IE-net Managing, handling, and modeling uncertainty in mechanical design

Nonintrusive probabilistic quantification of uncertainties with application to the management of manufacturing tolerances

Maarten Arnst

May 21, 2015

Manufacturing tolerances in metal forming

Raw materials variability:

- Material properties.

. . .

Process variability:

- Blank holder force.
- Initial dimensions.
- Friction.

. . .

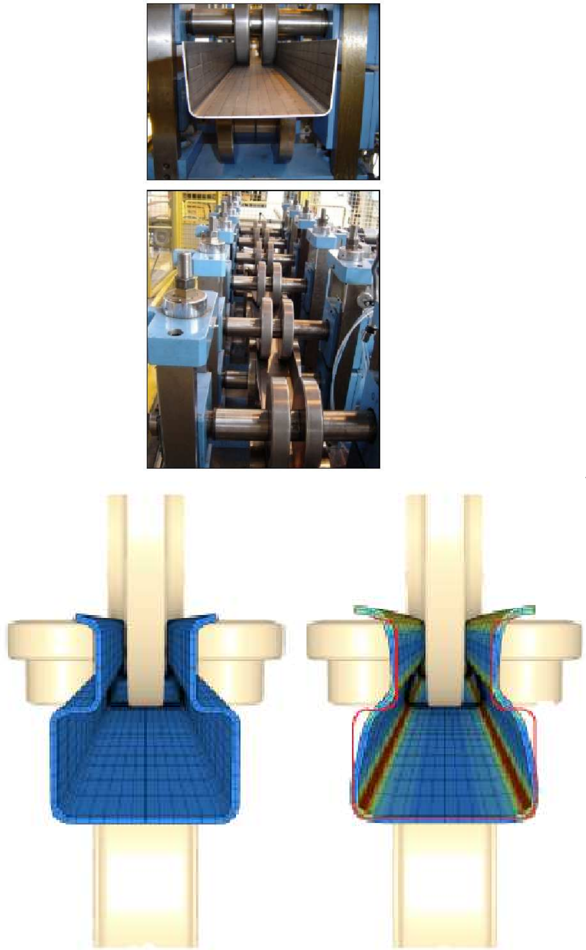
Modeling limitations:

- Constitutive model.
- FE discretization.

. . .

Input variables.

→



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Product variability:

- Final dimensions.
- Springback.

. . .

Prediction limitations:

- Numerical noise.

. . .

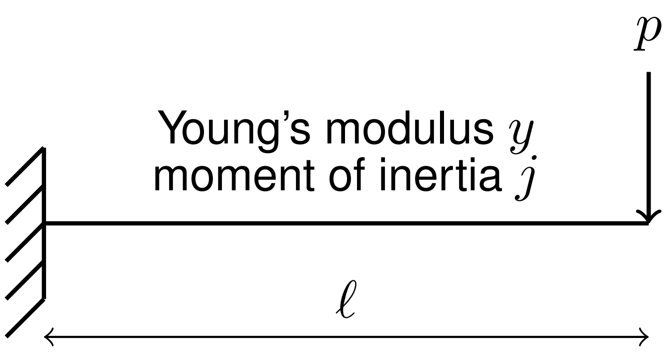
Output variables.

Outline

- Motivation.
- Outline.
- Context and current practice.
- New methods.
- Example: Metal forming.
- Conclusion and outlook.
- References.
- Contact information.

Selected elements from context and current practice

Example: Bending of a beam

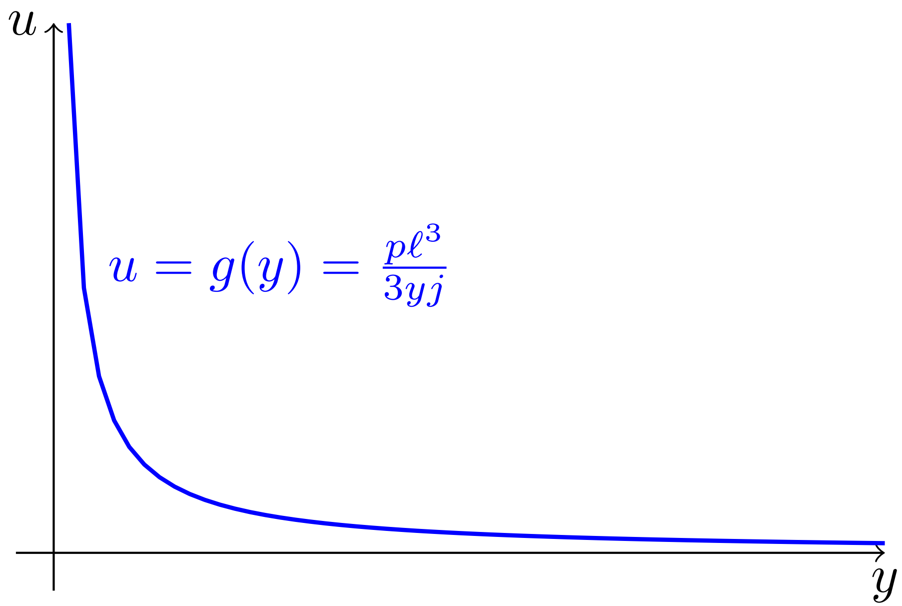


$$\underbrace{u}_{\substack{\text{output variable}\\\text{tip displacement}}} = \underbrace{g}_{\text{model}}(\underbrace{y, j, p, \ell}_{\text{input variables}}) = \frac{p\ell^3}{3yj}.$$

Let y be uncertain (e.g., imperfect knowledge at design time, imperfect manufacturing when compared to the design,...). Given uncertainty in y, what is the resulting uncertainty in u?

Example: Bending of a beam (continued)

- A probabilistic context effects the propagation of uncertainty from y to u as follows:



$$P\big(U \leq u\big) = P\big(g^{-1}(u) \leq Y\big)$$ because g is a decreasing function.

Example: Bending of a beam (continued)

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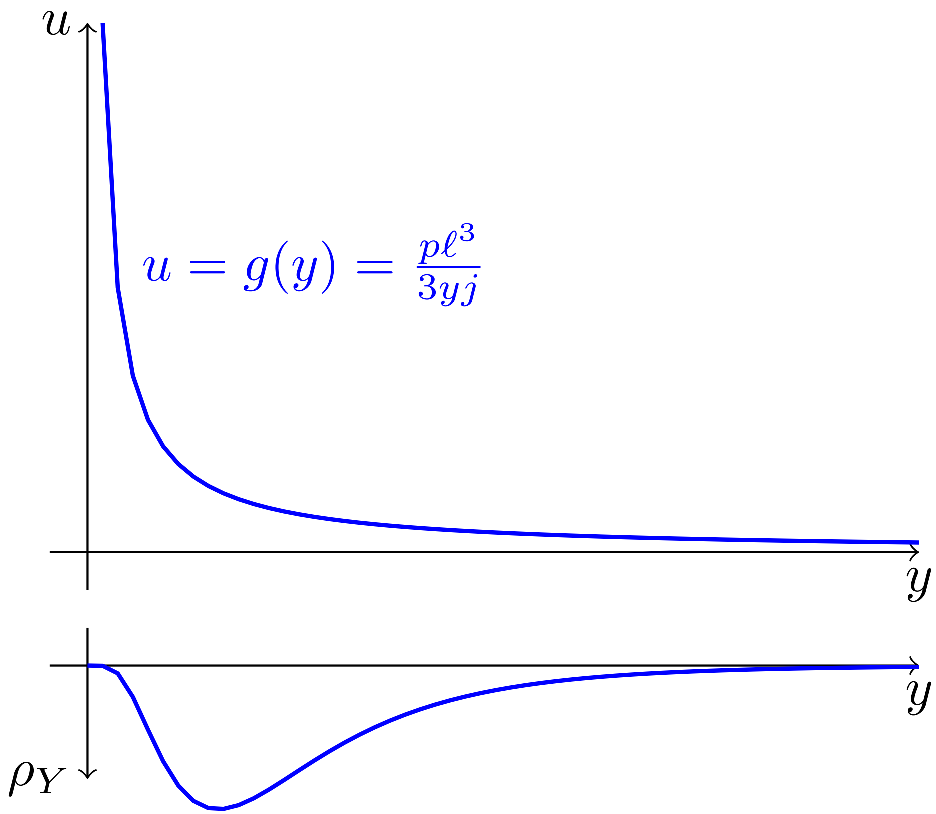

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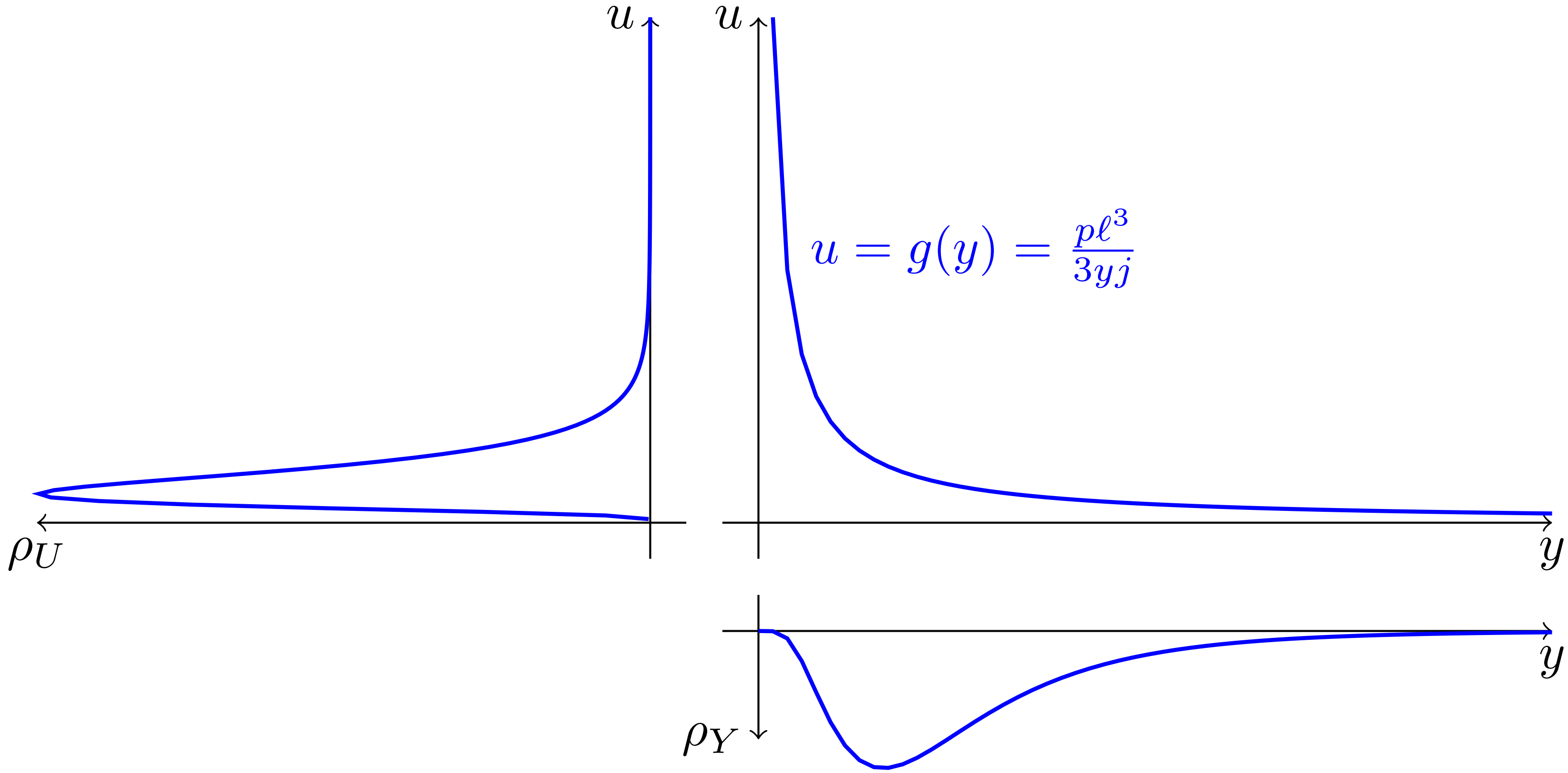

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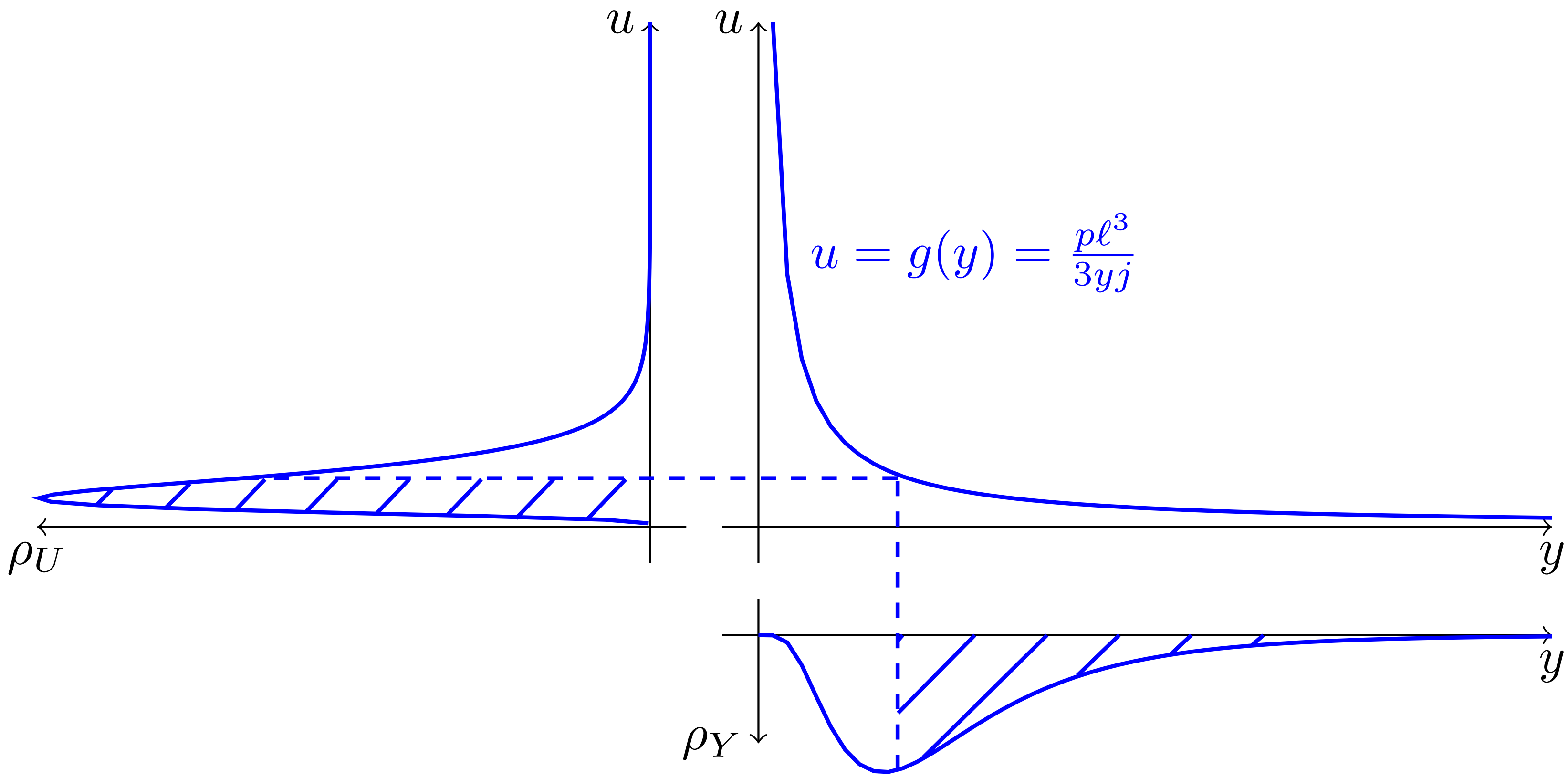


$$P\big(U \leq u\big) = P\big(g^{-1}(u) \leq Y\big)$$ because g is a decreasing function.

Example: Bending of a beam (continued)

- Elaborating this expression by means of the “changes of variables” formula,

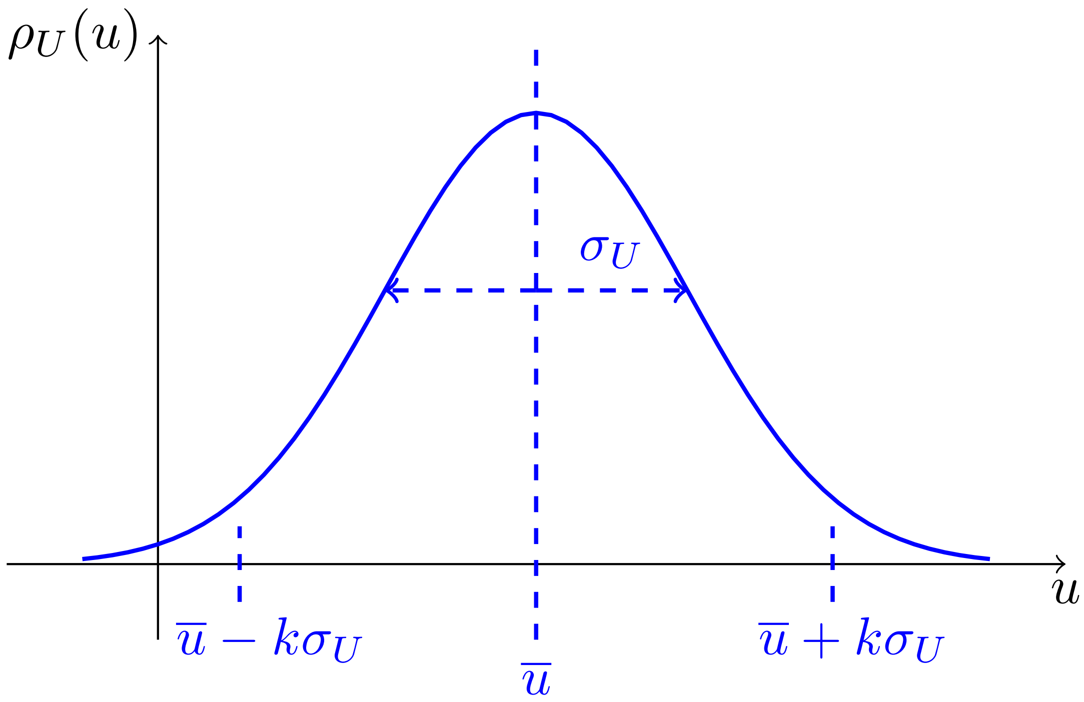
$$\begin{aligned}\int_0^u \rho_U(u)du &= \int_{g^{-1}(u)}^{+\infty} \rho_Y(y)dy \\ &= \int_u^0 \rho_Y\left(g^{-1}(u)\right)\frac{dg^{-1}}{du}(u)du \\ &= \int_0^u \rho_Y\left(g^{-1}(u)\right)\left|\frac{dg^{-1}}{du}(u)\right|du,\end{aligned}$$

  we find the following relationship between the probability density functions of the input and output variables, that is, of the Young’s modulus and the tip displacement:

$$\rho_U(u) = \rho_Y\left(g^{-1}(u)\right)\left|\frac{dg^{-1}}{du}(u)\right|.$$

Example: Bending of a beam (continued)

- Probability density function, mean, variance, and confidence interval:



  - Mean $\overline{u} = \int_{\mathbb{R}} u\rho_U(u)du$,
  - Variance $\sigma_U^2 = \int_{\mathbb{R}}(u - \overline{u})^2\rho_U(u)du$,
  - P_{c}-Confidence interval $[\overline{u} - k\sigma_U, \overline{u} + k\sigma_U]$ such that $\int_{\overline{u}-k\sigma_U}^{\overline{u}-k\sigma_U} \rho_U(u)du \geq P_{\text{c}}$.

Example: Bending of a beam (continued)

- Using the change-of-variables formula, we can deduce the following expression for the mean $\overline{u}$:

$$\begin{aligned}\overline{u} = \int_{\mathbb{R}} u\rho_U(u)du &= \int_{\mathbb{R}} u\rho_Y\left(g^{-1}(u)\right)\left|\frac{dg}{dy}\left(g^{-1}(u)\right)\right|^{-1} du \\ &= \int_{\mathbb{R}} g(y)\rho_Y(y)\left(\frac{dg}{dy}(y)\right)^{-1}\frac{dg}{dy}(y)dy \\ &= \int_{\mathbb{R}} g(y)\rho_Y(y)dy,\end{aligned}$$

  and we can deduce the following expression for the variance σ_U^2:

$$\begin{aligned}\sigma_U^2 = \int_{\mathbb{R}} (u-\overline{u})^2\rho_U(u)du &= \int_{\mathbb{R}} (u-\overline{u})^2\rho_U\left(g^{-1}(u)\right)\left|\frac{dg}{dy}\left(g^{-1}(u)\right)\right|^{-1} du \\ &= \int_{\mathbb{R}} (g(y)-\overline{u})^2\rho_Y(y)\left(\frac{dg}{dy}(y)\right)^{-1}\frac{dg}{dy}(y)dy \\ &= \int_{\mathbb{R}} (g(y)-\overline{u})^2\rho_Y(y)dy.\end{aligned}$$

- In conclusion, to determine the mean and the variance of the output, knowledge of the probability density function of the input and an integration method are required.

Example: Bending of a beam (continued)

- If the model is linearized,

$$u = g(y) \approx g(\overline{y}) + \left(\frac{dg}{dy}(\overline{y})\right)(y - \overline{y}).$$

then the expression for the mean $\overline{u}$ can be simplified as follows:

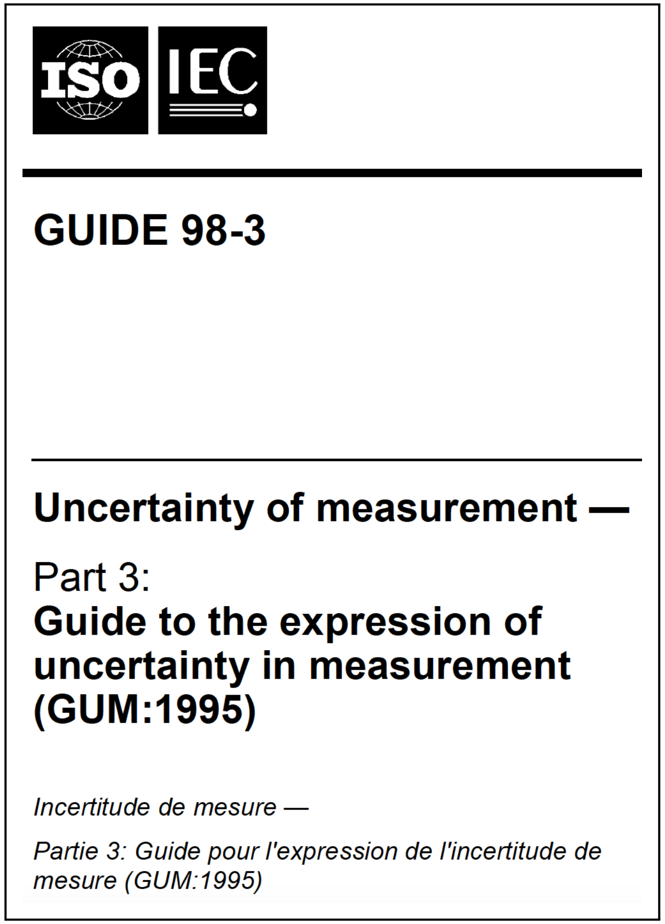
$$\overline{u} = \int_{\mathbb{R}} g(y)\rho_Y(y)dy \approx \int_{\mathbb{R}} \left(g(\overline{y}) + \left(\frac{dg}{dy}(\overline{y})\right)(y - \overline{y})\right)\rho_Y(y)dy = g(\overline{y}),$$

and the expression for the variance σ_U^2 can be simplified as follows:

$$\sigma_U^2 = \int_{\mathbb{R}} (g(y) - \overline{u})^2 \rho_Y(y)dy \approx \int_{\mathbb{R}} \left(\left(\frac{dg}{dy}(\overline{y})\right)(y - \overline{y})\right)^2 \rho_Y(y)dy = \left(\frac{dg}{dy}(\overline{y})\right)^2 \sigma_Y^2.$$

- In conclusion, linearising the model makes things much simpler!! Now, to approximate the mean and the variance of the output, knowledge of only the mean and variance of the input suffices.

Example: ISO 98


GUIDE 98-3

Uncertainty of measurement —

Part 3:
Guide to the expression of uncertainty in measurement (GUM:1995)

Incertitude de mesure —

Partie 3: Guide pour l'expression de l'incertitude de mesure (GUM:1995)

ISO 98: Guide to the expression of uncertainty in measurement.

Example: ISO 98 (continued)

5.1.2 The combined standard uncertainty $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$, which is given by

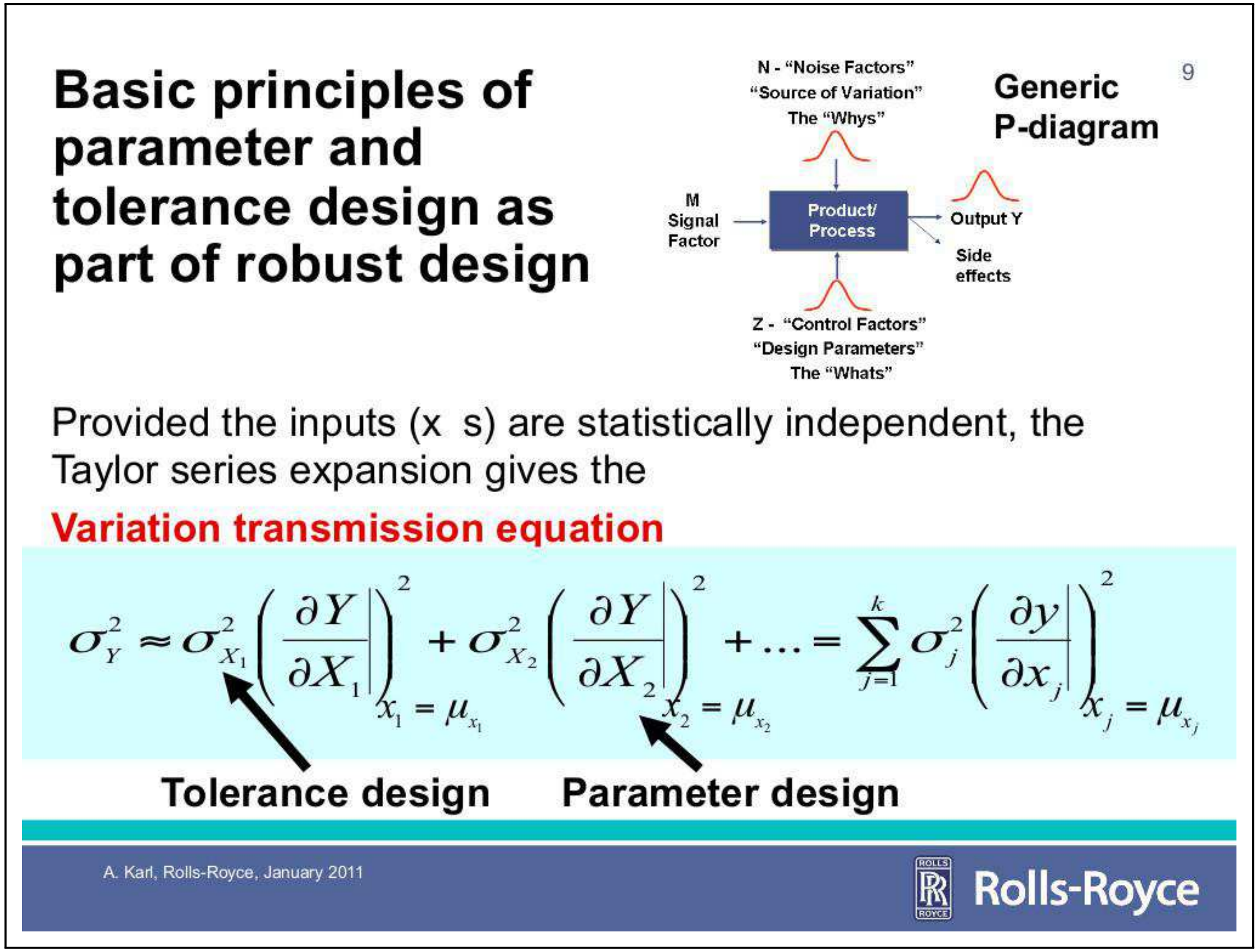
$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \quad (10)$$

where f is the function given in Equation (1). Each $u(x_i)$ is a standard uncertainty evaluated as described in 4.2 (Type A evaluation) or as in 4.3 (Type B evaluation). The combined standard uncertainty $u_c(y)$ is an estimated standard deviation and characterizes the dispersion of the values that could reasonably be attributed to the measurand Y (see 2.2.3).

ISO 98: Guide to the expression of uncertainty in measurement.

Context and current practice

Example: Robust design in aerospace engineering



From: A. Karl, B. Farris, L. Brown, and N. Metzger (Rolls-Royce). Robust design and optimization: Key methods and applications. Stanford, 2011.

Context and current practice

Some limitations associated with the approaches described so far...

- **Engineering problem**
  - Limited in scope to scalar uncertain quantities.
  - However, more complex uncertainties can be encountered in engineering problems, such as uncertain geometries, uncertain processes and fields, and uncertain matrices.

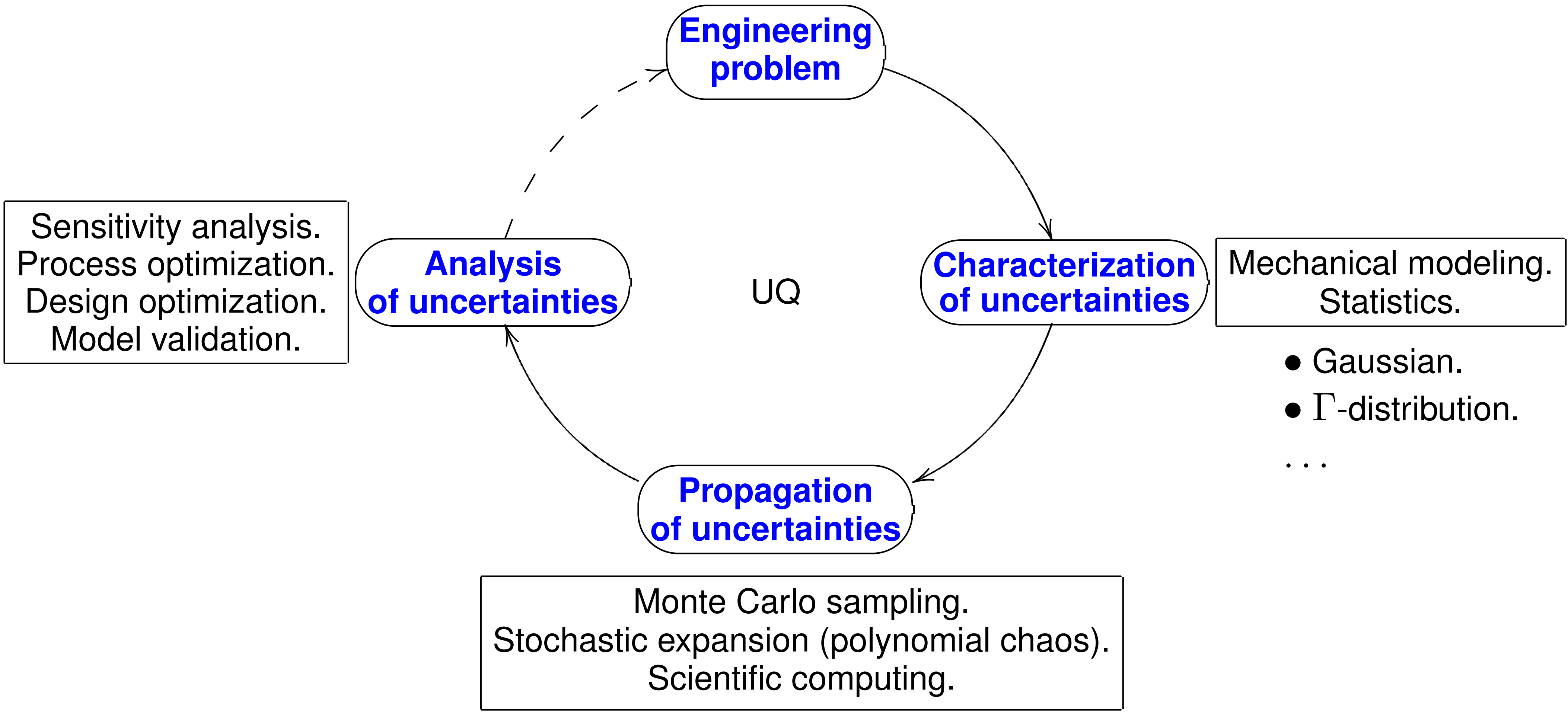
- **Characterization of uncertainties**
  - Limited to mean and variance.
  - No emphasis on constraints that can be imposed by mechanics and physics.

- **Propagation of uncertainties**
  - Approximation entailed by linearization of the model.

- **Sensitivity analysis of uncertainties**
  - Limited to local sensitivity analysis that is also encountered in deterministic problems.
  - However, global sensitivity analysis can also be of interest; and many new interesting questions can be asked in an uncertainty-quantification-enabled context.

Selected elements from new methods

Overview


Engineering problem
Characterization of uncertainties
Mechanical modeling.
Statistics.
• Gaussian.
• Γ-distribution.
. . .
Propagation of uncertainties
Monte Carlo sampling.
Stochastic expansion (polynomial chaos).
Scientific computing.
Analysis of uncertainties
Sensitivity analysis.
Process optimization.
Design optimization.
Model validation.
UQ

Characterization of uncertainties

- The objective of the characterization of uncertainties is to assign an appropriate **probability distribution** to the uncertain input variables.

- An appropriate probability distribution can be obtained by applying methods from **mathematical statistics** to the available information. In engineering, this available information typically consists not only of observed samples but also of applicable **mechanical and physical laws**.
  - Catalogs of probability distributions.
  - Principles of construction.
  - Methods for parameter estimation.
  - Methods for model selection.
  - . . .

INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING
Int. J. Numer. Meth. Engng 2008; **76**:1583–1611
Published online 2 July 2008 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/nme.2385

Construction of probability distributions in high dimension using the maximum entropy principle: Applications to stochastic processes, random fields and random matrices

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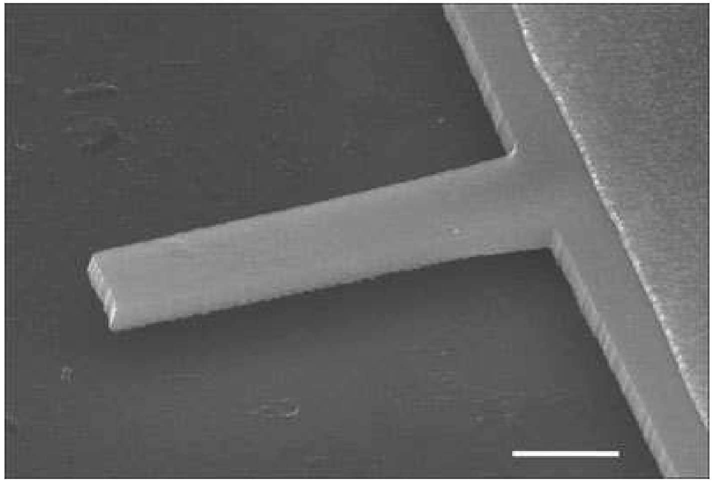
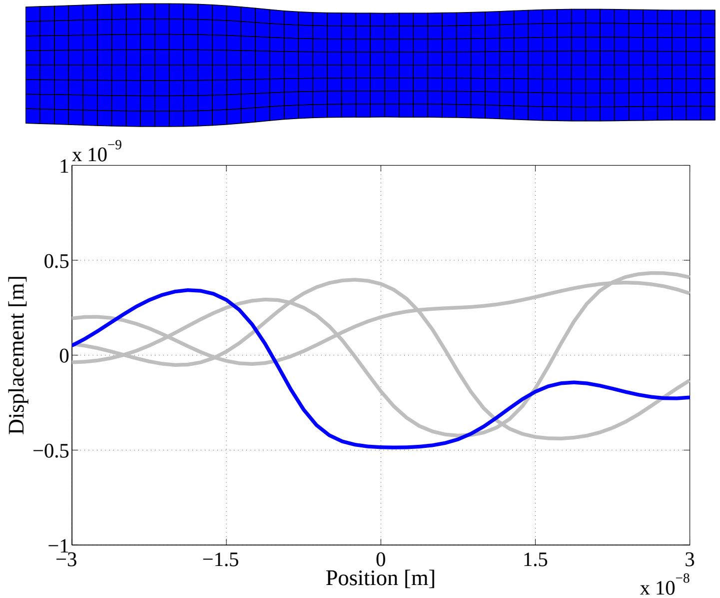
- If a sufficient amount of data is available, much of this can be automated.

- Current ressearch allows to consider as uncertain not only scalar input variables but also geometries, fields of mechanical and physical properties, matrix-valued input variables, etc.

Characterization of uncertainties (continued)

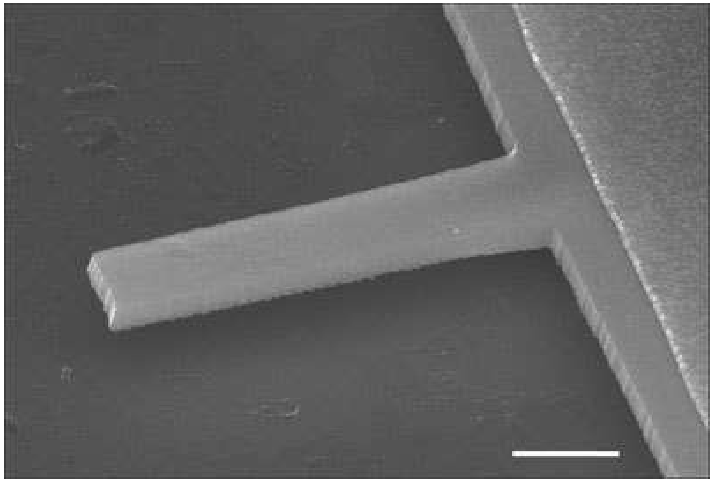


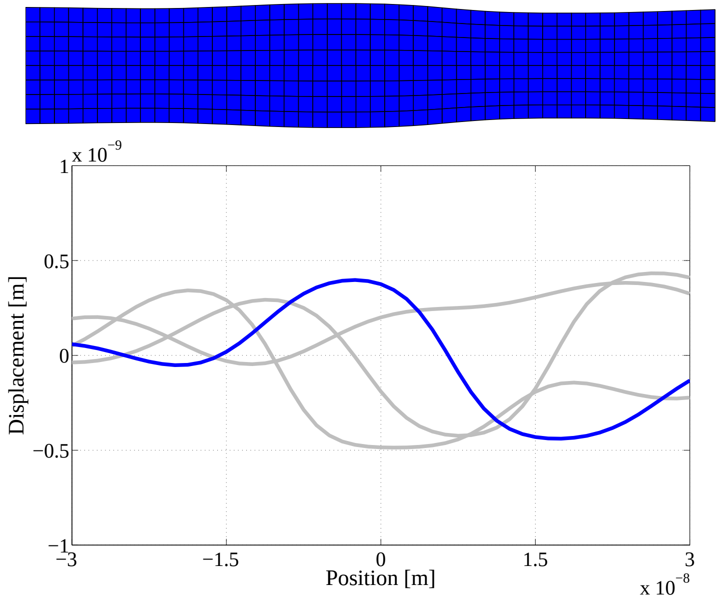


Random geometry.

From: M. Arnst and R. Ghanem. Probabilistic electromechanical modeling of nanostructures with random geometry. *Journal of Computational and Theoretical Nanoscience*, 6:2256–2272, 2009.

Characterization of uncertainties (continued)





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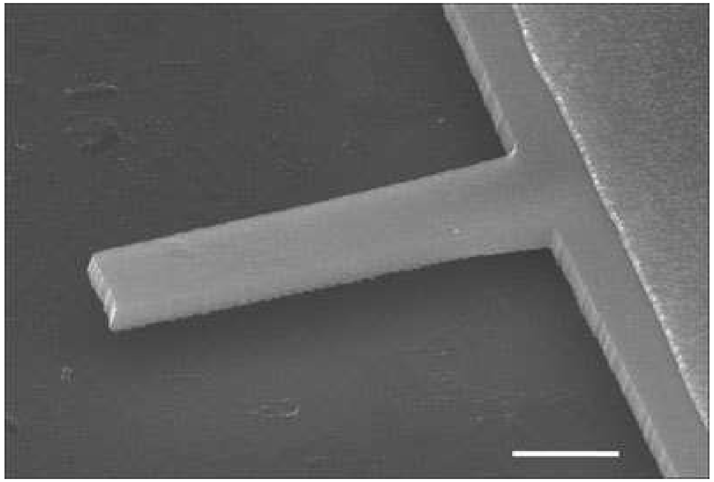

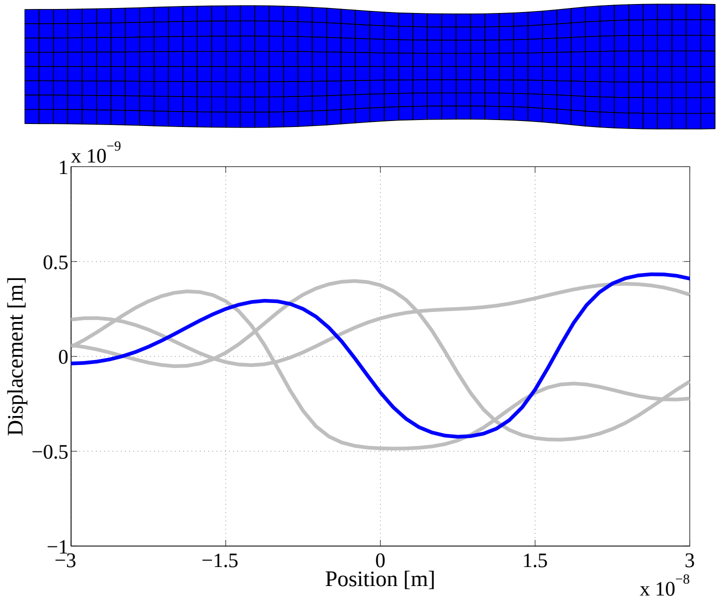

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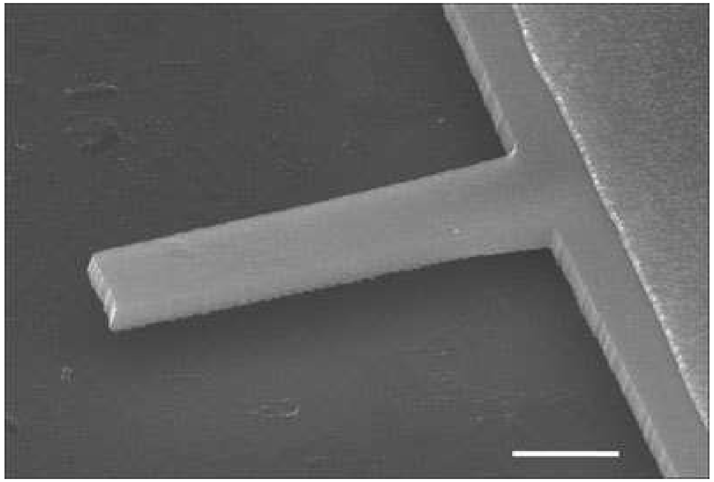

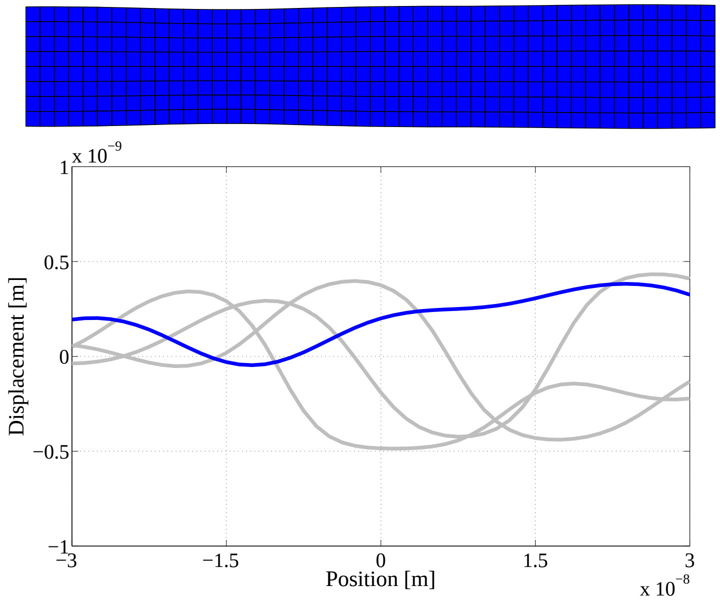

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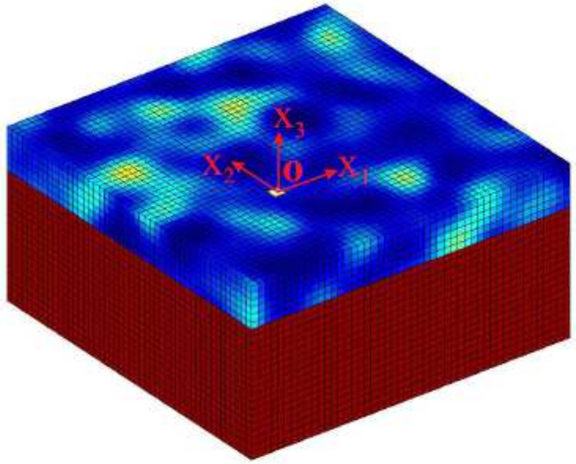
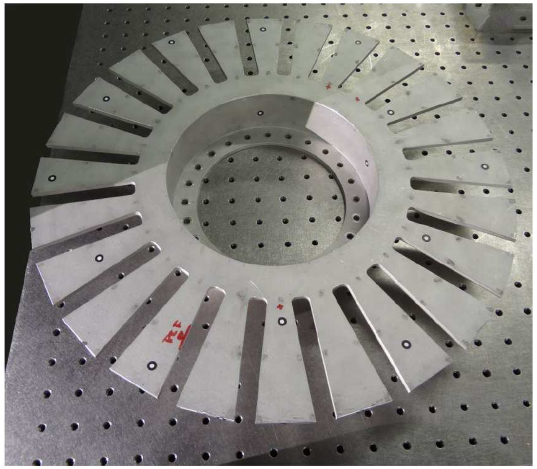
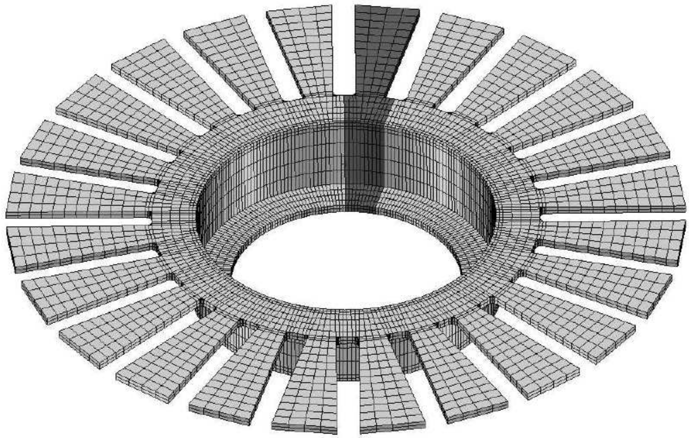


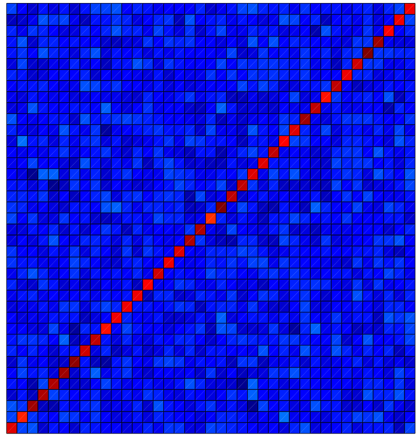
Random fields.

From: M. Arnst. Inversion of probabilistic models of structures using measured transfer functions. Thèse de Doctorat, Ecole Centrale Paris, France, 2007.

Characterization of uncertainties (continued)





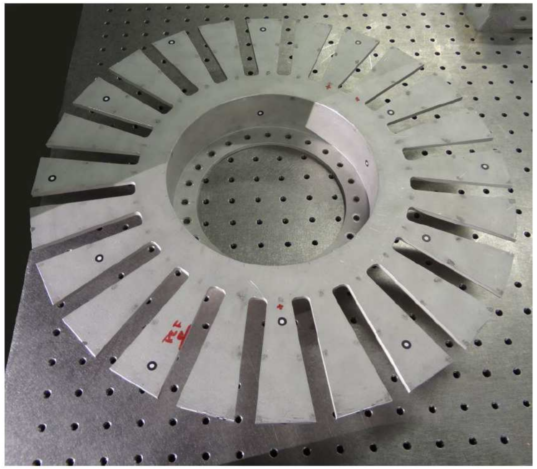


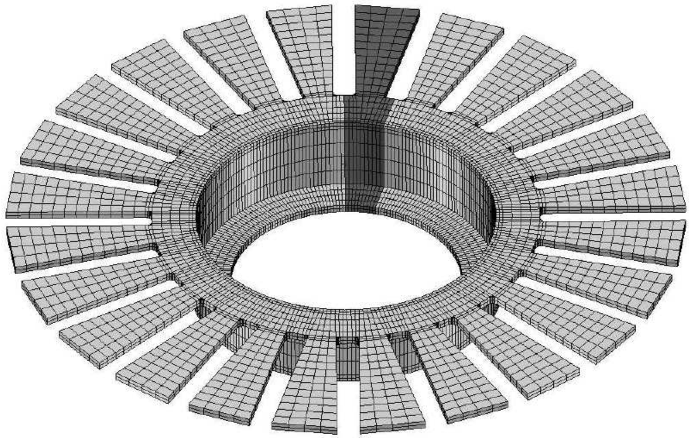
$$[K + i\omega D - \omega^2 M]\boldsymbol{u}(\omega) = \boldsymbol{f}(\omega).$$

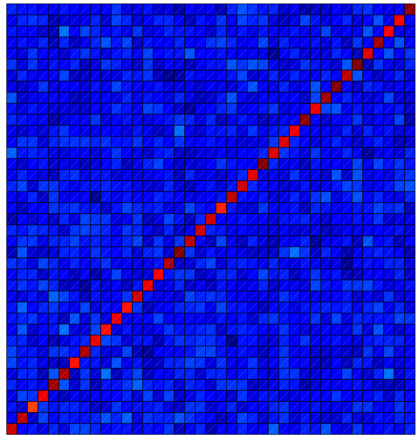
Random matrices.

From: F. Nyssen, M. Arnst, and J.-C. Golinval. Experimental modal identification of mistuning in an academic bladed disk and comparison with the blades geometry variations. ASME Turbo Expo, 2015.

Characterization of uncertainties (continued)







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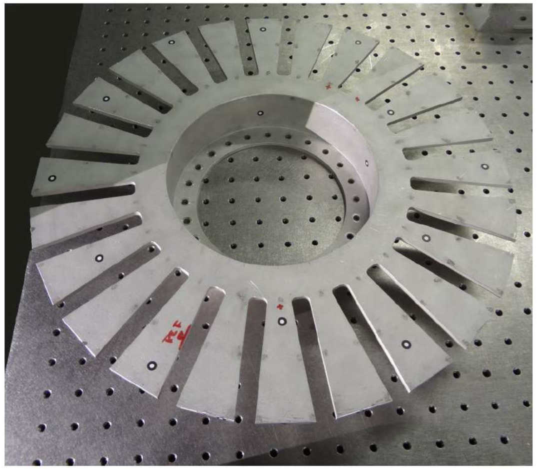

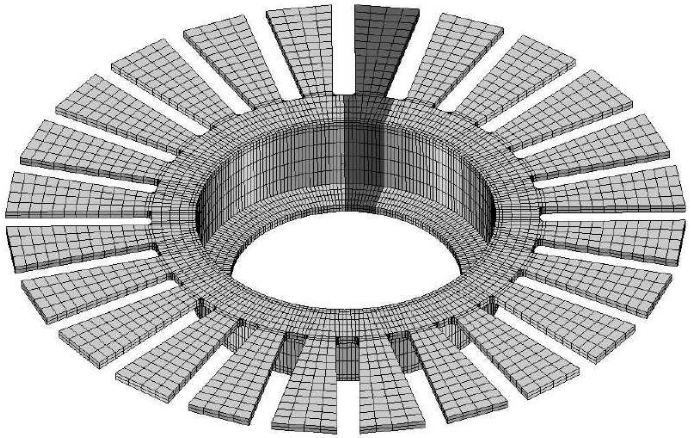

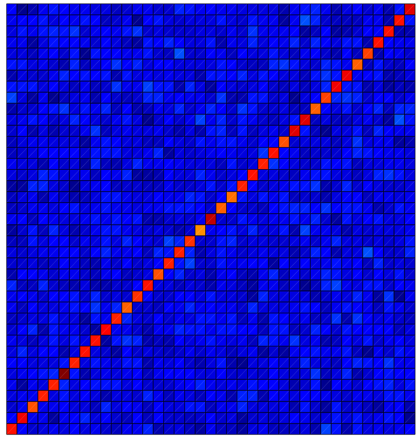

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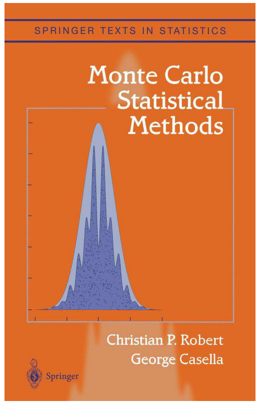
Propagation of uncertainties

- The next step is to propagate the uncertainties introduced in the input variables through the model to the output variable to quantify the impact of these uncertainties on this output variable.
- We apply the Monte Carlo method:



  - First, the Monte Carlo method involves generating an ensemble of i.i.d. samples with probability distribution $\rho_{\boldsymbol{X}}$:
$$\{\boldsymbol{x}_\ell,\ 1 \leq \ell \leq \nu\}.$$
  - Then, the computational model is used to map each sample of $\boldsymbol{X}$ into a sample of Y, that is,
$$y_\ell = g(\boldsymbol{x}_\ell).$$
to obtain the corresponding ensemble of i.i.d. samples of Y, written as follows:
$$\{y_\ell,\ 1 \leq \ell \leq \nu\}.$$
  - Finally, the second-order statistical descriptors of Y (if they exist) are approximated as
$$\overline{y} \approx \overline{y}^\nu = \frac{1}{\nu}\sum_{\ell=1}^{\nu} y_\ell \quad \text{and} \quad \sigma_Y^2 \approx (\sigma_Y^\nu)^2 = \frac{1}{\nu}\sum_{\ell=1}^{\nu}(y_\ell - \overline{y}^\nu)^2.$$
  - This can be extended to approximating the PDF (if it exists), quantiles, ... of Y.

Propagation of uncertainties (continued)

- The Monte Carlo method has the following advantages:
  - it is **nonintrusive**, that is, it requires only the repeated solution of the computational model for different values assigned to its input variables; the computational model need not be modified.
  - it is adapted to **parallel** computation.
  - **convergence** can be monitored during the computation.
  - the **rate of convergence** is independent of the number of input variables.

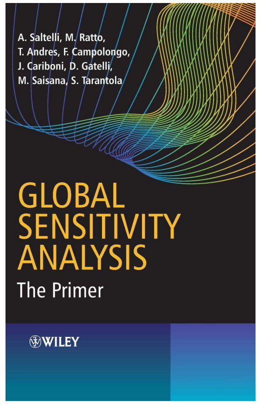
- Note that the Monte Carlo method can be improved using
  - advanced simulation procedures,
  - importance sampling,
  - multilevel approaches,
  - ...

- To gain efficiency, g can also be replaced by a surrogate model in the Monte Carlo method. This is the principle of stochastic expansion methods.

Sensitivity analysis of uncertainties

- The objective of the next step is to gain useful insight into how uncertainty in the input variables induces uncertainty in the output variable.

- There exist several types of sensitivity analysis of uncertainties:
  - methods involving scatter plots,
  - regression, correlation, and elementary effect analysis,
  - variance-based sensitivity analysis,
  - differentiation-based sensitivity analysis,
  - ...



- Variance-based sensitivity analysis leads to an “uncertainty budget:”

$$\underbrace{\sigma_Y^2}_{\substack{\text{variance}\\ \text{of output variable } Y}} = \underbrace{s_{X_1}}_{\substack{\text{contribution}\\ \text{from input variable } X_1}} + \ldots + \underbrace{s_{X_m}}_{\substack{\text{contribution}\\ \text{from input variable } X_m}} + \underbrace{\text{remainder}}_{\substack{\text{contribution}\\ \text{from interaction of } X_1,\ldots,X_m}} .$$

Example: Metal forming

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Engineering problem

Raw materials variability:
- Material properties.

. . .

Process variability:
- Blank holder force.
- Initial dimensions.
- Friction.

. . .

Modeling limitations:
- Constitutive model.
- FE discretization.

. . .

Input variables.

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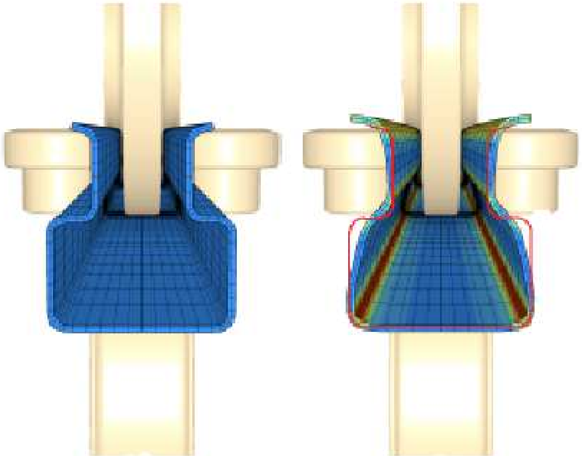

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Product variability:
- Final dimensions.
- Springback.

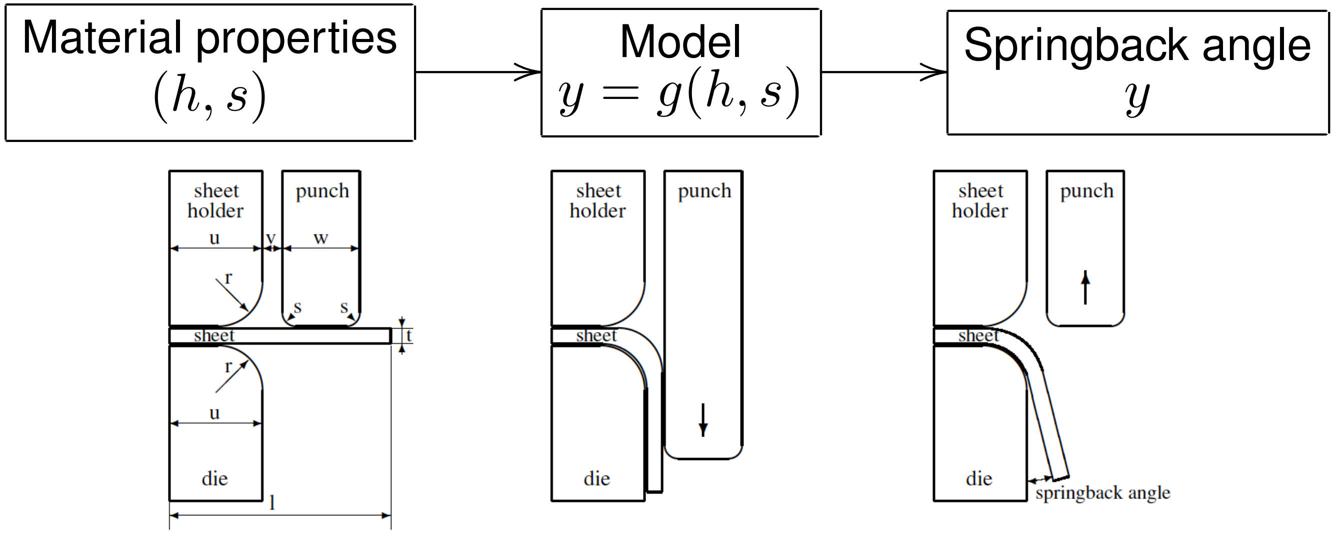
. . .

Prediction limitations:
- Numerical noise.

. . .

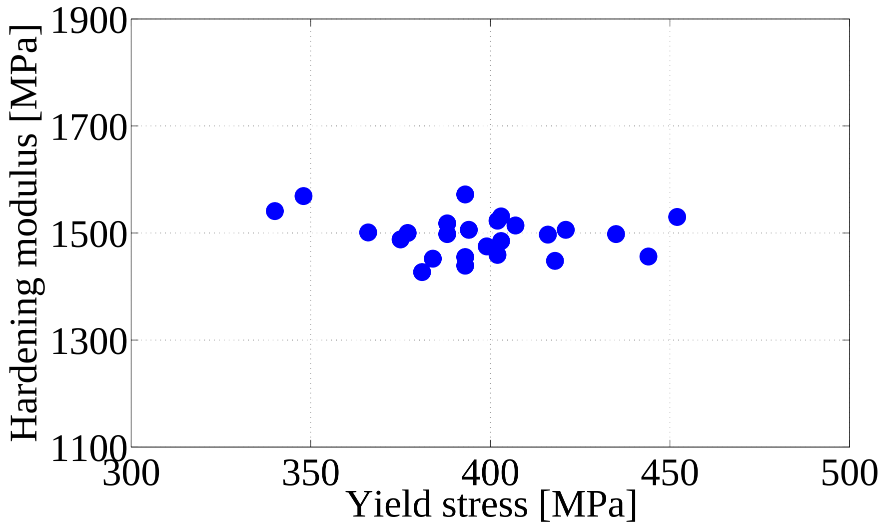
Output variables.

Engineering problem (continued)

- 

- Observed samples $(h_1^{\text{obs}}, s_1^{\text{obs}}), (h_2^{\text{obs}}, s_2^{\text{obs}}), \ldots, (h_n^{\text{obs}}, s_n^{\text{obs}})$.

h [MPa]	s [MPa]
1488	375
1485	403
1514	407
1500	377
…	…

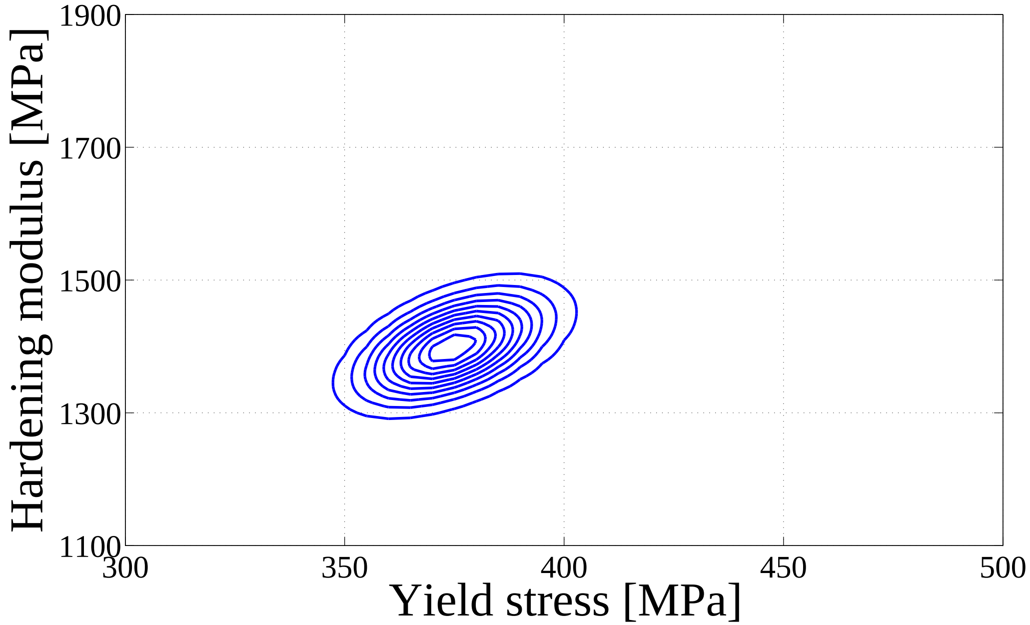


- Mechanics and physics impose that h and s be positive.

Characterization of uncertainties

- We select the bivariate gamma probability distribution

$$\rho_{(H,S)}(h,s;\underbrace{\overline{h},\sigma_H^2,\overline{s},\sigma_S^2,\rho}_{\text{parameters of the PDF}}) = \underbrace{\rho_\Gamma(h;\overline{h},\sigma_H^2)}_{\text{gamma marginal}}\underbrace{\rho_\Gamma(s;\overline{s},\sigma_S^2)}_{\text{gamma marginal}}\underbrace{\sigma\big(c_\Gamma(h;\overline{h},\sigma_H^2)c_\Gamma(s;\overline{s},\sigma_S^2);\rho\big)}_{\text{Gaussian copula}}.$$



- This probability distribution assigns vanishing probability to negative values of h and s.

Example: Metal forming

Characterization of uncertainties

- We select the bivariate gamma probability distribution

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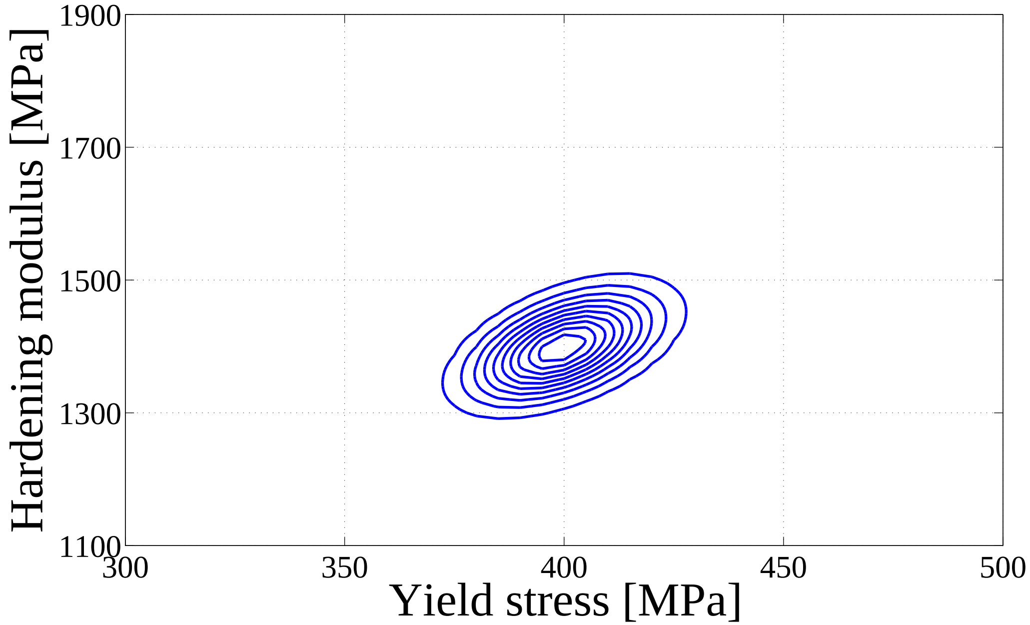

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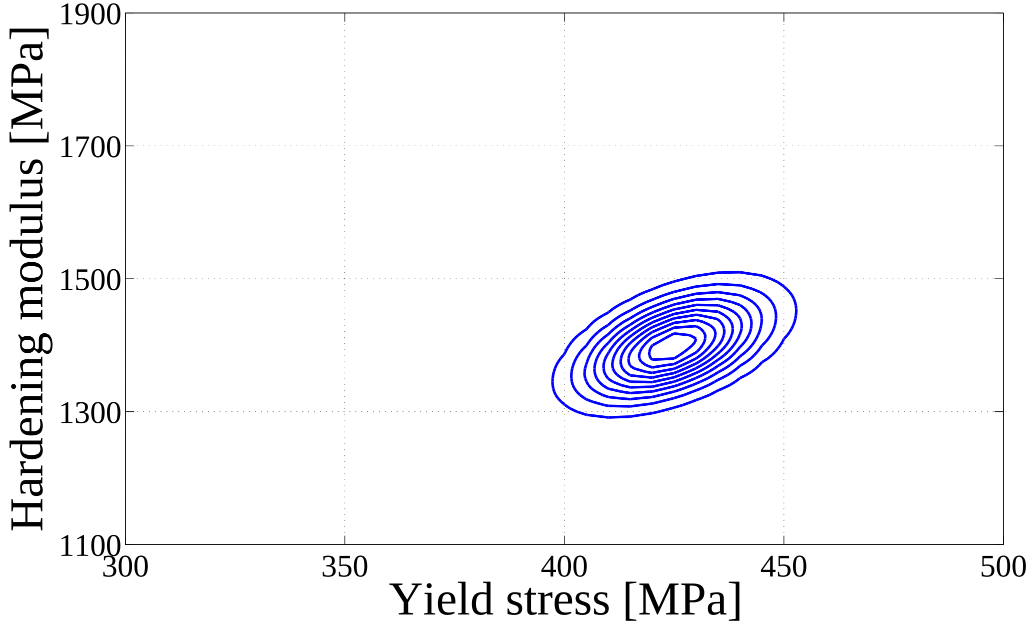

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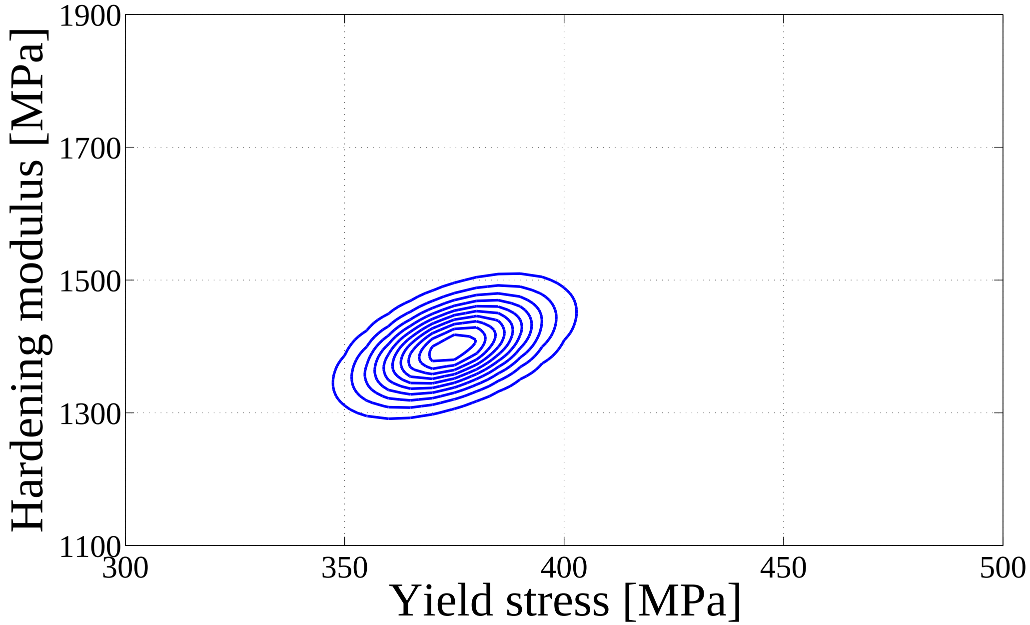

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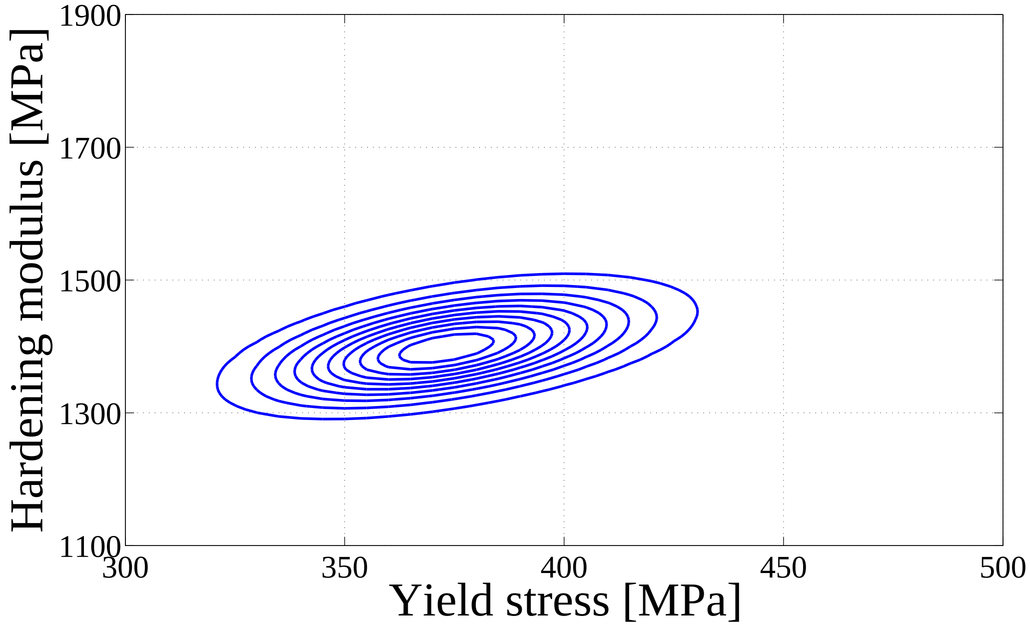

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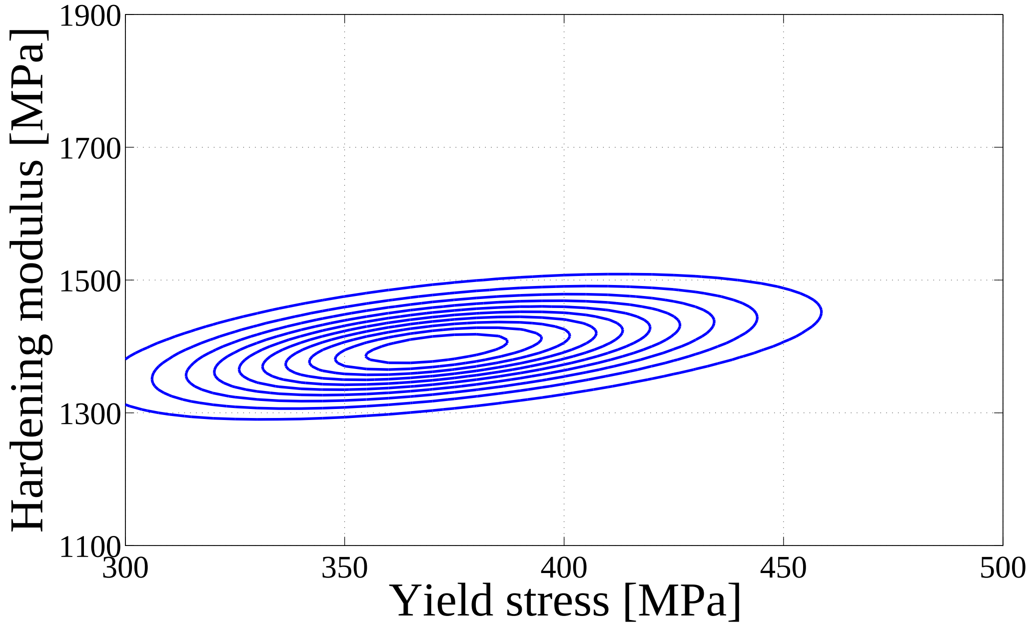

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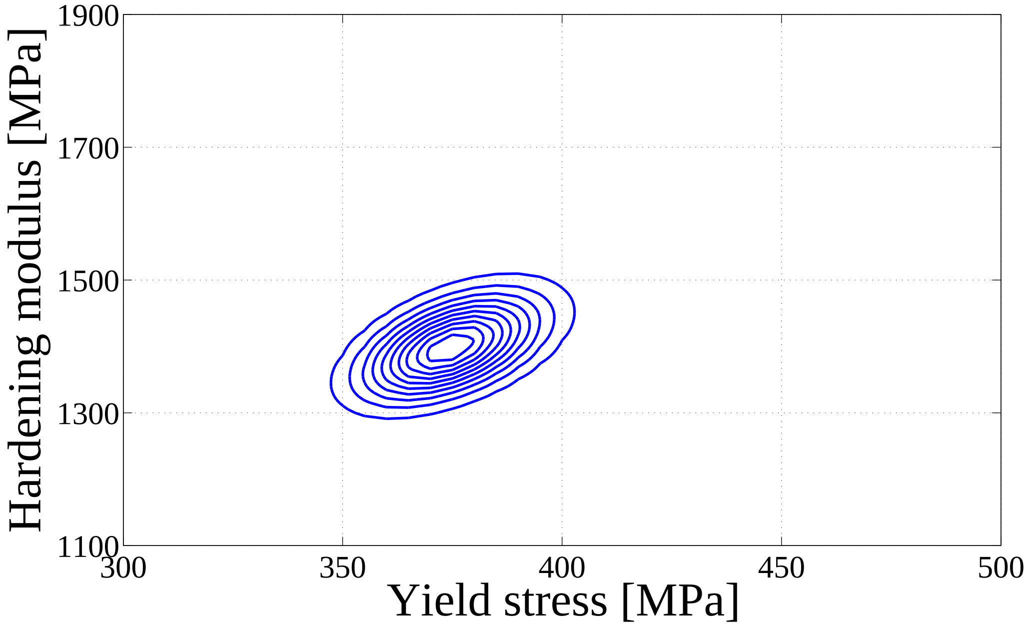

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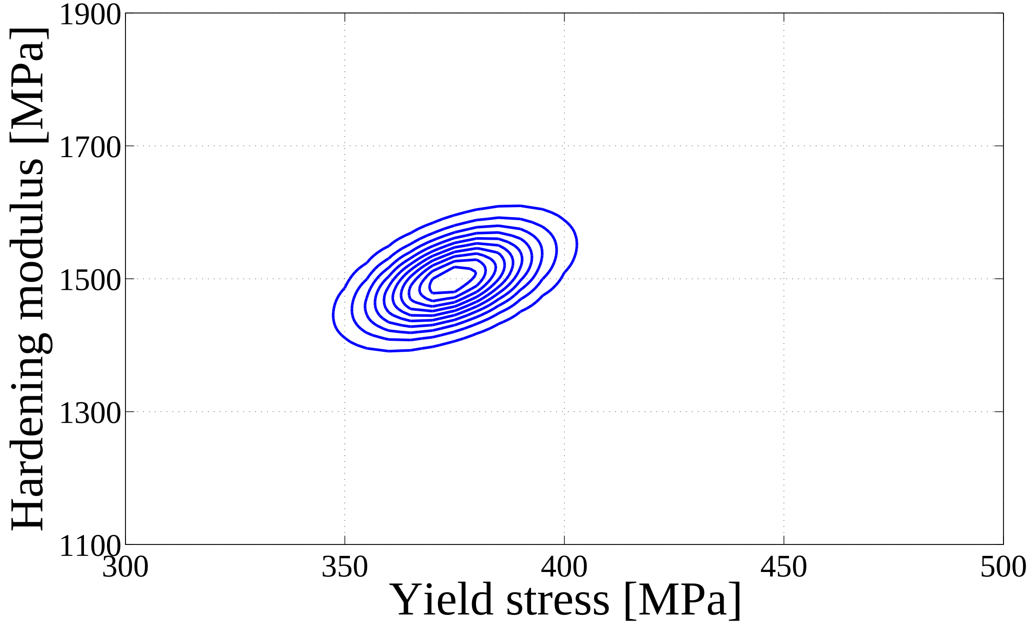

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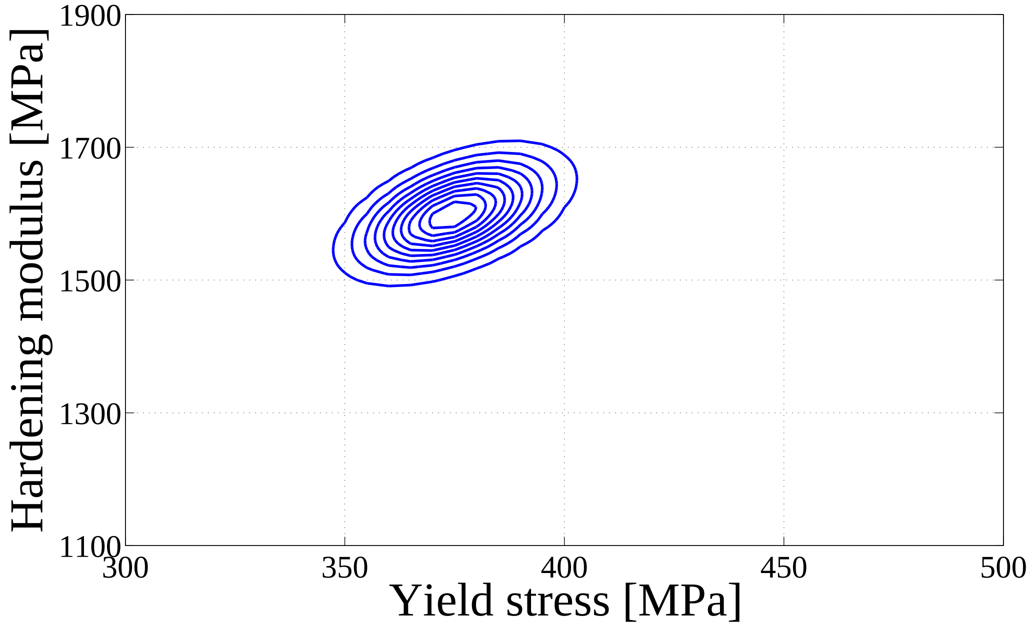

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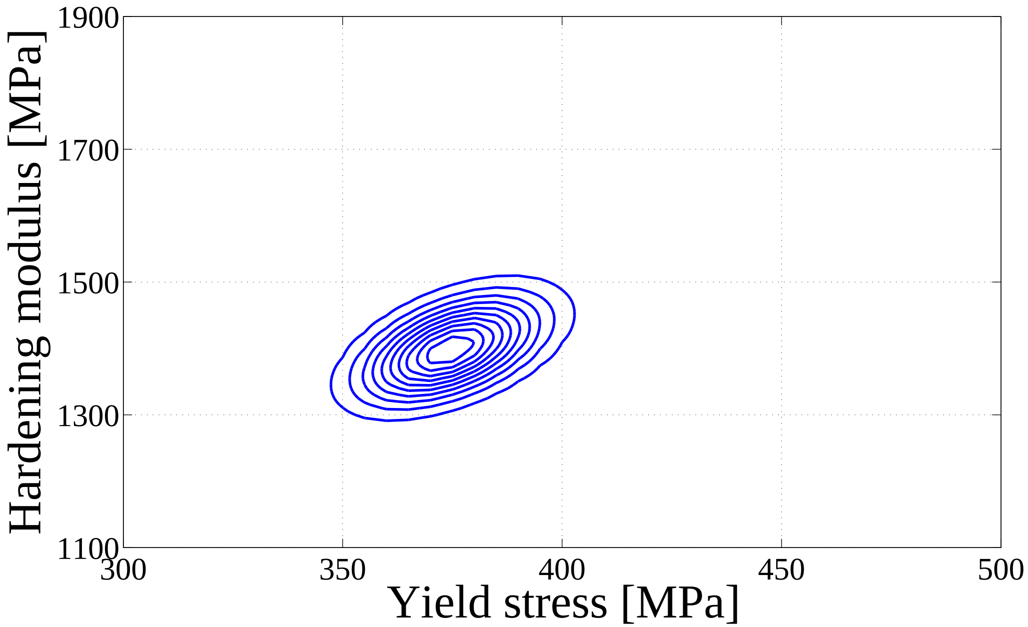

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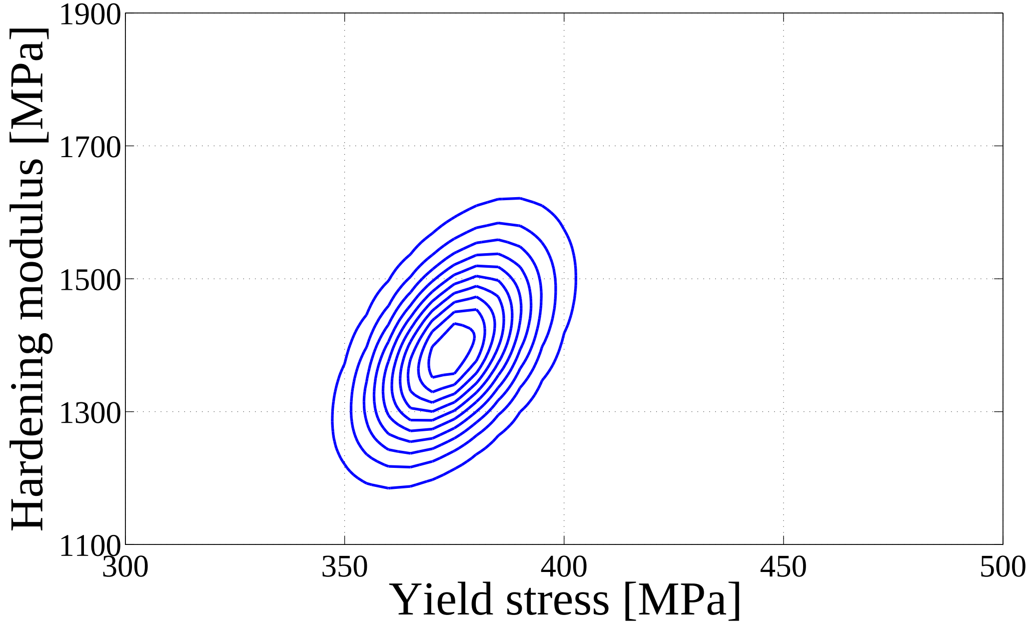

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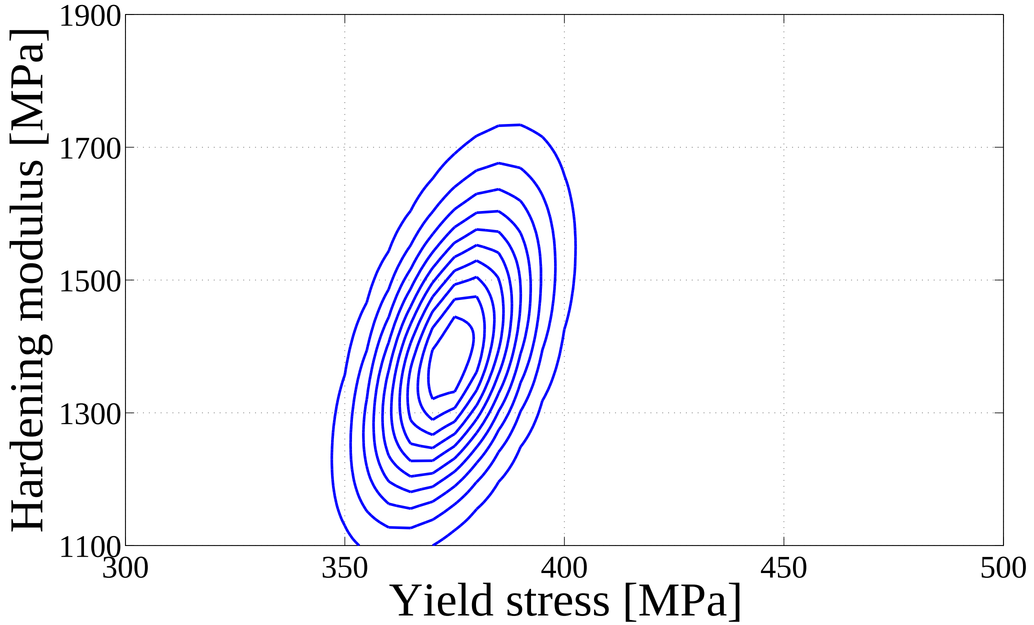

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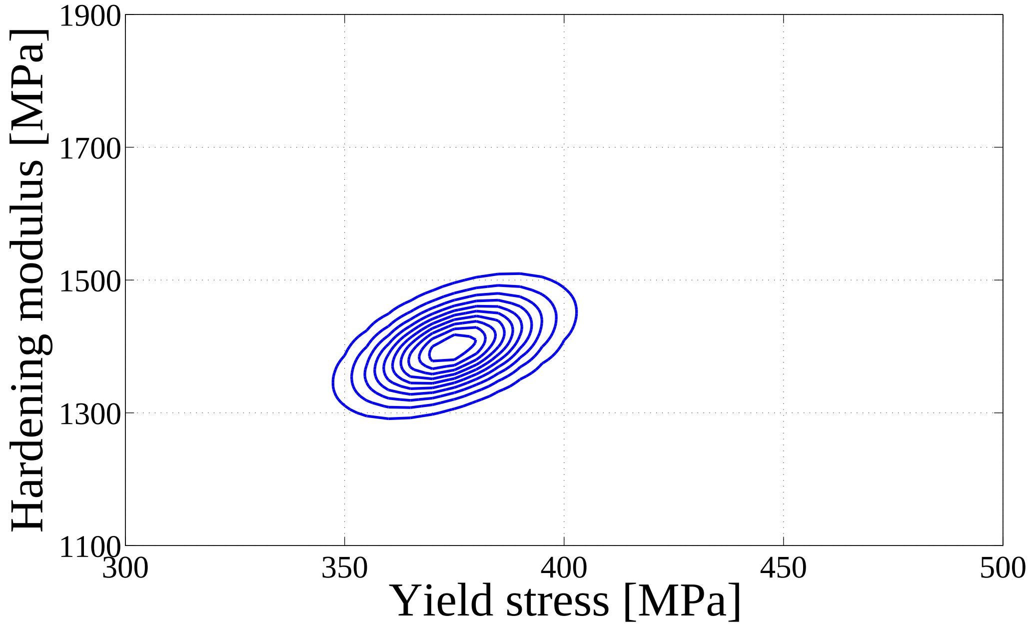

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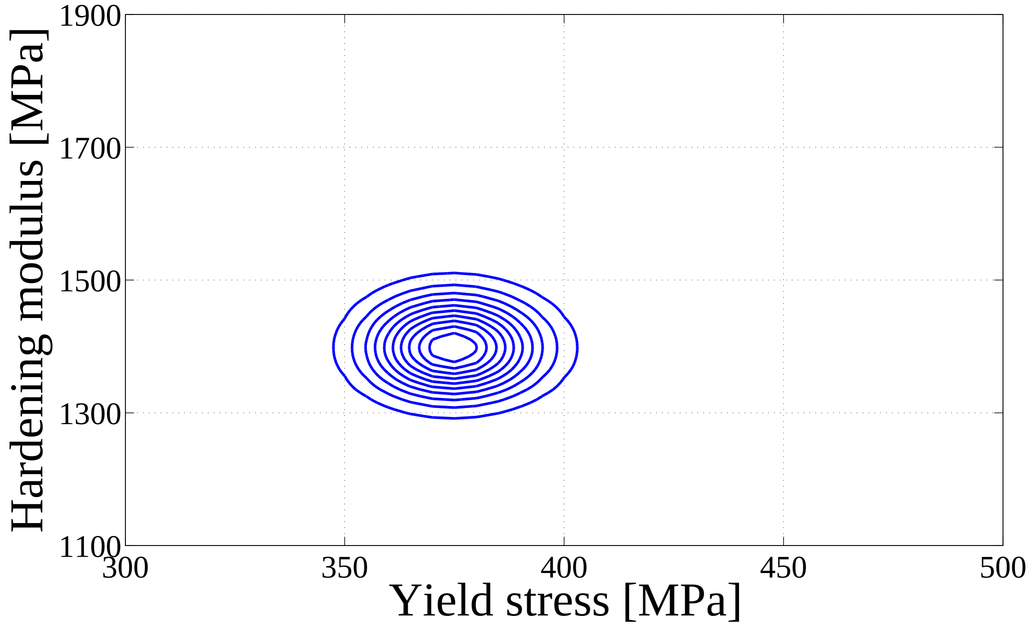

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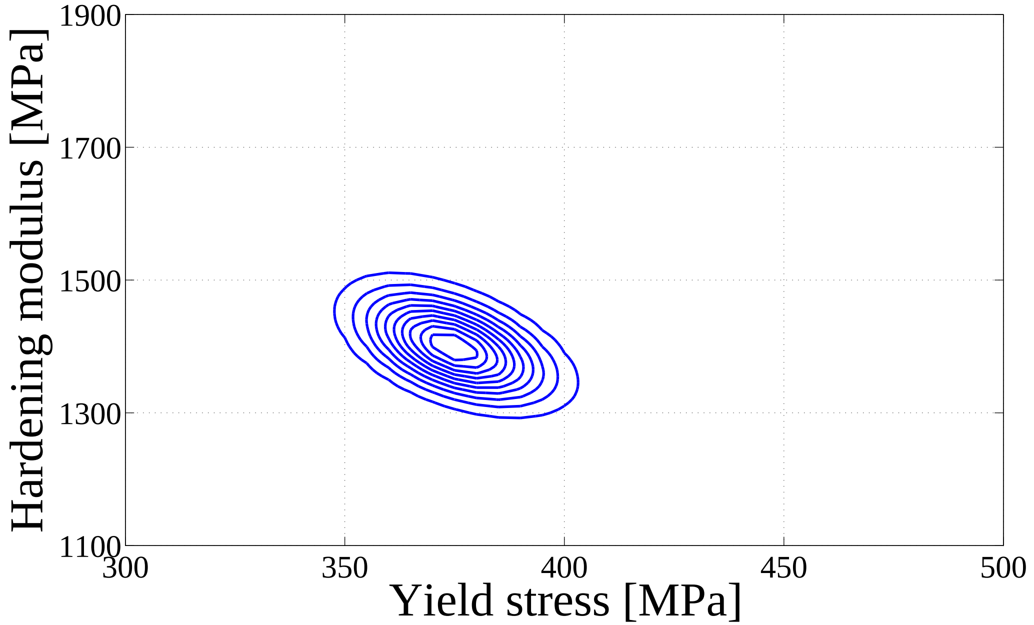

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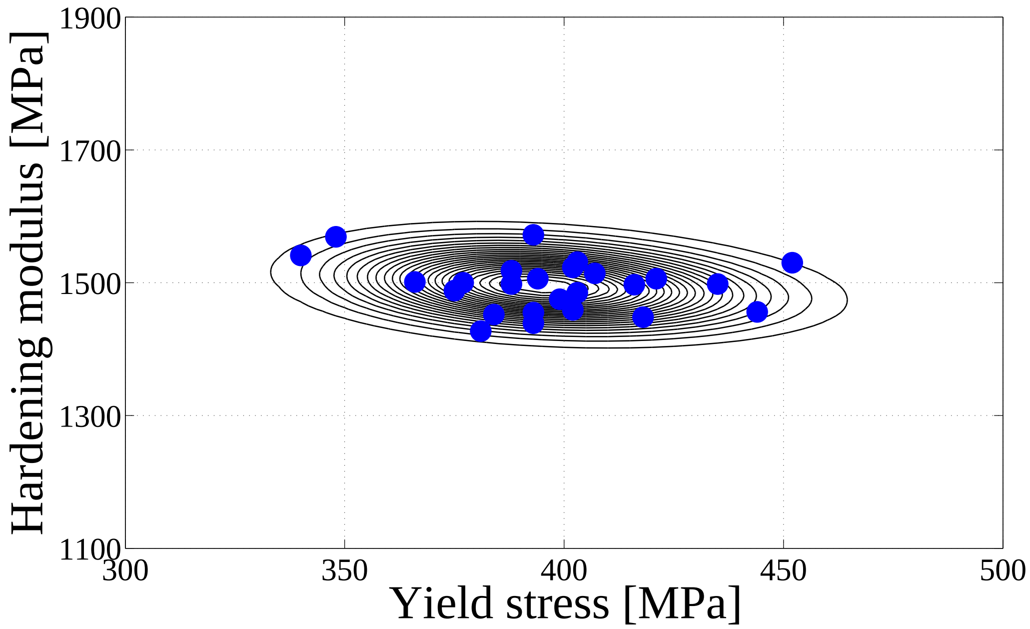
Characterization of uncertainties (continued)

- We estimate adequate values for the parameters of the bivariate gamma probability distribution by using the method of maximum likelihood as follows:

$$(\hat{\overline{h}}, \hat{\sigma}_H^2, \hat{\overline{s}}, \hat{\sigma}_S^2, \hat{\rho}) = \text{solution of} \max_{(\overline{h},\sigma_H^2,\overline{s},\sigma_S^2,\rho)} l(\overline{h}, \sigma_H^2, \overline{s}, \sigma_S^2, \rho),$$

where the likelihood of the parameters $\overline{h}$, σ_H^2, $\overline{s}$, σ_S^2, and ρ is given by

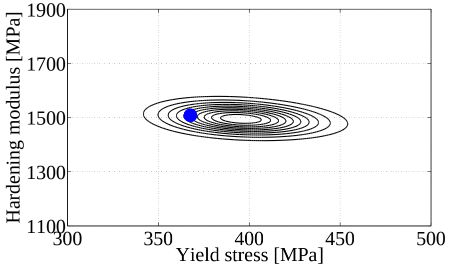
$$l(\overline{h}, \sigma_H^2, \overline{s}, \sigma_S^2, \rho) = \prod_{\ell=1}^{n} \rho_{(H,S)}(h_\ell^{\text{obs}}, s_\ell^{\text{obs}}; \overline{h}, \sigma_H^2, \overline{s}, \sigma_S^2, \rho).$$



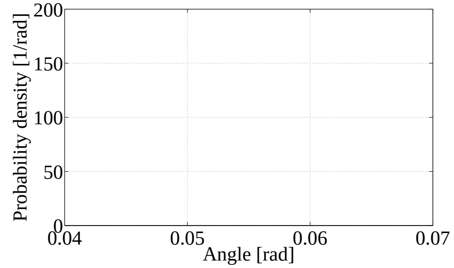
Example: Metal forming

Propagation of uncertainties

- **Monte Carlo method**:



→ Metafor nonlinear Finite Element Software LTAS-MN²L →



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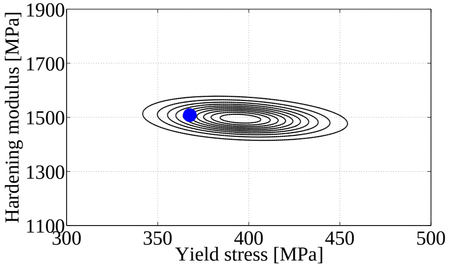

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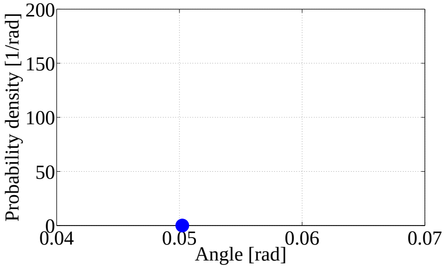

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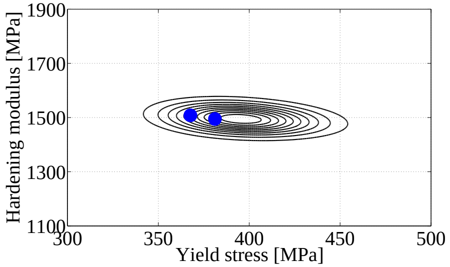

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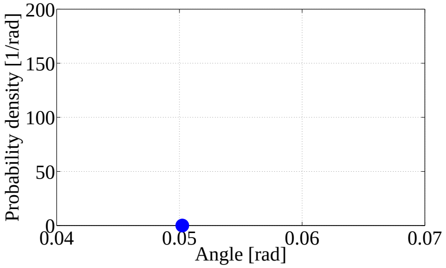

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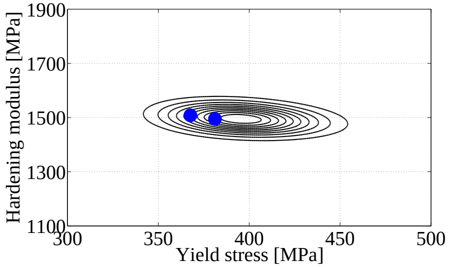

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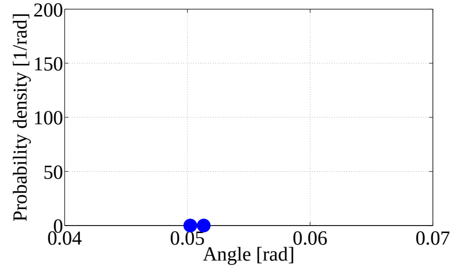

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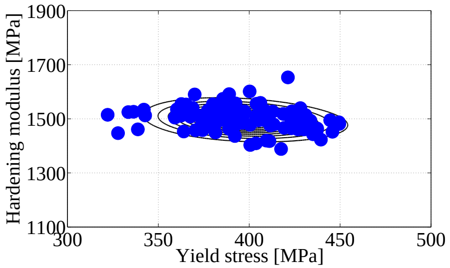

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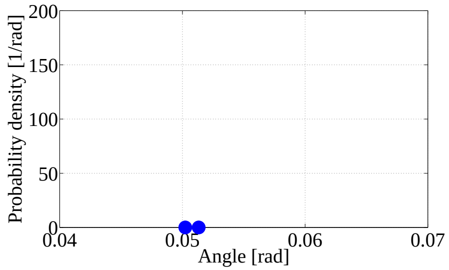

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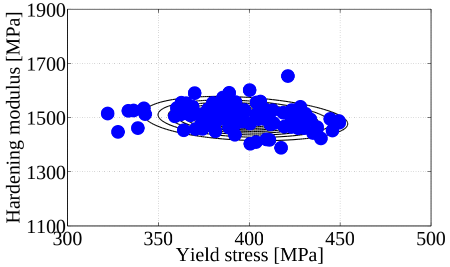

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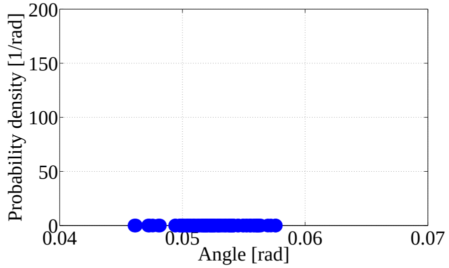

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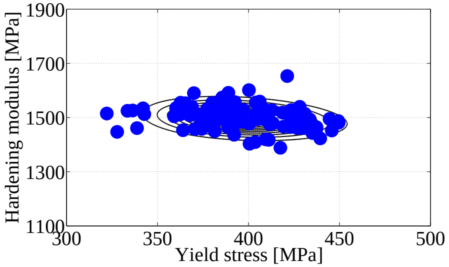

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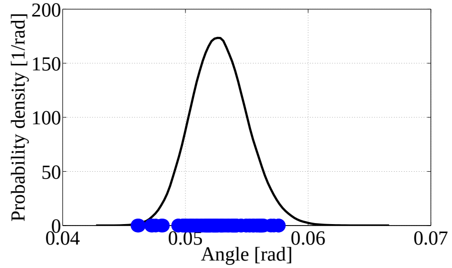

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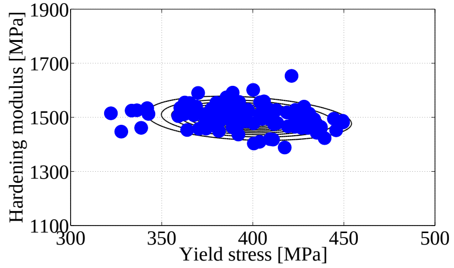

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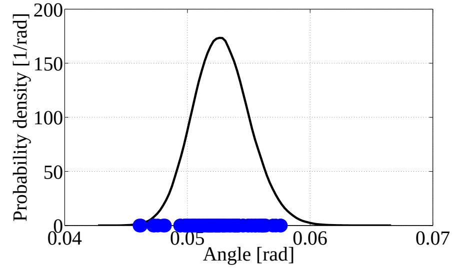

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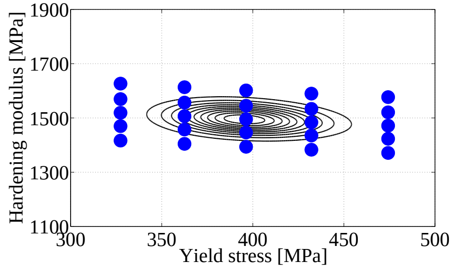

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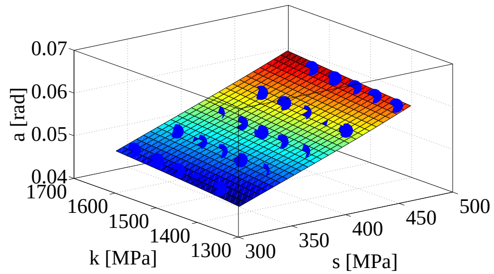

Computationally inexpensive surrogate model

Propagation of uncertainties

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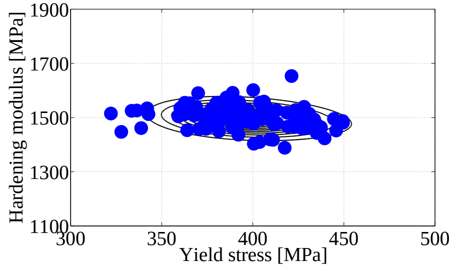

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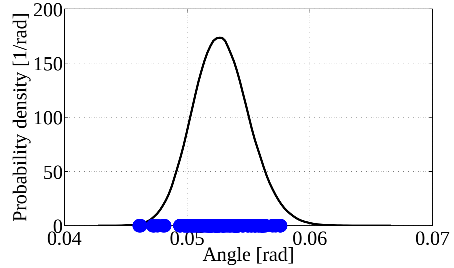

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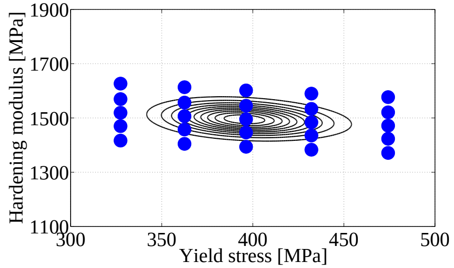

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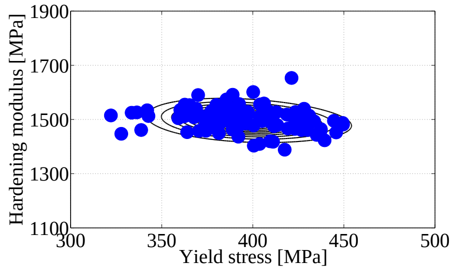

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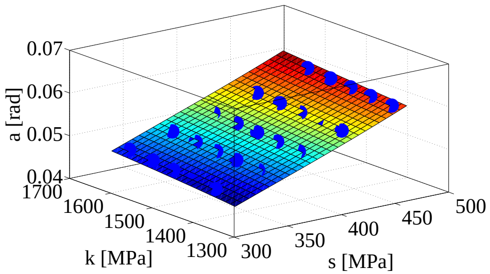

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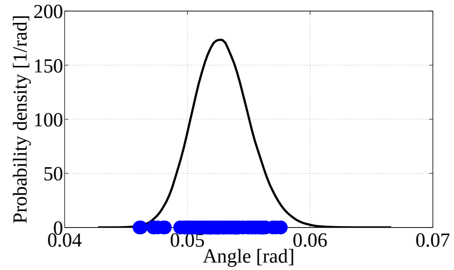
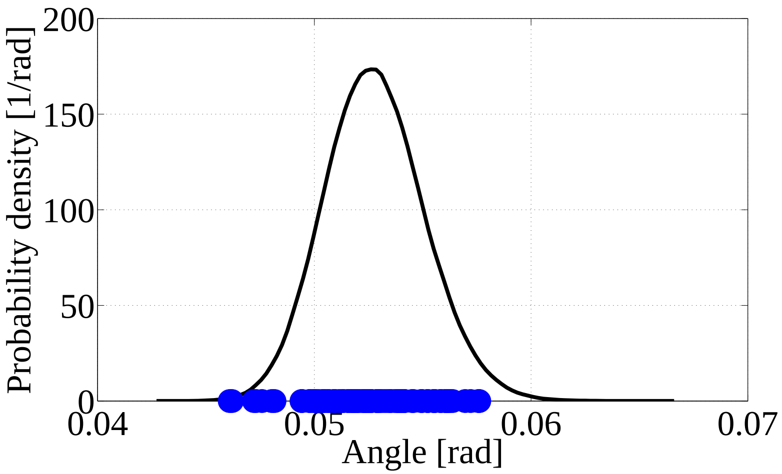


Computationally inexpensive surrogate model

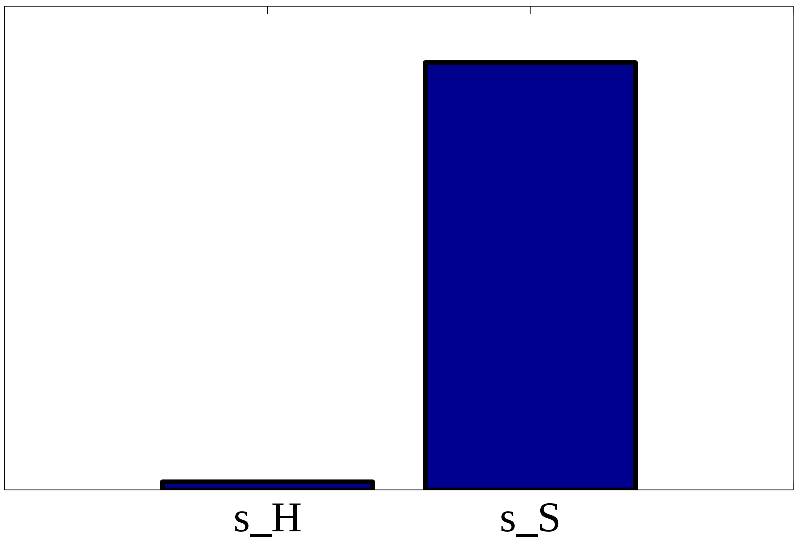
Sensitivity analysis of uncertainties



• solid: PDF of output.

$[0.046\,\text{rad},\ 0.059\,\text{rad}]$.
95%-confidence interval.

Propagation.



s_H, s_S represent significance of inputs in inducing uncertainties in output.

Sensitivity analysis.

Conclusion and outlook

- Context and current practice are rich but entail limitations.

- New methods are emerging for characterizing, propagating, and analyzing uncertainty:
  - characterization of uncertain geometries, uncertain fields, and uncertain matrices.
  - propagation of uncertainties by using Monte Carlo sampling.
  - sensitivity analysis to guide resource allocation towards reducing uncertainty, robust design, robust control,...

- These new methods are easily usable by engineers. Because they are nonintrusive, these new methods can be easily and effectively integrated with key tools (CAD, FEM,...) used in industry.

References

- The methods in this presentation are described in greater detail in the following paper:

International Journal for Uncertainty Quantification, 4 (5): 387–421 (2014)

AN OVERVIEW OF NONINTRUSIVE CHARACTERIZATION, PROPAGATION, AND SENSITIVITY ANALYSIS OF UNCERTAINTIES IN COMPUTATIONAL MECHANICS

Maarten Arnst & Jean-Philippe Ponthot*

Université de Liège, Département d'Aérospatiale et Mécanique, B-4000 Liège, Belgium

- Recent books on uncertainty quantification:
  - R. Ghanem and P. Spanos. Stochastic finite elements: a spectral approach. Dover, 2003.
  - O. Le Maître and O. Knio. Spectral methods for uncertainty quantification: with application to computational fluid dynamics. Springer, 2010.
  - D. Xiu. Numerical methods for stochastic computations: a spectral method approach. Princeton University Press, 2010.
  - M. Grigoriu. Stochastic systems: uncertainty quantification and propagation. Springer, 2012.
  - R. Smith. Uncertainty Quantification: Theory, Implementation, and Applications. SIAM, 2013.
  - C. Soize. Stochastic models of uncertainties in computational mechanics. ASCE, 2014.

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