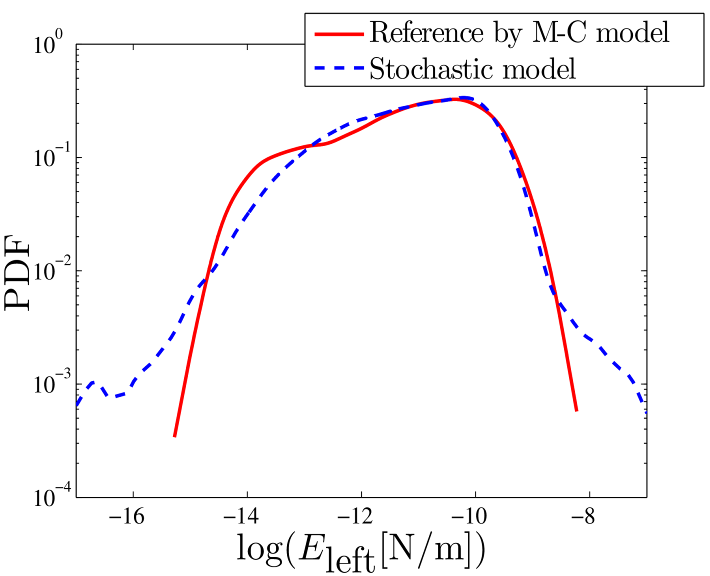
(**) F Poirion, C Soize, *Numerical Methods and Mathematical Aspects For Simulation of Homogeneous and Inhomogeneous Gaussian Vector Fields*

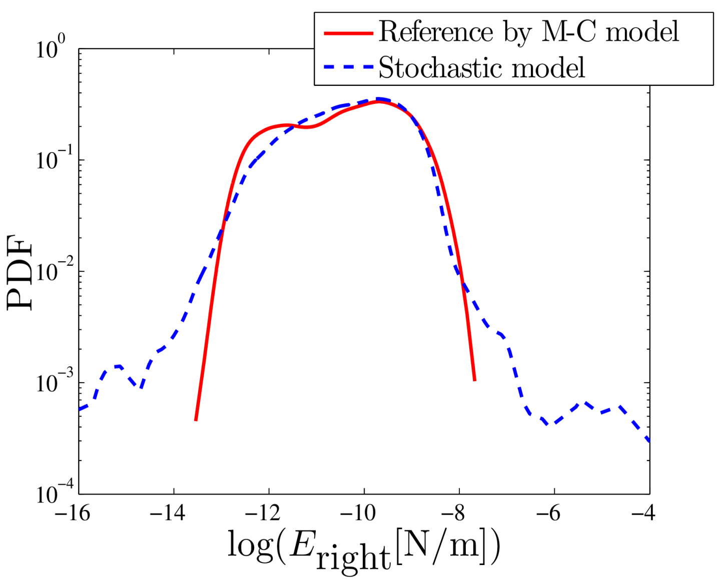
Numerical results: Meso-scale contact laws

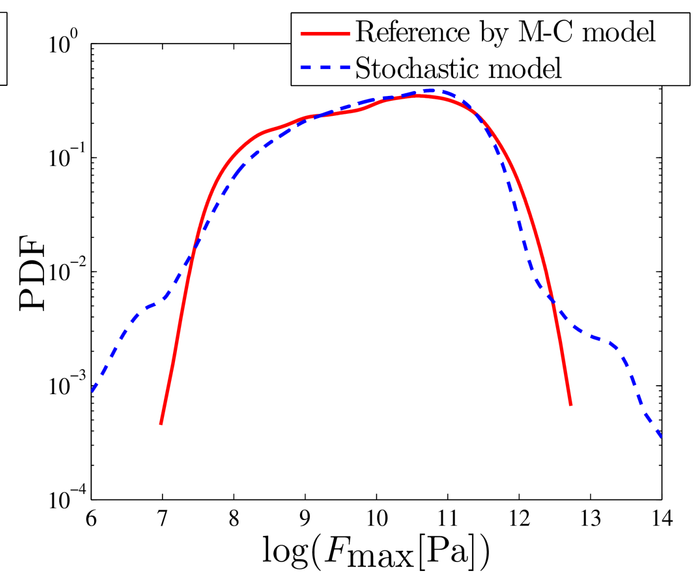
- Comparison of the distributions of reduced parameters of random meso-scale contact laws obtained
  - By full Monte Carlo method and
  - By the stochastic model of random meso-scale contact laws





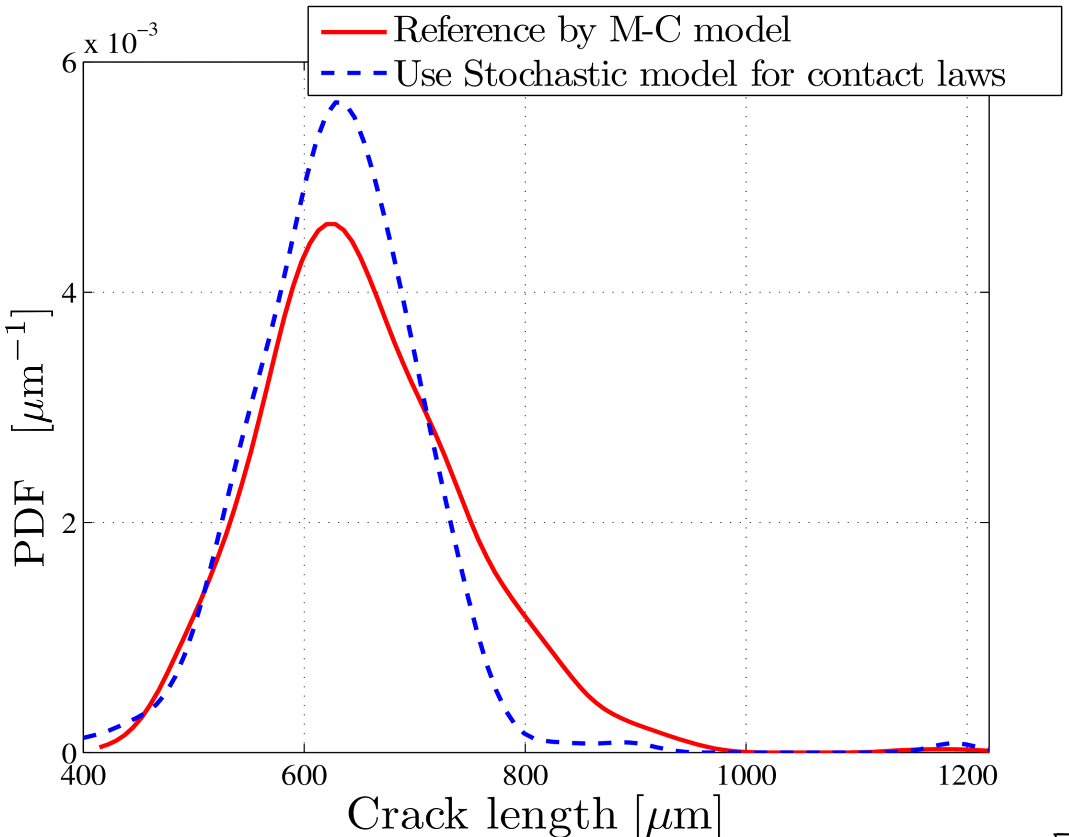


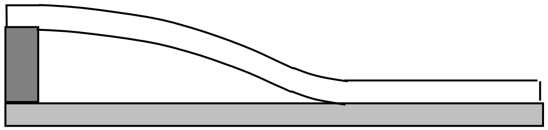


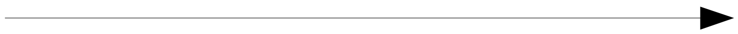


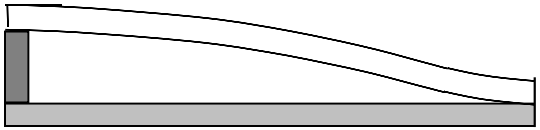
Numerical results: Macro-scale stiction level

- Comparison of the distribution of crack lengths obtained by **SMM** with two different methodologies
  - Using direct method (full Monte Carlo method) as the reference and
  - Using indirect method through the stochastic model of random meso-scale contact laws



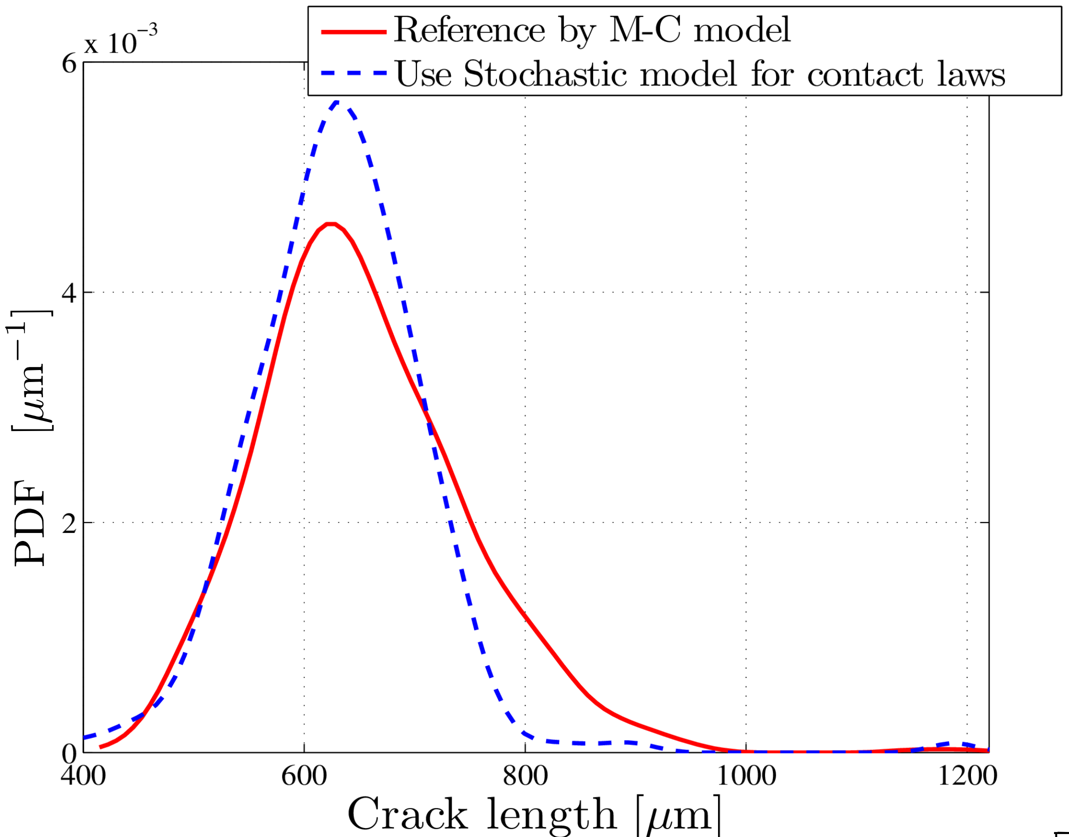


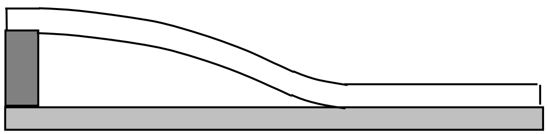


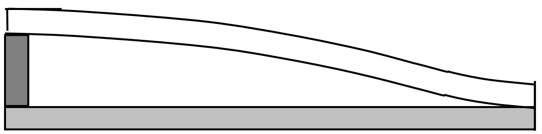


Numerical results: Macro-scale stiction level

- In case of **SMM** using stochastic model of random contact laws the crack lengths are shorter, the adhesive energies are higher.
  - Due to the magnifying of the error resulting from the logarithm scaling
- Improvements:
  - Increase the order of PCE; or
  - Adapt the probability distribution of the Ξ random variables.







Longer crack length

Conclusions

- We construct a Stochastic Multi-scale Model (SMM) for stiction problems taking the surface topology into account by
  - Using multi-scale approach with the introduction of the meso-scale contact laws
  - Applying PCE to build a stochastic model of the random meso-scale contact laws
    - to reduce efficiently the computational cost
- The stochastic model of meso-scale contact laws needs to be improved
  - Increasing the order of PCE; or
  - Adapt the probability distribution of the Ξ random variables.
- Experimental validation

References

- TV Hoang et *al*., *A probabilistic model for predicting the uncertainties of the humid stiction phenomenon on hard materials*
- L Wu et *al., A micro–macro approach to predict stiction due to surface contact in microelectromechanical systems*
- F Poirion, C Soize, *Numerical Methods and Mathematical Aspects For Simulation of Homogeneous and Inhomogeneous Gaussian Vector Fields*
- A Clément, C Soize, J. Yvonnet*, Uncertainty quantification in computational stochastic multiscale analysis of nonlinear elastic materials*
- C Soize, R Ghanem*, Physical Systems with Random Uncertainties: Chaos Representations with Arbitrary Probability Measure*
- C Desceliers*,* R Ghanem, C Soize*, Maximum likelihood estimation of stochastic chaos representations from experimental data*
- M Arnst, R Ghanem, C Soize*, Identification of Bayesian posteriors for coefficients of chaos expansions*
- D W Scott*, Multivariate Density Estimation: Theory, Practice, and Visualization*

Q&A

- Thank you for your attention