# The Nonlinear Tuned Vibration Absorber, Part II: Robustness and Sensitivity Analysis

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## Introduction

Nonlinear vibration absorbers, including the autoparametric vibration absorber [1], the nonlinear energy sink (NES) [2] and other variants [3, 4], can absorb disturbances in wide ranges of frequencies due to their increased bandwidth. However, the performance of existing nonlinear vibration absorbers is known to exhibit marked sensitivity to motion amplitudes. For instance, there exists a well-defined threshold of input energy below which no significant energy dissipation can be induced in an NES [2]. In the companion of the present paper [5], a Nonlinear Tuned Vibration Absorber (NLTVA) is introduced to address these problems by extending Den Hartog's linear tuning rule [6] to the nonlinear domain.

While excellent performance of the NLTVA is observed at low and moderate energy levels, some peculiar phenomena can arise at higher energies, such as the apparition of detached resonance curves (DRCs), and quasiperiodic solutions; this is the focus of the present study.

In order to identify the working range of the NLTVA in the presence of such adverse dynamics, the absorber's robustness has to be assessed. The analysis proposed in this study builds upon numerical methods such as the continuation of codimension-1 bifurcations in parameter space, and the identification of the basins of attraction of the periodic and quasiperiodic solutions. It is shown that some variations of the absorber parameters can improve its robustness without deteriorating the performance significantly.

## Computation of the adverse dynamics of the NLTVA

The dynamics of a Duffing oscillator with an attached NLTVA is considered:

$$\begin{aligned} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + k_{nl1}x_1^3 + c_2(\dot{x}_1 - \dot{x}_2) + g(x_1 - x_2) &= F\cos\omega t \\ m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) - g(x_1 - x_2) &= 0 \end{aligned} \qquad (1)$$

where $x_1(t)$ and $x_2(t)$ are the displacements of the primary system and of the NLTVA, respectively. The application of the tuning procedure proposed in [5] gives a restoring force for the NLTVA with both linear and cubic stiffness, i.e., $g(x_1 - x_2) = k_2(x_1 - x_2) + k_{nl2}(x_1 - x_2)^3$.

The system described in (1) exhibits linear-like dynamics in an important range of forcing amplitudes but, for high values of $F$, some nonlinear phenomena can arise and possibly alter the performance of the NLTVA. As an illustration of the adverse dynamics, Figure 1(a) depicts the frequency response of the Duffing oscillator for a forcing amplitude $F = 0.15$ N. One first observes that the main frequency response verifies Den Hartog's criterion, with two resonance peaks of same amplitude, which validates the effectiveness of the absorber. In this case, however, one can also detect the presence of pairs of fold and Neimark-Sacker bifurcations, with stable quasiperiodic oscillations emanating from the latter through a combination resonance. This can lead to higher oscillation amplitudes than what was expected from the study of the main frequency response, but a careful investigation shows that they remain acceptable. For higher frequencies, one notices the

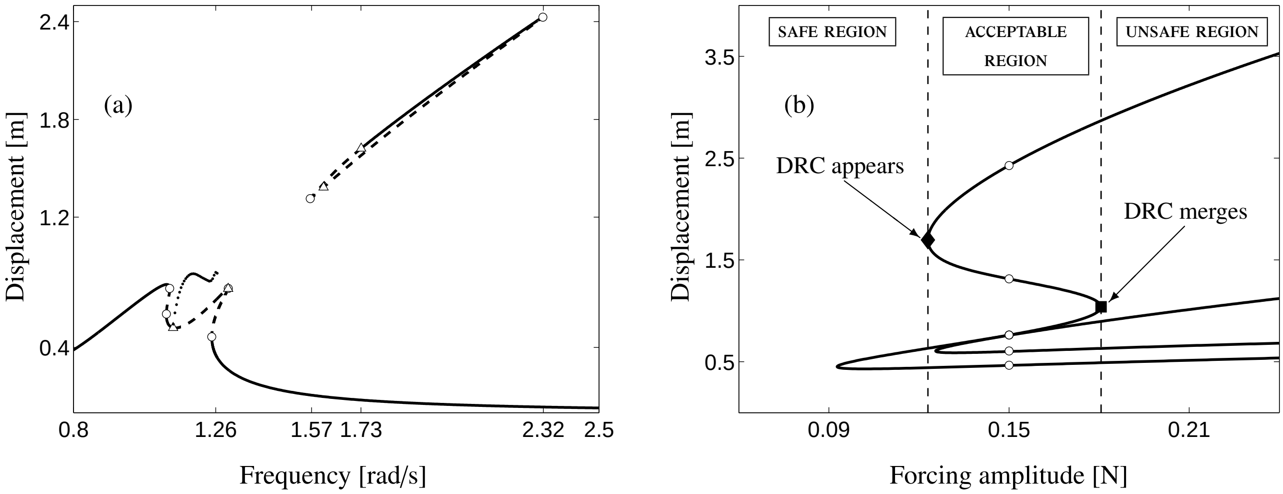


Figure 1: Adverse dynamics and robustness analysis of the NLTVA. (a) Frequency response of the Duffing oscillator with an attached NLTVA for $F = 0.15$ N. The solid and dashed lines represent stable and unstable solutions, respectively. The dots represent the amplitude of stable quasiperiodic oscillations. Fold and Neimark-Sacker bifurcations are depicted with circle and triangle markers, respectively. (b) Projection of the branches of fold bifurcations on the $F$-displacement plane. The circle markers highlight fold bifurcations at the forcing amplitude of interest.

presence of a DRC with stable parts coexisting with periodic solutions of lower amplitude.

In order to assess the robustness of the NLTVA with respect to these phenomena, one of the methods we propose consists in tracking fold bifurcations against the forcing parameters, $F$ and $\omega$. This is performed in Figure 1(b) which shows the projection of branches of fold bifurcations in the $F$-displacement plane. Interestingly, together with the bifurcations on the main frequency response, the branches indicate the presence of bifurcations on the DRC, which gives valuable information about this isolated solution. From the upper turning point, one can identify the forcing amplitude at which the DRC appears, one can then quantify its growth, and, from the lower turning point, one can eventually evaluate a forcing amplitude at which it merges with the main frequency response. This forms the basis of the concept of safe, acceptable and unsafe regions, for the performance of the NLTVA (see Figure 1(b)). The present work is based on the study of the evolution of the regions for variations of the NLTVA parameters, and shows the compromises that can be made to enlarge the working range of the NLTVA and enhance its practical applicability.


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