ENSO forecast using a wavelet-based mode decomposition

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Abstract

We introduce a new method for forecasting major El Niño/ La Niña events based on a wavelet mode decomposition. This methodology allows us to approximate the ENSO time series with a superposition of three periodic signals corresponding to periods of about 31, 43 and 61 months respectively with time-varying amplitudes. This pseudo-periodic approximation is then extrapolated to give forecasts. While this last one only resolves the large variations in the ENSO time series, three years hindcast as retroactive prediction allows to recover most of the El Niño/ La Niña events of the last 60 years.

1. Method

a) We perform the wavelet transform of the Niño 3.4 time series f:

$$Wf(a,t) = \int f(t)\bar{\psi}\left(\frac{x-t}{a}\right)\frac{dx}{a}$$

where ψ is very similar to the Morlet wavelet, $\bar{\psi}$ is the complex conjugate of ψ, t stands for the time, and $a > 0$ is the scale parameter.

b) We compute the wavelet spectrum Λ associated to f:

$$\Lambda(a) = E\,|Wf(a,.)|$$

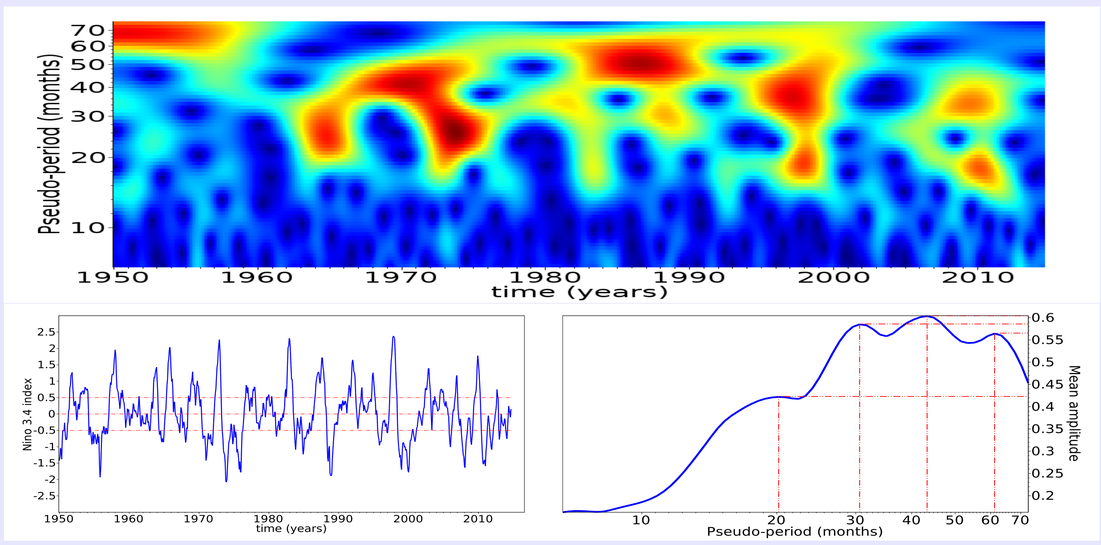
where E denotes the mean over time and we look for the scales $a_1, ..., a_J$ for which Λ reaches a maximum.

c) A good approximation of f is given by

$$f(t) \approx \sum_{j=1}^{J} |Wf(a_j,t)| \cos(\arg Wf(a_j,t)).$$

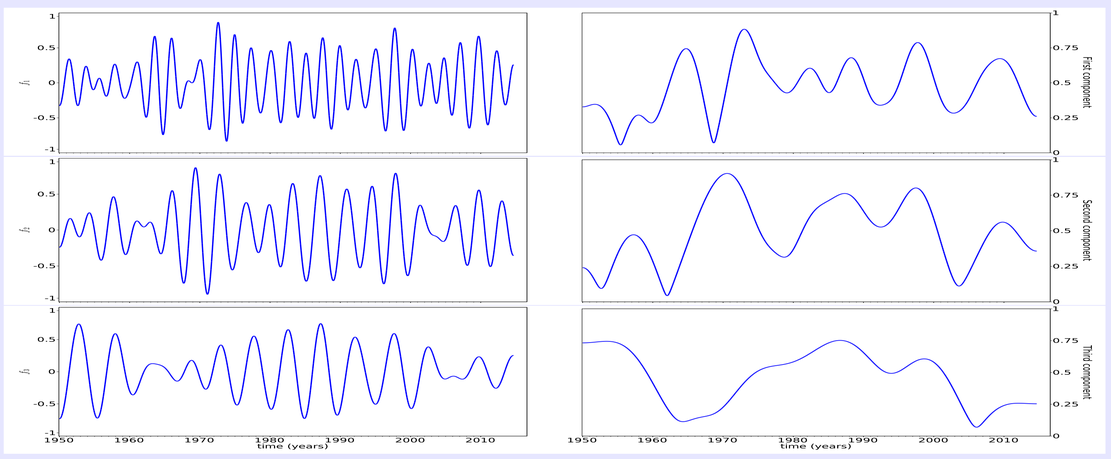
d) Since $\cos(\arg Wf(a_j,t))$ roughly corresponds to a cosine with a period proportional to a_j, forecasts of the reconstructed signal can be obtained with smooth extrapolations of the amplitudes $|Wf(a_j,t)|$.

2. Wavelet transform and spectrum

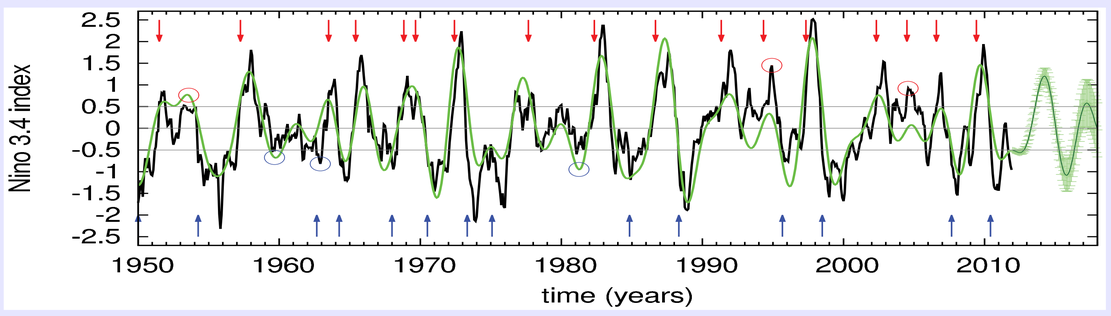


Top: Modulus of the wavelet transform of the Niño 3.4 time series. Bottom left: The original Niño 3.4 time series (SST anomalies, monthly sampled data, NOAA ERSST.V3B). Bottom right: Wavelet spectrum of the ENSO time series. One can see that 4 maxima appear, but the first is clearly weaker thus will be ignored. The others correspond to periods of about 31, 43 and 61 months.

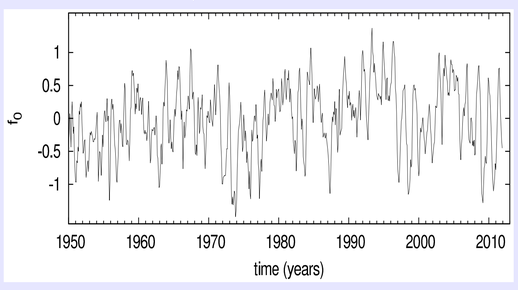
3. Reconstruction



Left: the 3 components extracted from the Nino 3.4 time series corresponding to the periods of 30.62, 43.61, and 61.21 months respectively. Right: The time-varying amplitude associated to each component.

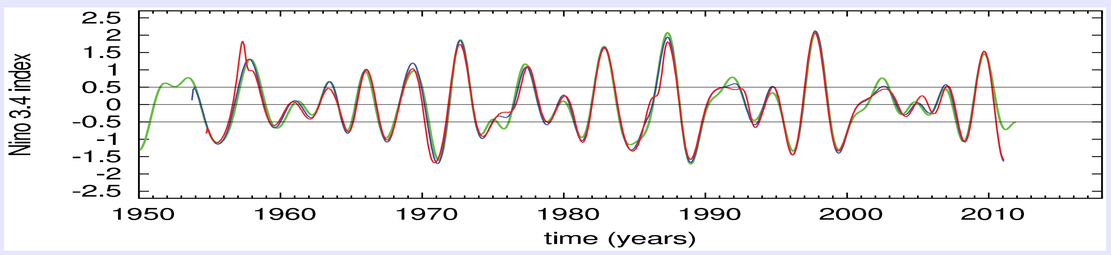


The signal Nino3.4 (black) and the reconstructed signal (green) based on the 3 components. We can notice that 28 out of 31 major events (El Niño and La Niña) are recovered and 3 events are erroneously predicted.

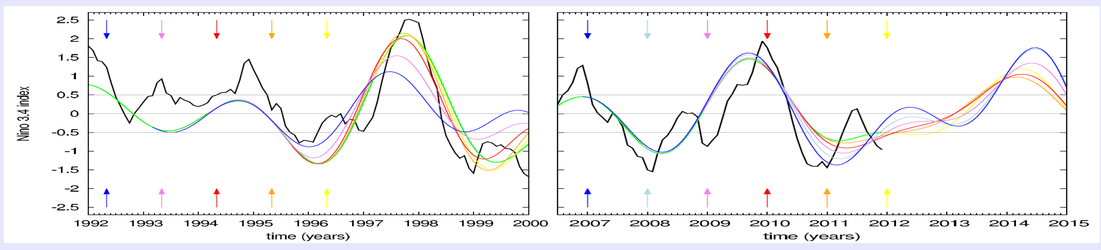


The difference between the original and the reconstructed signal, i.e. the part of the Nino 3.4 index that can not be explained in terms of oscillatory components.

4. Retroactive probing forecasts



The reconstructed Nino 3.4 time series (green) with retroactive probing forecasts for 12 (blue) and 24 (red) months. These hindcasts respectively predict 27 and 24 major events, but 4 and 5 false positive detections. Note that 36 months hindcast still predicts 23 major events and 5 false positive. The RMSE between these signals and the original Niño 3.4 time series remains lower than 0.6.



Left: Forecasts starting 1,...,5 years before the 1997-1998 El Niño event. One can see that our method is competitive for long-term predictions of ENSO. Right: Forecasts starting 0,...,5 years before February 2011 and predicting an El Niño event in 2014-2015.

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