A Global Optimization Method for Structural Design

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Outline

The design a ship structure consists of

- drawing the steel structure of the ship and
- sizing all its main components (plates, piping, etc.)
- given some overall size specifications.

At the end of the design study,

- the dimensions of all plates and constitutive elements of the structure are decided,

as well as its overall weight and production cost.

Structural design

Structural design

Structural design

A ship is represented as prismatic structure.
Only midship sections are considered, excluding the bow, the stern and superstructures.

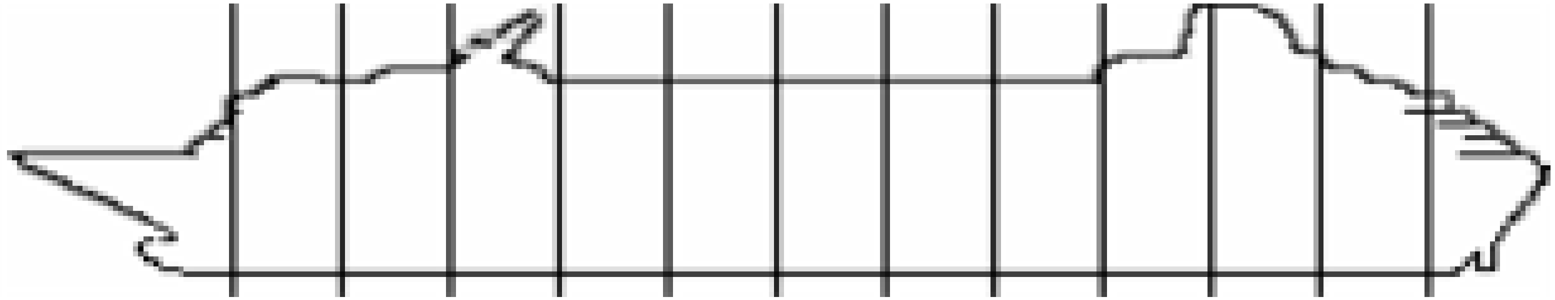


Figure : Cruise Ship

Cross sections of the structure are considered as the basic structure to be optimized.

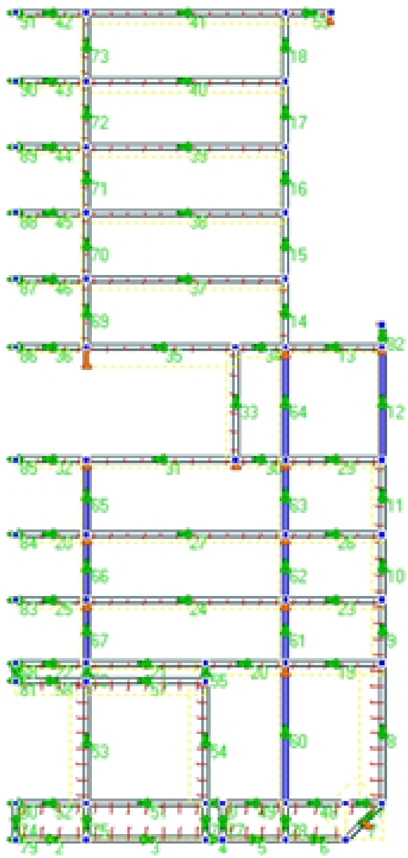


Figure : Transversal Section

A ship structure is composed of stiffened panels.

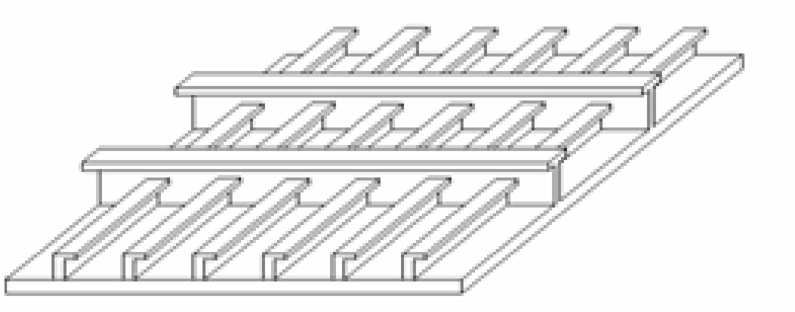
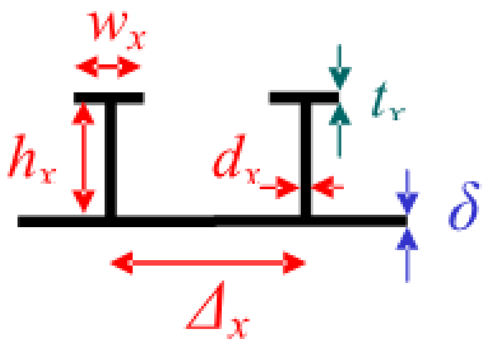


Figure : Stiffened Panel

The design variables are the spacing and scantlings of the structural members: plate thickness, member dimensions and spacing.



The objective of the preliminary design is to determine the optimal hull scantling according to a given objective.

Design variables of a structure are the spacing and scantlings of the structural members: plate thickness, member dimensions and spacing.

Objective function $f(x, y)$ can be the weight, the cost, the moment of inertia or a combination of these functions.

Global Constraints :
constraints on the moment of inertia, the section modulus of the hull girder and the gravity center shift.
These constraints relate to the entire structure and involve all the variables of the model, they are referred to as "global constraint" as opposed to "local constraints" that apply locally on each panel of the structure.

Local Constraints :
structural constraints must be verified at various points of each panel of the structure, they are defined according to IACS and Bureau Veritas requirements.

Constraints of discreteness, equality, bounds and geometry are also considered.

We consider the discrete nonlinearly constrained problem $P(x, y)$:

$$\begin{array}{lll} \min & f(x,y) & \\ \text{s.t.} & g_j(x,y) \leq 0 & j = 1, ...J \\ \text{with} & x \in X, \ y \in Y & \end{array}$$

$x = (x_1, ...x_K)$ is the stiffeners spacing (vector of continuous variables)
$y_k = (\delta_k, h_k, d_k, w_k)$: plate thickness and stiffeners dimensions of a panel k: web height, web thickness, flange height and flange thickness

$X = \{x \mid x \in \Re^n, x^L \leq x \leq x^U\}$
$Y \subset \Re^m$ of the form $Y = Y(\delta) \times Y(h) \times Y(d) \times Y(w)$
where each Y is a finite set of reals multiple of a given quantity *step*

N.B. :
Y is a set of preferable values for y variables but choosing intermediate values in $[x^L, x^U]$ has a physical meaning, and all the functions of the problem and their derivatives exist everywhere on these intervals.

$$F_W = \gamma \, \mathrm{L} \sum_p \mathrm{B}^{\mathrm{p}} \left\{ \delta^p + \frac{h_x^p d_x^p + w_x^p \mathrm{t}_{\mathrm{x}}^{\mathrm{p}}}{\Delta_x^p} \right\} = \sum_p m^p \quad (1)$$

$$GC = \frac{\sum_p m^p r^p}{\sum_p m^p} <= GC_{min} \quad (2)$$

$$I_{xx} = \sum_p m^p r^p > I_{xx\ min} \quad (3)$$

$$\frac{I_{xx}}{d} > I_{xx/v\ min} \quad (4)$$

- ◦ *ic10* : Bending strength of plate: $\sigma_x \leq \sigma_a$ (5)
- ◦ *ic19* : Shear strength: $\tau_{xy} \leq \tau_a$ (6)
- ◦ *ic14* : Compressive buckling of plate: $\sigma_a \leq \sigma_C$ (7)
- ◦ *ic20* : Shear buckling of plates: $\tau_a \leq \tau_C$ (8)
- ◦ *ic37* : Compressive buckling of stiffeners: $\sigma_a \leq \frac{1}{\beta}\sigma_C$ (9)
- ◦ *ic38* : Bending strength of stiffeners: $\sigma \leq \frac{R_y}{\gamma_R \gamma_m}$ (10)

The model suggests the decomposition into two levels:

- a system level relative to the beam ship
- a subsystem levels relative to panels.

At the system level:
optimization is performed to obtain the optimal weights of each panel.

At the subsystem level:
optimization aims at finding a discrete scantling of each panel (members dimensions and positions)
such that the optimal panel weight at the subsystem level is as close as possible to the panel's optimal scantling found at the system level.

Problem Decomposition

We develop a method that combines problem decomposition and direct search method.

Problem decomposition tackles the high dimension issue by generating subproblems of lower dimensions.

This method have been successfully applied for nonlinear optimization in the early seventies by Kirsch et al. (1972), Schmit and Ramanathan (1973) and have been further developed in numerous papers since then.

In Sobieszczanski (1993) decomposition is applied to structural design and the authors propose efficient formulations for the coupling of subproblems.

We develop a **Direct Search Method** for the subproblems generated:
those problems have continuous and discrete variables, general constraints and offer a complete knowledge of derivatives.

For mixed integer nonlinear sub-problems with a high number of discrete variables Kravanja et al. (2005) extends the multilevel optimization approach to a linked multilevel hierachical strategy.

Our Direct Search algorithm:

- *iterates on rational lattice (or mesh) to consider only discrete values of the variables*
- *makes use of the derivative information available for the objective and the constraints.*

We define an aggregate variable χ_k to denote the weight of a stiffened panel k, where $\chi_k \in X$ is a continuous variable defined on a finite interval in $\Re$.
χ_k writes as follows:

$$\chi_k = \delta_k + \frac{h_k.d_k + w_k.t_k}{\Delta_k} \tag{11}$$

$$ \tag{12}$$

We formulate the Master Problem $P(\chi)$:

$$\begin{array}{lll} \min & f(\chi) & \\ \text{s.t.} & g_j(\chi) \leq 0 & j = 1, ...J \\ \text{with} & \chi = (\chi_1, ..., \chi_K) & \chi \in \Re \end{array}$$

$P(\chi)$ is a relaxation of the original problem $P(x, y)$ as the optimization variable χ is an aggregate variable (i.e. a nonlinear combination of continuous and discrete variables, $x \in X$ and $y \in Y$)

We have:

$$v_S = \Phi_{v_S}\left(\frac{\sum_k \chi_k v_k}{\sum_k \chi_k}\right) \tag{13}$$

$$i_S = \Phi_{i_S}\left(\sum_k c{\chi_k}^3 + \chi_k (v_k - v_S)^2\right) \tag{14}$$

the vertical position (v_S) of the center of inertia and the inertia (i_S) of the beam ship, and v_k the vertical position of the center of inertia of panel k, which is a given data.

The gravity center GC function becomes

$$g_1(\chi) = \sum_k \Phi_1(\chi_k, (v_k - GC_{max})) \tag{15}$$

The moment of inertia I_{xx} of the beam ship becomes

$$g_2(\chi) = \sum_k \Phi_2(\chi_k, (v_k - v_S)) \tag{16}$$

The section modulus $\frac{I_{xx}}{d}$ of the beam ship is

$$g_3(\chi) = \sum_k \Phi_3(\chi_k, (v_k - v_S)) \tag{17}$$

The structural constraints are computed for each panel k and each load case l. Four types of constraints (numbered j=4,5,6,7) express admissible normal stress σ (bending and buckling constraints: with references ic10, i14, ic37 and ic38 and three types of constraints (j=8,9,10) are related to shear stress τ (with reference ic19 and ic20.

The dependencies on the variables x_k and y_k are defined as :

$$g_j^{k,l}(x_k, y_k) = \Phi_4(x_k, y_k, \frac{v_S}{i_S}) \qquad j = 4, 5, 6, 7 \quad (18)$$

$$g_j^{k,l}(x_k, y_k) = \Phi_5(\sum_k (x_k, y_k), v_S) \qquad j = 8, 9, 10 \quad (19)$$

The objective function (weight or cost) has a generic formulation:

$$f(\chi) = \sum_k \Phi_f(\chi_k) \qquad (20)$$

We define the subproblems $S_k(x, y)$:

find for each panel an admissible scantling such that the panel weight is as close as possible to the optimal value of the weight (χ_k) computed at the system level.

Stated in terms of the design problem, the objective of a subproblem is to minimize the difference between

- the panel weight obtained at the master level : optimization of the beam ship in continuous variables considering all constraints, and
- the weight of a panel constituted of discrete size elements, compliant with the set of constraints defined for this panel.

The subproblems are independent of each other and related to the master problem through a coordinating function C.

$$
\begin{array}{lll}
\min & C(x_k, y_k) & \qquad\qquad\qquad\qquad S(x_k, y_k) \\
\text{s.t.} & \tilde{g}_j^{k,l}(x_k, y_k) \leq 0 & j = 4, \ldots, 10 \quad l = 1...L \\
\text{with} & x_k \in X, \; y_k \in Y & \\
\text{and} & \tilde{g}_j^{k,l}(x_k, y_k) = g_j^{k,l}(x_k, y_k) & \text{with fixed values of } \; v_S, i_S
\end{array}
$$

with k the subproblem number and panel number

L is the number of load cases defined and active for the subproblem,
for each load case l, a subset of the constraints j=4,...,10 is defined.

The objective function aims at minimizing the gap between the components of the solution of the continuous master problem χ_k^{opt}, and the optimal solutions (x_k, y_k) (or equivalently $\Delta_k, \delta_k, h_k, d_k, w_k$) of the subproblems.

$$
Min\ C_{(x_k, y_k)} = ((\chi_k)^{opt} - (\delta_k + \frac{h_k.d_k + w_k.t_k}{\Delta_k}))^2 \qquad (21)
$$

The algorithm is an iterative process with the following steps:

Step 1
The master nonlinear problem $P(\chi)$ is solved with a subset K of active panels. An optimal weight χ_k^i of the panels $k \in K$ is found
(*this solution is generally not feasible for the discrete problem*).
The moment of inertia i_S^i and the vertical position of the neutral axis v_S^i are computed and the values of χ_k^i, v_S^i, i_S^i are fixed for step 2.

Step 2
for each panel $k \in K$ a discrete subproblem is solved using a direct search method. The solution (x_k^i, y_k^i) is a discrete scantling of the subproblem $S_k(x, y)$ such that the panel weight is as close as possible to the weight χ_k^i fixed at step 1.

If the solution is feasible then the values of the panel variables are fixed for the rest of the execution and k is removed of the subset K of panels active in the next iterations.

At the end of step 2
a feasibility test is performed. The master problem objective f and constraints g are computed using the optimal discrete solutions (x_k^i, y_k^i) of the K subproblems.
If theses solutions form a feasible solution (x, y) for the master problem then the algorithm terminates and returns this solution.

Otherwise the iteration terminates and a new iteration starts at step 1, with the current solution as the initial point. This iterative procedure is performed i_{max} times at most.

Master Problem :
we use the convex linearization method associated with a dual procedure developed by Fleury et al. to find an initial solution to the nonlinear master problem (Fleury and Braibant (1986), Fleury (1989a,b), Zhang and Fleury (1997)).

The authors describe an efficient sequential convex programming (SCP) method based on *Conlin* that iterates the following steps:

- a convex linearization of the objective and constraints generates the approximation
- a dual resolution method is used to find an optimal solution to the approximation.

The solution of the approximation is the starting point of the next iterate.

DS methods are characterized by a sequential examination of trial solutions and a strategy to determine what the next trial solution will be.

The original motivation underlying the development of Direct Search (DS) methods is to use the values of the objective function only, with no knowledge of derivative information (Audet and Dennis 2006, Abramson et al. 2009).

Although developed as derivative free methods, GPS Methods have been adapted to take advantage of derivative information when available... Kolda et al. (2003), Abramson et al. (2004)

Each iterate lies on a mesh built using a finite set of directions called positive basis or generating set, that positively spans the solution space : such that any vector of the space can be written as a positive combination of the directions in the set.

Our interest in DS methods is to **exploit the mesh structure in order to iterate on feasible discrete solutions only.**

Mesh Definition

- Using discretization information
  the definition set of each variable is used to set the minimal discretization unit of the mesh, so that the discreteness of the variables is respected.

Search Directions

- Exploit Objective function structure
  the objective function is used to build the set of search-generating directions before the start of the algorithm
- Using constraints information
  at each polling iteration the active constraints values and derivatives are used to prune this set in order to exclude those leading to a deterioration of the objective or that increase the infeasibility.
- Using discretization information
  the definition set of each variable is also used to prune the set of search directions so that the domain definition of the variables is respected.

The industrial test case is a passenger ship composed of 118 stiffened plate elements and 19 pillars, 14 decks, a 80 m breadth and a 45 m height.

Five scantling design variables are defined for each stiffened plate element: plate thickness, flange width, web height and thickness of longitudinal stiffeners and spacing between two longitudinal stiffeners.
The domain definition of the discrete design variables is defined with a step of 1 mm for the thicknesses and 10 mm for the web height and flange width, the stiffener spacing is continuous.
The size of the solution space for the discrete variables is roughly $38.\ 10^{439}$.

Four global constraints on the hull girder are considered, they are the weight (bounded above), the section modulus and moment of inertia (bounded below) and the gravity center shift.

Ten load cases were considered in the calculation:

- two “IACS load cases” (hogging & sagging): still water bending + wave bending with a probability of exceedence = 10^{-8}
- eight “BV load cases” (hogging & sagging)

The Decomposition and Direct Search (DDS) algorithm has been tested with the following parameter sets :

- DDS4-P : the initial poll step size $\Delta_p = 4$ and pruning is active.
- DDS4-NP : the initial poll step size $\Delta_p = 4$ and no pruning is performed based on the constraints.
- DDS1-P : the poll step size $\Delta_p = 1$ is constant over the entire algorithm execution, and pruning is active.
- DDS1-NP : the poll step size $\Delta_p = 1$ is constant over the entire algorithm execution, and no pruning is performed based on the constraints.

Table : Experiments: Decomposition and Direct Search Algorithm applied on a cruise ship.

	DDS4-P	DDS4-NP	DDS1-P	DDS1-NP
$W(P(\chi))$	6551117	6551117	6551117	6551117
$Z(P(x,y))$	7056146	7023685	6928474	6928474
$W\%Z$	7.71	7.21	5.76	5.76
Iterations	41	124	22	18

The Decomposition and Direct Search (DDS) method has been developped for large scale structural optimization with discrete variables. The method has a limited number of parameters (mesh and application of Direct Search), whose values may be fixed automatically from the instance data.

DDS has been applied to real ship design problems, based on a "‘Beam ship"’ 2-dimensional model commonly used for certification purposes.
DDS provides optimal discrete solutions of good quality with a small number of structural analysis and computing time.

The method has been included in the preliminary design analysis and optimization software LBR-5 (Rigo 2001a,b).

More numerical experiments on truss topology or strucutral engineering problems need to be performed to better assess the quality of the algorithm.

References

M. Abramson, C. Audet, and J.E. Dennis. Generalized pattern searches with derivative information. *Mathematical Programming Series B*, 100(1):3–25, 2004.

M. Abramson, C. Audet, J. Chrissis, and J. Walston. Mesh adaptive direct search algorithms for mixed variable optimization. *Optimization Letters*, 3(1):35–47, 2009.

C. Audet and J.E. Dennis. Mesh adaptive direct search algorithms for constrained optimization. *SIAM Journal on Optimization*, 17(1):188–217, 2006.

C. Fleury. Conlin: an efficient dual optimizer based on convex approximation concepts. *Structural Optimization*, 1 (2):81–89, 1989a.

C. Fleury. First and second order convex approximation strategies in structural optimization. *Structural Optimization*, 1(1):3–10, 1989b.

C. Fleury and V. Braibant. Structural optimization: a new dual approach using mixed variables. *International Journal for Numerical Methods in Engineering*, 23:409–428, 1986.

U. Kirsch, M. Reiss, and U. Shamir. Optimum design by partitioning into substructures. *Structural of Structural Division ASCE*, 98(ST1):249, 1972.

T.G. Kolda, R.M. Lewis, and V. Torczon. Optimization by direct search: new perspectives on some classical and modern methods. *SIAM Review*, 45(3):385–482, 2003.

S. Kravanja, S. Lilih, and Z. Kravanja. The multilevel minlp optimization approach to structural synthesis: the simultaneous topology, material, standard and rounded dimension optimization. *Advances in Engineering Software*, 36(9):568 – 583, 2005.

Ph. Rigo. A module oriented tool for optimum design of stiffened structures – part I. *Marine Structures*, 14: 611–629, 2001a.

Ph. Rigo. Least cost structural optimization oriented preliminary design. *Journal of Ship Production*, 17(4):202–215, November 2001b.

L.A. Schmit and R.K. Ramanathan. A multilevel approach to minimum weight design including buckling constraints. *AIAA Journal*, 16(2):97–104, 1973.

J. Sobieszczanski. Two alternative ways for solving the coordination problem in multilevel optimization. *Structural Optimization*, 6:205–215, 1993.

W.H. Zhang and C. Fleury. A modification of convex approximation methods for structural optimization. *Computers & Structures*, 64(1–4):89–95, 1997.