V. Denoël
E. Detournay

Recent Advances in the Modeling of a Drillstring Inside a Curved Borehole

Vincent Denoël[a], Emmanuel Detournay[b]

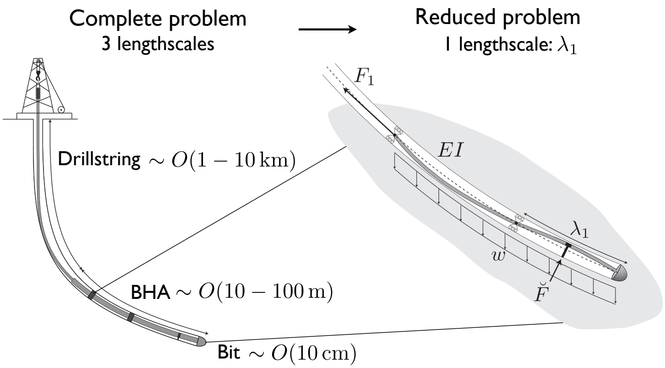
[a] University of Liège, Belgium
[b] University of Minnesota, USA; Commonwealth Scientic and Industrial Research Organization, Australia

Non-linear dynamics and control of deep drilling systems
Eindhoven May 15-16 2012

Context

From Luc Perneder's talk, Tuesday 15 May 2012, Eindhoven:

Directional Drilling Model



Thursday, May 10, 12

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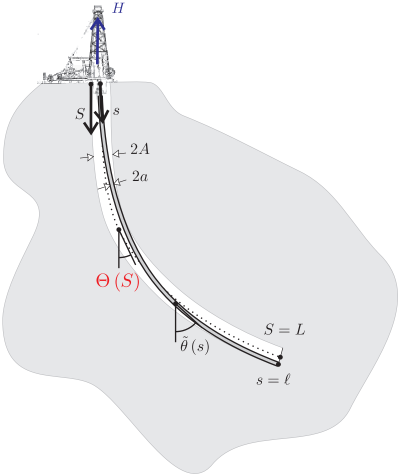
The Model

The Problem



- Given borehole geometry ($\Theta(S)$, L)
- Rigid boundaries
- Given drillpipe geometry EI, a
- Given end conditions
- Unknown length of drillpipe (ℓ ?)

→ Consider insertion of drillpipe

Deformed configuration, Axial, Bending and Shear Stresses in the drillstring ?

Number, position & type of contacts ?

Drillpipe-Borehole Interactions

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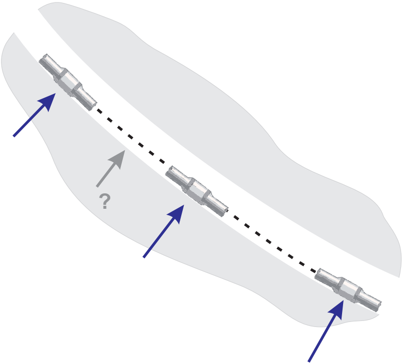
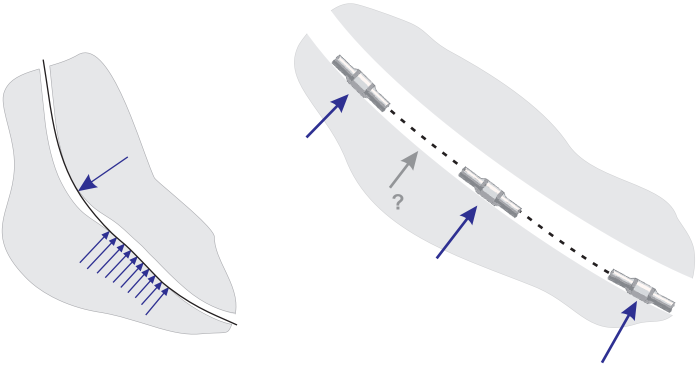


Contact at pipe connections ?

Drillpipe-Borehole Interactions



→ discrete and continuous contacts (reduce # of unkn. - simplicity/complexity)

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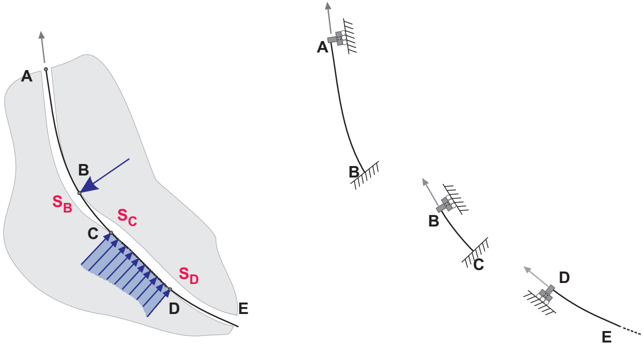
Segmentation

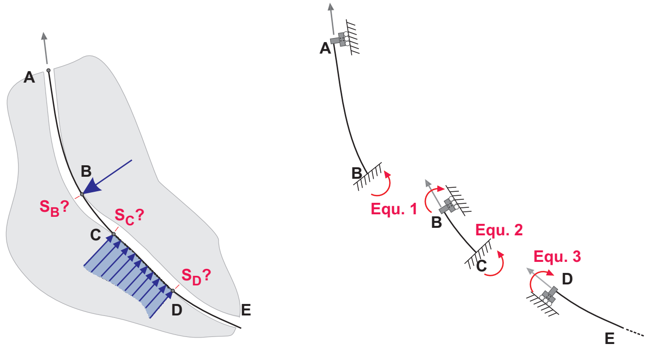
Imagine that the contact pattern and the contact positions are known...



(i) simple solutions between contacts (limit behaviours)
(ii) trivial solutions where contact is continuous

Segmentation

Imagine that the contact pattern is known; now tune the contact positions ...



→ set of nonlinear equations (solved with a quasi-Newton method)
NB: contacts positions do not move significantly

Pattern Switcher

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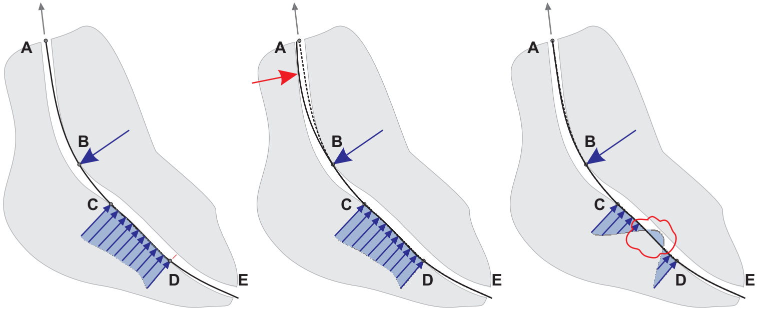
Finally make sure the contact pattern is admissible...



Architecture of the Model

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1. Consider a contact pattern
2. Guess the position or extent of contacts

Establish the system of governing equations

Estimation of contact positions

Transmission of axial force from top to bottom

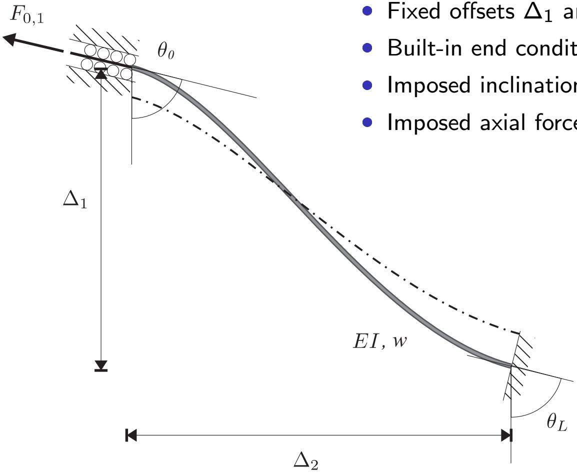
Solution of Auxiliary problem for each segment
Compute unbalanced bending moments
(Compute shear forces bending moments)

Loop until satisfaction of reconnetion equations

Evaluate validity of contact pattern (> Switcher)

(i) limited number of unknowns
(ii) use of efficient numerical techniques (simple & robust)
(iii) good initial estimates of solution (if insertion/incremental loading)
(!) number and kind of contacts a priori unknown & heavy checking of penetration (mode switching)

The Auxiliary Problem



- Given EI, w
- Fixed offsets Δ_1 and Δ_2
- Built-in end conditions
- Imposed inclination at top
- Imposed axial force at top $F_{0,1}$

The Auxiliary Problem: Governing Equations

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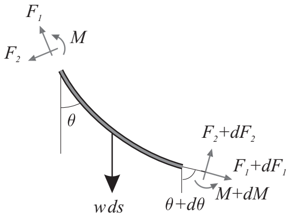
Equilibrium equations:

$$\begin{aligned} F_1\theta' + F_2' - w\sin\theta &= 0 \\ F_2\theta' - F_1' - w\cos\theta &= 0 \\ M' + F_2 &= 0 \end{aligned}$$

Constitutive equation:

$$M = EI\theta'$$



Classical linear beam theory $\rightarrow F_2' - w\sin\theta = 0$
Cable theory $(F_2 = 0) \rightarrow F_1\theta' - w\sin\theta = 0$

$\rightarrow$ Nonlinear beam theory with large rotations and large displacements

(elastica / rod / cosserat)

The Auxiliary Problem

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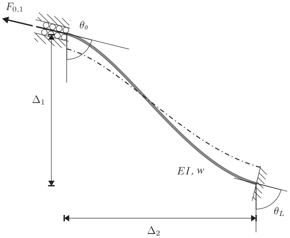
$$\left(\mu\ddot{\theta}+\right) EI\theta'' + F_{2,0}\cos\left(\theta-\theta_0\right) - F_{1,0}\sin\left(\theta-\theta_0\right) + w\, s\sin\theta = 0$$



with

$$\theta(0) = \theta_0 \quad ; \quad \theta(\ell) = \theta_L$$

and

$$\int_0^{\ell} \cos\theta(s)\, ds = \Delta_1 \quad ; \quad \int_0^{\ell} \sin\theta(s)\, ds = \Delta_2$$

as $F_{2,0}$ and ℓ are unknown

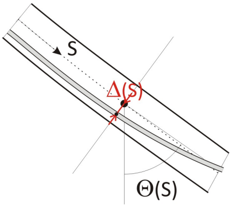
(Second order nonlinear ODE, with isoperimetric constraints) $\rightarrow$ Shooting (?)

nb: Non-penetration condition: compute
$x(s) = \int_0^s \cos\theta(\tilde{s})\, d\tilde{s}; \quad y(s) = \int_0^s \sin\theta(\tilde{s})\, d\tilde{s}$

The Auxiliary Problem: Eulerian Formulation

Motivations for an Eulerian Approach, i.e. borehole related

- Contacts do not move much along the conduit but well along the drillstring
- ℓ appears in domain of integration & restraining conditions ($\rightarrow$ shooting)
- Elastica solution is not always unique
- Ill-conditionness due to ℓ, more generally to $s \simeq S$, but $s \neq S$



Signed Distance Function $\Delta(S)$

Δ is a one-to-one function
$\rightarrow$ no more solutions with curl
$\rightarrow$ appropriate formulation for penetration detection

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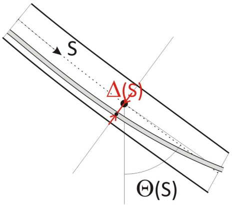

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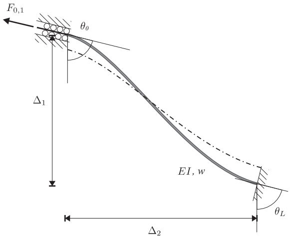
$$EI\,\theta''(s) + F_{2,0}\cos(\theta(s)-\theta_0) - F_{1,0}\sin(\theta(s)-\theta_0) + w\,s\sin\theta(s) = 0$$

$$\rightarrow \mathscr{F}\left[\Delta(S), \Delta'(S), \ldots, \Delta'''(S), \vartheta(S)\right] = 0$$



and the former BC (& isoperimetric)

$$\theta(0) = \Theta_p \quad ; \quad \theta(\ell) = \Theta_q$$

$$\int_0^\ell \cos\theta(s)\,ds = \Delta_1 \quad ; \quad \int_0^\ell \sin\theta(s)\,ds = \Delta_2$$

become

$$\Delta(0) = \Delta_0 \quad ; \quad \Delta(L) = \Delta_L$$

$$\Delta'(0) = 0 \quad ; \quad \Delta'(L) = 0$$

$\rightarrow$ definitely got rid of ℓ, classical BC for BVP (FD, FE, ...)

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Numerical Implementation

Description of Borehole

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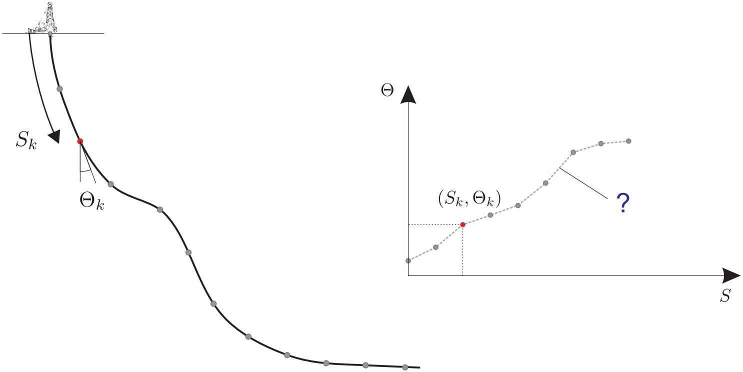
Most simple & realistic description in 2-D: (depth, inclination)



How to interpolate ?

Description of Borehole

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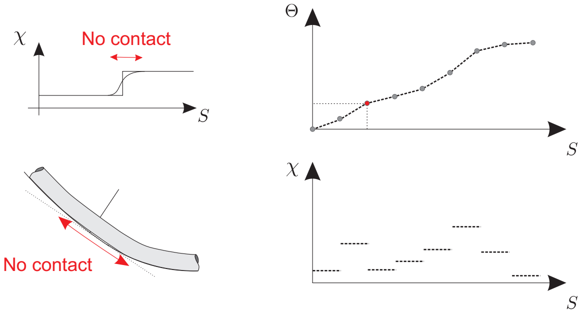

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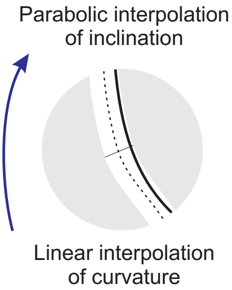
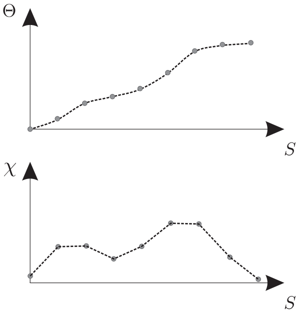
Description of Borehole





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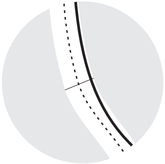
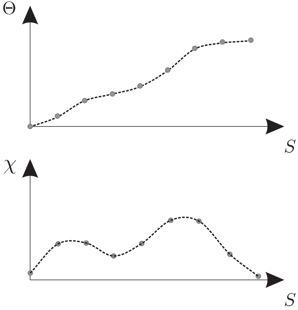
Description of Borehole

Spline interpolation with (S_k, Θ_k) and (S_k, χ_k)





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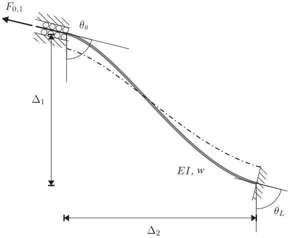
The Auxiliary Problem: Eulerian Formulation

$$\mathscr{F}\left[\Delta,\Delta',...,\Delta''',\Theta;F_{0,2}\right]=0\rightarrow\mathscr{G}\left[\Delta,...,\Delta'''',\Theta\right]=0$$



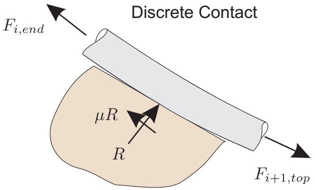
with 4 boundary conditions

$$\Delta(0)=\Delta_0 \quad ; \quad \Delta(L)=\Delta_L$$
$$\Delta'(0)=0 \quad ; \quad \Delta'(L)=0$$

Solve with a Bubnov-Galerkin approach with a limited number of elements ($N\simeq 8-10$)

$$\mathbf{K}(\boldsymbol{\Delta})\,\delta\boldsymbol{\Delta}=\mathbf{w}$$

Transmission of Axial Force through Contacts



$$F_{i+1,top} = F_{i,end} - \mu R$$

Transmission of Axial Force through Contacts

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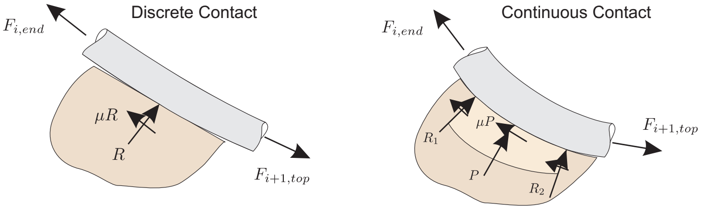

Discrete Contact:

$$F_{i+1,top} = F_{i,end} - \mu R$$

Continuous Contact:

$$F_1^{'} - \mu \Theta' F_1 + EI\left(\Theta'' + \mu \Theta'''\right) + w\left(\cos\Theta + \mu \sin\Theta\right) = 0$$

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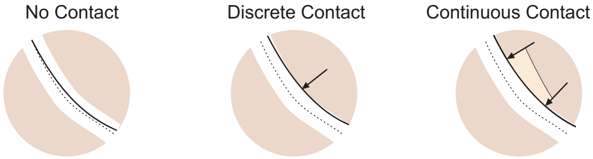
Pattern Switcher



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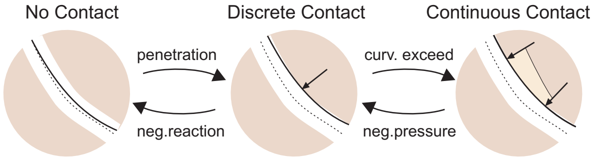

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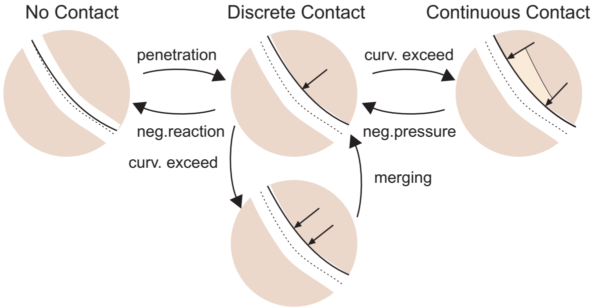

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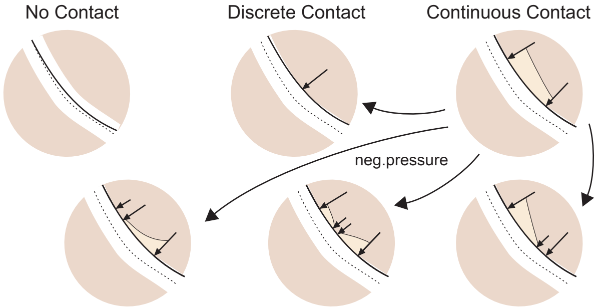

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Illustration

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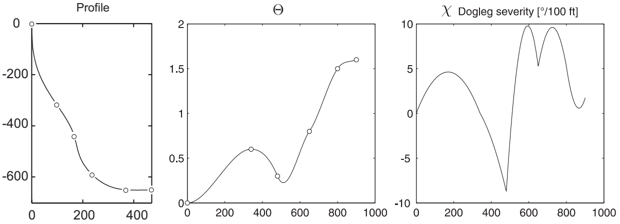

S(m)	0	340	480	650	800	900
Θ(rad)	0	0.6	0.3	0.8	1.5	1.6
χ (°/100 ft)	0	0	-8.7	5.2	5.2	1.7

- Borehole diameter: 0.203m (8")
- Drillstring: continuous pipe with $\phi_o = 11.4$ cm (4.5") and $\phi_i = 9.2$ cm (3.64") $\rightarrow w = 292N/m$
- Boundary condition: clamped at both ends (rotary table, BHA)
- Hook load: equal to the total weight of inserted pipes

Results

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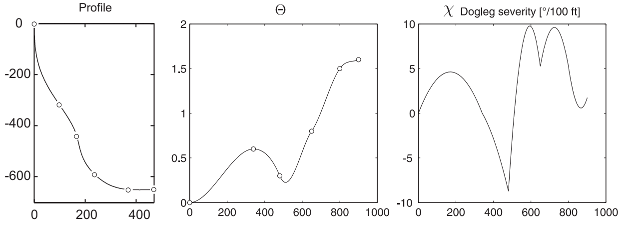

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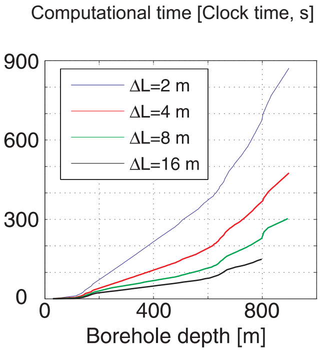
The beam is inserted with a pushing step $\Delta L = 2m$ (!)

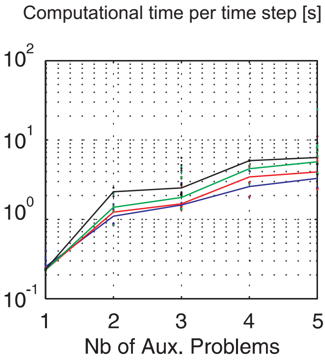
- $N = 10$ elements

Animation - Results

Computation time

Implemented with Matlab routines



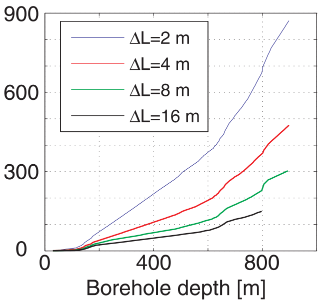


Extrapolation to a dynamic analysis ?
Computation time $\mathcal{O}$(1sec) per time step.

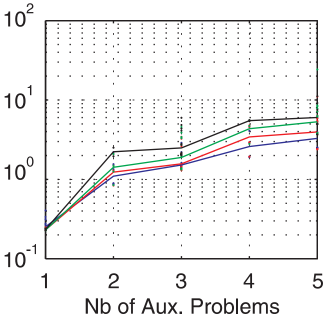
Computation time

Implemented with Matlab routines

Computational time [Clock time, s]



Computational time per time step [s]



Extrapolation to a dynamic analysis ?
Computation time $\mathcal{O}(1\text{sec})$ per time step.

An appropriate Framework for further developements...

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Eulerian formulation:

- simple implementation of $A(S)$
- simple implementation of hydrostatic pressure

Few DOFs and fast:

- Sensibility, Parametric, Reliability or other Stochastic Analyses
- Dynamic Analysis

Start from an accurate basline solution:

- Stability & FRF Analysis
- Vibration Analysis

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Summary

The components of the model:

- Segmentation of the drillstring between contacts
- The Auxiliary Problem
- Eulerian formulation
- Signed-Distance Function $\Delta(S)$

Implementation for 2-D planar boreholes:

- B-G solution of the auxiliary problem (Eulerian version)
- Pattern Switcher for automatic managment of contacts

What we are busy on:

- Validation of the 2-D model with field measurements
- Extension to $A(S)$, $EI(S)$, hydrostatic pressure
- Extension to 3-D boreholes and vibration analysis

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Thank you for listening ...

Read out more: www.orbi.ulg.ac.be
Contact me: v.denoel@ulg.ac.be

- Denoël V., Detournay E., Eulerian Formulation of a Drillstring Constrained inside a Curved Borehole, joint IEEE/SPE session at the MSC-2011 IEEE Multi-Conference on Systems and Control, Denver CO, USA, 8 p. (2011).
- Denoël V. and Detournay E., Eulerian formulation of constrained elastica. International Journal Of Solids And Structures, 2011. 48(3-4): p. 625-636.
- Denoël V. and Detournay E. The Problem of a Drillstring Inside a Curved Borehole: a Perturbed Eulerian Approach. in First International Colloquium on Non-Linear Dynamics of Deep Drilling Systems. 2009. Liège, Belgium: 89-95.
- Denoël V., Advantages of a semi-analytical approach for the analysis of an evolving structure with contacts. Communications in Numerical Methods in Engineering, 2008. 24(12): p. 1667-1683.