A stable numerical scheme for shallow flow modelling taking into account bottom curvature

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Accurately modelling free surface flows on strongly vertically curved bottoms, such as for instance over a spillway (see picture: spillway of the Nisramont dam), is a challenge for any depth-averaged model. This type of computation requires the use of axes properly inclined along the mean flow direction in the vertical plane (Figure 1) and a modelling of curvature effects. The herein presented generalized model performs such computations by means of suitable curvilinear coordinates in the vertical plane (Figure 2) [1], leading to the possibility of conducting unified simulations of the flows in the upstream reservoir, on the spillway, in the stilling basin and in the downstream river reach within a single computation domain. The velocity profile is generalized in comparison with the uniform one usually assumed in the classical shallow water equations. The pressure distribution is modified as a function of the bottom curvature and is thus not purely hydrostatic but accounts for effects of centrifugal forces. The new model constitutes a two-dimensional extension of the pioneering work carried out by Dressler [2] in 1978.

This enhanced mathematical modelling framework has been implemented within the finite volume numerical model *WOLF 2D*. A specific *Flux Vector Splitting* (FVS) technique has been developed and demonstrated to be stable for any flow regime, by means of a von Neuman analysis. The upwind scheme offers the advantage of being dependent only on the sign of the bottom curvature. For a vanishing bottom curvature, the new model converges smoothly towards the conventional shallow-water equations.

Besides describing the mathematical and numerical extended model, the presentation will detail several validation examples, as well as an application of the model to design a large hydraulic structure.

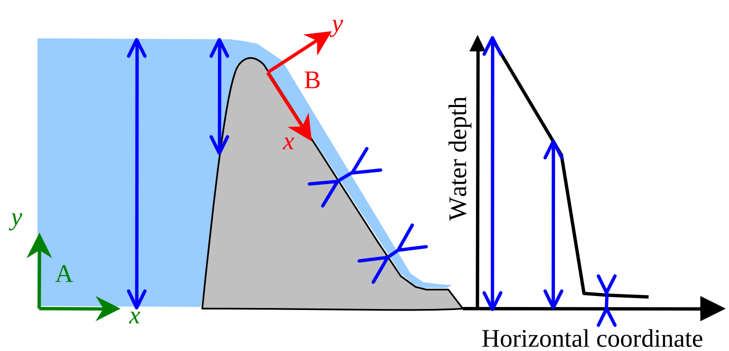

Figure 1: Frame of reference locally inclined along the mean flow direction (A vs B).

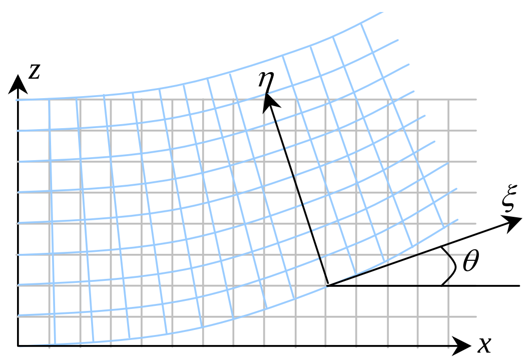

Figure 2: Cartesian (x, y) vs curvilinear (ξ, η) frames of reference.

References

[1] B.J. Dewals, S. Erpicum, P. Archambeau, S. Detrembleur and M. Pirotton. Depth-integrated flow modelling taking into account bottom curvature. *J. Hydraul. Res.*Vol. 44(**6**), 2006.

[2] R.F. Dressler. New nonlinear shallow flow equations with curvature. *J. Hydraul. Res.* 1978.