

# Optimization of Surface Utilization Using Heuristic Approaches

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## 1 Introduction

We present a new method to solve a scheduling problem that arises in factories producing large building blocks, namely a shipyard workshop producing prefabricated keel elements. The factory is divided in equal size areas. The blocks produced in the factory are very large. Once a building block is placed into the factory, it cannot be moved until all processes on the building block are finished. The blocks cannot overlap. The objective is to maximize the number of building blocks produced in the factory during a certain time window.

More precisely, we are given a set of  $n$  rectangular-shaped blocks. Each block is characterized by its geometric dimensions (width  $w_j$ , length  $l_j$  and height  $h_j$ ) but also by processing information such as its processing time  $t_j$ , its ready time  $r_j$  and its due date  $d_j$  ( $j \in \{1, \dots, n\}$ ). We are also given a number  $A$  of identical two-dimensional areas, having width  $W$  and length  $L$ . Time is considered as a third dimension. The areas are fully dedicated to the production of the blocks.

The problem consists of orthogonally ordering the blocks into the areas, while respecting the time constraints, and with the objective to produce the largest number of building blocks. In practical terms, we have to assign six variables for each block  $j$ :

- $p_j = \{0,1\}$  indicating whether the block  $j$  is produced or not
- the name  $a_j = \{1, \dots, A\}$  of the area where block  $j$  is to be produced
- $x_j$  and  $y_j$  coordinates, representing the position of the upper left corner of the block  $j$  in the area
- an orientation  $o_j = \{0,1\}$  (either horizontally or vertically) for block  $j$
- a starting date  $s_j$

A solution will be considered as feasible if the individual and the collective constraints are met. We call individual constraints those which focus on one block only, regardless of the other blocks. Major individual constraints are:

- blocks must fit within the width of an area ( $x_j \geq 0$  and  $x_j + [o_j w_j + (1 - o_j) l_j] \leq W$ )
- blocks must fit within the length of an area ( $y_j \geq 0$  and  $y_j + [o_j l_j + (1 - o_j) w_j] \leq L$ )
- blocks must fit in their time windows ( $s_j \geq r_j$  and  $s_j + t_j \leq d_j$ )

Collective constraints focus on the interaction between the positions of different blocks. In a first step, the only collective constraint considered is that we need to prevent the blocks from overlapping.

We assume initially that there exists at least one feasible solution for the set of blocks initially given. I.e. all the constraints can be satisfied when  $p_j = 1$  for all  $j \in \{1, \dots, n\}$ .

## 2 Analogy to the 3D-BPP

In the three-dimensional bin packing problem (3D-BPP), we are given a set of  $n$  rectangular-shaped items, each characterized by width  $w_j$ , height  $h_j$ , and depth  $d_j$  ( $j \in \{1, \dots, n\}$ ) and an unlimited number of identical three-dimensional containers (bins) having width  $W$ , height  $H$ , and depth  $D$ . The 3D-BPP consists of orthogonally packing all the items into the minimum number of bins.

The major difference between 3D-BPP and our initial problem is that, in the former, items/blocks must fit into the container height ( $z_j \geq 0$  and  $z_j + h_j \leq H$ ), whereas they must fit into their time window in the latter ( $s_j \geq r_j$  and  $s_j + t_j \leq d_j$ ). One can compare a bin in the 3D-BPP to a timeline of the two-dimensional representation of an area, which gives us a three-dimensional representation of the problem. The differences between minimizing the number of bins and maximizing the number of blocks will not complicate the formulation, since we will assume (initially) that there exists at least one feasible solution for a fixed number of block and of areas/bins.

The 3D-BPP is strongly NP-hard. (See *Garey and Johnson (1979)* for more information about the complexity of combinatorial optimization problems. Indeed, it is a generalization of the one-dimensional bin packing problem (1D-BPP), in which a set of  $n$  positive values  $w_j$  has to be partitioned into the minimum number of subsets so that the total value in each subset does not exceed a given bin capacity  $W$ . 1D-BPP is the special case of 3D-BPP for  $h_j = H$  and  $d_j = D$  for all  $j \in \{1, \dots, n\}$  and it has been proven that the 1D-BPP is NP-Hard, *Coffman et al. (1997)*. For such difficult problems, one way to contain combinatorial explosion is to allow algorithms to reach fairly good solutions, without guaranteeing that the best possible solution is reached. Local search heuristics use this strategy.

*Faroe et al. (2003)* proposed a new heuristic for 3D-BPP. Their method is very flexible allowing to adapt it to various additional constraints. Therefore it fits perfectly to our problem, as to many other real-life problems.

## 3 Finding feasible solutions

### 3.1 General approach

The local search heuristic proposed to find a feasible schedule strictly enforces the individual constraints only. Then, penalties linked to the collective constraints are summed up in an objective function that is minimized. With no additional real-life collective constraints, the objective function value of a given solution is the total pairwise overlap between the blocks. Therefore, with a randomly generated unfeasible solution where blocks can overlap, searching for a feasible solution is equivalent to minimizing the objective function, since an objective value of zero indicates that also the collective constraints are met. For any solution  $X$ , let  $overlap_{ij}(X)$  be the overlap (in square meters days) between blocks  $i$  and  $j$ . The objective function can now be formulated as

$$f(X) = \sum_{i < j} overlap_{ij}(X) \quad (1)$$

Given a solution  $X$ , we can redefine the neighborhood  $v(X)$  proposed by *Faroe et al (2003)* as 'the set of all solutions that can be obtained by translating any single block along the coordinates axes and the timeline, or by a move to the same position in another area, or by a  $\pm 90^\circ$  rotation of a block around one of its four corners.' A neighbor of  $X$  is therefore constructed by assigning a new value to one of the variables  $x_i, y_i, s_i, a_i, o_i$ . This definition of a solution space includes all feasible schedules and that there is a path of moves between every pair of solution.

A typical local search procedure proceeds by moving from the current solution  $X_p$  to a neighboring solution  $X_{p+1} \in v(X_p)$  whenever this move improves the value of the objective function. This may lead to two types of difficulties. First, the solution may settle in a local minimum. Several standard methods, such as the Simulated Annealing, *Aarts and Korst (1989)* or the Tabu Search, *Glover (1990)*, exist to avoid this shortcoming of local search procedures. Secondly, the neighborhood of any given solution may be quite large (even if continuous, variables like  $x_i, y_i$  or  $s_i$  can be discretized for practical purposes). Therefore, exploring the neighborhood to find an improving move can be very-costly in computing time. To deal with these issues, we present in this paper an application of the Guided Local Search (GLS) heuristic, and its accompanying neighborhood reduction scheme called Fast Local Search (FLS).

### 3.2 Guided local search

The Guided Local Search Heuristic (GLS) has its root in a Neural Network architecture named GENET, Wang and Tsang (1991), which is applicable to a class of problems known as Constraint Satisfaction Problems. The actual GLS version with its accompanying FLS has been first shown by Voudouris (1997), Voudouris and Tsang (1997,1999), and applied to the 3D-BPP by Faroe et al (2003).

Basically, GLS augments the objective function of a problem to include a set of penalty terms and considers this function, instead of the original one, for minimization by the local search procedure. Local search is confined by the penalty terms and focuses attention on promising regions of the search space, Voudouris and Tsang (1999). Iterative calls are made to a local search procedure, denoted as  $LocalOpt(X)$ . Each time  $LocalOpt(X)$  gets caught in a local minimum, the penalties are modified and local search is called again to minimize the modified objective function. In a certain measure, the heuristic may be classified as a tabu search heuristic; it uses memory to control the search in a manner similar to Tabu Search.

GLS is based on the concept of 'features', a set of attributes that characterizes a solution to the problem in a natural way. In our adaptation of the model, features are the overlaps between the blocks, and the indicator  $I_{ij}(X) = \{0,1\}$  denotes whether blocks  $i$  and  $j$  overlap or not. In a particular solution, a feature with a high overlap is not attractive and may be penalized. As a result, the value of  $overlap_{ij}(X)$  can measure the impact of a feature on a solution  $X$  ('cost function' in Faroe et al. (2003)).

$p_{ij}$  denotes the number of times a feature has been penalized,  $p_{ij}$  is initially zero. We want to penalize the features with the maximum overlap, that have not been penalized too often in the past. The source of information that determines which features will be penalized should thus be the overlap and the amount of previous penalties assigned to the features. For this purpose, we define a utility function  $\mu(X) = overlap_{ij}(X)/(1+p_{ij})$ . After each  $LocalOpt(X)$  iteration, the procedure adds one to the penalty of the pairs with maximum utility.

After incrementing the penalties of the selected features, they are incorporated in the search with an augmented objective function

$$h(X) = f(X) + \lambda \sum_{i,j} p_{ij} \cdot I_{ij}(X) = \sum_{i < j} overlap_{ij}(X) + \lambda \sum_{i,j} p_{ij} \cdot I_{ij}(X) \quad (2)$$

$\lambda$  is the only parameter in this method that has to be chosen experimentally. Thus, when local search has found a solution  $X^* = LocalOpt(X)$ , overlaps with maximum utility are penalized and become undesirable. In a sense, the search procedure is commanded to set a priority on these features and, for this reason, it jumps out of the local minimum.

### 3.3 Fast local search

The 'fast local search' (FLS) procedure, Voudouris and Tsang (1997), Faroe et al. (2003) is used to transform a current solution  $X_{cur}$  into a local minimum  $X^* = LocalOpt(X_{cur})$ . It allows to reduce the size of the neighborhood with a selection of the moves that are likely to reduce the maximum utility overlaps.

We define the sets  $v_m(X)$  as subsets of the neighborhood  $v(X)$  where all solutions in  $v_m(X)$  only differ from  $X$  by the value of the variable  $m$  ( $m = \{x_i, y_i, t_i, a_i, o_i\}$  with  $j \in 1..n$ ). (In the case of  $m = \{o_1, \dots, o_n\}$ ,  $v_m$  also includes the particular change in  $x_j$  and in  $y_j$  that considers a rotation around the four corners of a block. To simplify the explanation, this technical issue is not detailed.)

The neighborhood  $v(X)$  is thus divided into a number of smaller sub-neighborhoods that can be either 'active' or 'inactive'. Initially, only some sub-neighborhoods are active. FLS now continuously visits the active sub-neighborhoods in a random order. If there exists a solution  $X_m$  within the sub-neighborhood  $v_m(X_{cur})$  such that  $f(X_m) < f(X_{cur})$ , then  $X_{cur}$  becomes  $X_m$ ; otherwise we suppose that the selected sub-neighborhood will provide no more significant improvements at this step, and thus it becomes inactive. When there is no active sub-neighborhoods left, the FLS procedure is stopped and  $X_{cur}$ , the best solution found, is returned to GLS. From a less formal point of view, FLS selects at random a variable  $m$  within a list of active variables, as long as this list is not empty. Then, it searches within the domain of  $m$  any improvement of the objective function. If it does not exist the variable  $m$  becomes inactive and is removed from the list. By doing so, we focus specially on variables open for improvement.

The size of the sub-neighborhoods related to the  $a_i$  and the  $o_i$  variables is relatively small: 5 in the first case, 5 in the second (initial + four corners). Therefore FLS is set to test all the neighbors of these sets. But, on the other hand, using an enumerative method for the translations along the  $x$ ,  $y$  and  $t$  axis would become very expensive in terms of computing time, if areas and/or time windows are large. However, only certain coordinates of such neighborhoods need to be investigated. If  $m$  represents  $x_j$ , changes in the overlap function only depend on  $x_j$  ( $h(X) = h(x_j)$ ). Most of the terms of this function are constant, thus, since we want to compare values, only the few terms dependent on  $x_j$  should be computed. Furthermore, an overlap is the product of four partial overlaps (three for the overlaps on each of the  $x$ ,  $y$  and  $z$  axis, and the fourth equals one if  $a_i = a_j$ ; zero otherwise). Since we know that only the partial overlaps for the  $x$  axis depend on  $x_j$ , computing efforts can be reduced to their smallest size. Also,  $overlap_{ij}(X) = overlap_{ji}(X)$ , so that the computing time of one solution is linear ( $n$ ) instead of quadratic ( $n^2$ ). Additionally, all functions  $overlap_{ij}(x_j)$  are piecewise linear functions, and therefore the functions will attain their maximum in one of their breakpoints (or at the limits of their domains). As a result, FLS only needs to compute the values of  $f(x_j)$  with  $x_j$  at breakpoints or at extreme values. In fact, there are at most four breakpoints for each function, and only the first and the last one are evaluated. Indeed, in regard to the analogy with the 3D-BPP, a good packing intuitively supposes that the boxes touch each other.

FLS represents a relatively fast procedure that leads to a local minimum if the amount of active sub-neighborhoods is relatively small. Remember that  $LocalOpt(X)$  is called iteratively by GLS, and that penalties are changed with an objective of escaping local minima. Activation of sub-neighborhoods should therefore allow moves on penalized features. The following reactivation scheme is used, *Faroe et al. (2003)*: (1) moves on the two blocks  $i$  and  $j$ , corresponding to the penalized features, are reactivated. (2) We reactivate the moves on all blocks that overlap with blocks  $i$  and  $j$ . The latter reactivation is added to allow FLS to pay attention not only to the two overlapping blocks but also to the whole area around the penalized feature.

#### 4 Selecting the blocks

In the previous chapter, we described a method that minimizes the collective constraints under restriction of the individual constraints and we supposed that there exists at least one feasible solution for the set of blocks initially given. Let us denote this procedure by  $GlobalOpt(X)$ . If  $GlobalOpt(X)$  is efficient, it should find a solution with an objective function of zero after a certain time and this solution would be one of the feasible solutions. However, in the initial formulation of the problem, we do not know whether a set of blocks is feasible or not. The combination of GLS and FLS can be used anyway if we rely on the following heuristic assumption: there exists no feasible solution if none is found within a certain amount of computing time  $T$ . Consequently, the search heuristic  $GlobalOpt(X, T)$  is utilized as a test of feasibility and gives the correspondent schedules if a feasible solution is identified within  $T$ .

Several methods have been tested using this concept. The objective was to remain as close as possible to the working methods and habits used in the factory under study. From this point of view, an efficient approach for the industrial application is to start GLS with a randomly generated solution  $X_0$  that includes the entire set of blocks ( $p_j = 1$  for all  $j = \{1, \dots, n\}$ ). After a search of  $T$  seconds, the algorithm is stopped and returns  $X_1 = GlobalOpt(X_0, T)$ , the best solution found (in terms of overlap). One of the blocks with the highest overlap is removed from the set ( $X_1 \rightarrow X_1'$ ) and the heuristic  $GlobalOpt(X_1', T)$  is restarted. The entire procedure ends if a solution  $X_n$  with zero overlap is found.

A variant procedure is to start with an empty set  $X_0$  ( $p_j = 0$  for all  $j = \{1, \dots, n\}$ ). At each iteration, if the solution  $X_{n+1} = GlobalOpt(X_n, T)$  is feasible, then an additional block is inserted in the set; otherwise an overlapping block is removed. This procedure is stopped after a certain computing time, or by any more sophisticated stopping criterion, and returns the solution with the largest collection of blocks. Fig.1 shows the iterative process of this procedure.

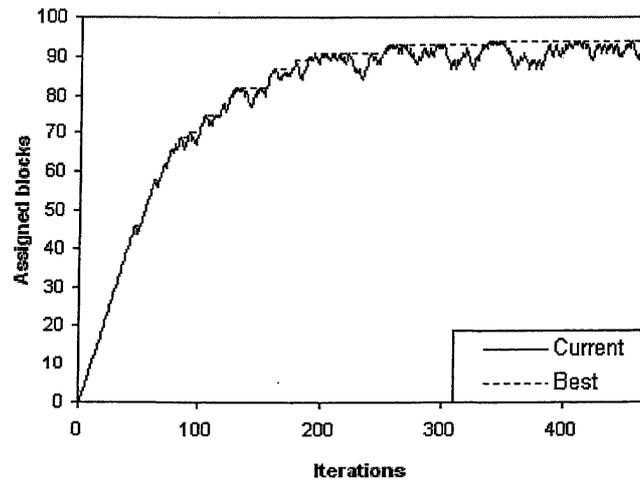


Fig.1: Test instance ( $T = 1$  s, Time limit = 600 s)

Both approaches suffers from one major default: they are likely to have aversion for the largest blocks. Indeed, we do not have an appropriate weighting scheme to evaluate the preferences between blocks, and, since small blocks generally provide smaller overlaps, they are preferred to larger ones. In the real-life situation, when the entire set of block cannot be produced, the person in charge of scheduling can either subcontract specific blocks in other factories, or change some temporal parameters (e.g. intensify the workforce to reduce processing times or postpone due dates). No formal information can describe all the aspects of these choices. For this reason, the operator should be able to change manually the collection of blocks to be produced. Starting from our "fairly good" feasible solution  $X_n$ , iterative  $X_{n+1} = GlobalOpt(X_n)$  calls are ordered manually after deliberate changes ( $X_n \rightarrow X_n'$ ) in the assignment. In addition, a last procedure provides a list with each block that is not assigned even though a feasible solution that includes the block can be found.

By not regenerating solutions on a random basis, some of the information from previous solutions is preserved. A drawback to this approach is that the structure of a previous solution can confine GLS to an area of the solution space that can be difficult to escape. (Faroe et al. (2003) suggest a similar problem in their approach for 3D-BPP.) We may therefore not reach the very best solution. However, the modus operandi described in this section is developed for a daily industrial use. In that setting, the above drawback may actually be viewed as an advantage. Indeed, it may be very costly for the company to mix up the schedules over and over again. Traditionally, methods for problems of similar classes utilize a construction algorithm during the search. A slight improvement may disturb the whole solution; with GLS, non-problematic regions are not perturbed. Murata et al. (1996) developed a tricky construction technique based on partial-orders coding scheme for the 2D-BPP. Imahori et al. (2005) adapted this approach for a problem very close to our's.

## 5 Additional real-life issues

Additional constraints may occur in any firm-specific situation. The tool proposed here is easily customizable to most of them. For example, we may need to restrict or force the position of a block (e.g. a tool is only available in one area or the block is already in process). Integrating those constraints is trivial: restricted positions are not generated and unfeasible neighbors simply do not exist. As a result, the end-user may fix the value of any variable (including  $p_j$ ) or reduce its domain.

Specific collective constraints may also appear in the wording of a problem. In our case, the areas of the factory have one single door, and the crane bridge can only carry blocks up to a certain height. As a result, a large block may obstruct a door, and some blocks might not be deliverable in time because there is no route to transport them out. We dealt with this issue in the same way as for overlaps. For each generated solution  $X$ , we add to the objective function  $h(X)$  a new term accounting for exit difficulties:

$$g(X) = h(X) + ExitProblems(X) = \sum_{i < j} overlap_{ij}(X) + \lambda \cdot \sum_{i,j} p_{ij} \cdot I_{ij}(X) + ExitProblems(X) \quad (3)$$

The *ExitProblems* values should somehow be scaled to the  $h(X)$  values, but there is no need to find a precise weight for this term, because the procedure is designed to find a zero objective function and that weights are less relevant in this case. Other collective constraints may be included in the algorithm using this approach.

## 6 Conclusion

We presented a sophisticated local-search heuristic based on the GLS method. *Faroe et al (2003)* showed that their algorithm outperforms other approaches for the 3D-BPP. Solutions for our industrial problem are indeed found within a few seconds. More importantly, the algorithm offers much flexibility for handling the constraints allowing easy adaptation to many real-life cases. In addition, it focuses the search on promising parts of the solution space and previous schedules are not fully perturbed at each iteration.

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