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ASTRONOMY AND ASTROPHYSICS

# Radiative Forces in Expanding Envelopes

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**Summary.** The escape probability method introduced by V. V. Sobolev is applied to derive expressions for the radiative forces acting on atoms by the resonance scattering of radiation in envelopes which expand spherically with positive or negative radial velocity gradients. In the latter case, it is shown that the contribution of the whole stellar envelope to the radiative force acting on individual atoms has the same effect as the one due to the stellar core radiation, i.e. a radial outward-acceleration. Numerical applications to a two-level atom model illustrate the behaviour of radiative forces in outward-accelerating and outward-decelerating stellar envelopes.

**Key words:** moving stellar envelopes — radiative forces — radiative transfer

## 1. Introduction

The following derivation of radiative forces exerted on atoms by absorption of radiation in spectral lines is entirely based on a previous work (Surdej, 1977a, referred to below as Paper I) which contains a general development of the theory about spectral line formation in outward-accelerating and outward-decelerating stellar envelopes. It was there shown that in the case of envelopes which expand spherically with a negative radial velocity gradient an atom can interact with radiation from the stellar core, from its neighbouring atoms and from atoms situated at distant points. We recall here that two points which are separated by more than a distance in which the Doppler displacement at the line frequency changes by an amount equal to the line width and between which atoms are able to interact radiatively with each other are called "distant points". In contrast with the case of outward-accelerating flows one can wonder whether the contribution to the radiative force due to spectral line radiation emitted by distant atoms will act on individual atoms as an outward or an inward directed force.

The expressions for radiative forces respectively due to radiation emitted from neighbouring atoms, the stellar core and distant atoms in rapidly expanding envelopes are obtained in Section 2. Lucy (1971) and Castor (1974, 1975) derived quite similar expressions for the force exerted on gas in the case of outward-accelerating flows. We refer the reader to the excellent review paper by Castor (1974) about the force associated with absorption of spectral line radiation in static or expanding atmospheres. The approximate analytic results derived there are not considered in our present paper. Without any assumption of, a priori, optical depth smaller or bigger than unity we present in Section 3 results of calculations concerning the behaviour of radiative forces in outward-accelerating (later referred as A.E.) and outward-decelerating (later referred as D.E.) envelopes. These numerical applications were solved in a quite similar way to those used for solving the statistical equilibrium equation involving a two-level atom model in moving media (Surdej, 1977b, referred to below as Paper II).

Section 4 deals with discussions about these applications.

Last section outlines some general conclusions.

## 2. Radiative Forces

Let us first recall the general hypotheses and definitions we assume (cf. Paper I) in the escape probability method later used for deriving the expressions of radiative forces in rapidly expanding envelopes. We consider spherically expanding envelopes in which the level populations of different atoms have reached a steady state. $\Phi(\nu-\nu_{ij})$ represents an arbitrary function describing the line profile, normalized to unity when integrated over frequency and equal to zero outside the interval $[\nu_{ij}-\Delta\nu/2, \nu_{ij}+\Delta\nu/2]$. $\nu_{ij}$ denotes the central frequency of the line and $\Delta\nu$ the maximum width of the line profile due to chaotic motions of the atoms. The volume absorption and emission coefficients for an observer moving with the atoms are respectively

$$\alpha_{ij}=(n_iB_{ij}h\nu_{ij}/4\Pi)\,(1-g_in_j/g_jn_i), \tag{1}$$

$$\varepsilon_{ij}=n_jA_{ji}h\nu_{ij}/4\Pi, \tag{2}$$

$n_i$ and $n_j$ being respectively the volume populations of the levels $i$ and $j$, and $g_i$, $g_j$ their statistical weights. $B_{ij}$ and $A_{ji}$ are Einstein-Milne transition probabilities and $h$ is Planck's constant. The absorption coefficient for one atom is then given by

$$k_{ij}=\alpha_{ij}/n_i. \tag{3}$$

When considering only line processes in the transition $i\rightarrow j$ and under the assumption of a complete redistribution of the radiation with frequency and direction, the source function $S_{ij}$ of the line transition in the frame of the atom is

$$S_{ij}=\varepsilon_{ij}/\alpha_{ij}. \tag{4}$$

If $\partial v_s/\partial s$ denotes the velocity gradient along a direction $\boldsymbol{l}$, $\boldsymbol{l}$ being a unit vector, we define the fictive optical depth

$$|\tau_{ij}|=\left|\alpha_{ij}c\Big/\left(\nu_{ij}\frac{\partial v_s}{\partial s}\right)\right|, \tag{5}$$

$c$ being the light velocity.

In Paper I we obtained general expressions for the specific intensity $I_\nu$ defined at the local frequency[1] $\nu$ of the radiation which can interact along a direction $\boldsymbol{l}$ with an atom situated at a point $C_0$.

The vectorial radiative force $\boldsymbol{F}_{ij}$ acting on an atom by absorption of radiation in the line transition $i\rightarrow j$ is given by

$$\boldsymbol{F}_{ij}=\frac{\Pi}{c}\,k_{ij}\int_{\nu_{ij}-\Delta\nu/2}^{\nu_{ij}+\Delta\nu/2}\Phi(\nu-\nu_{ij})\boldsymbol{F}_\nu d\nu, \tag{6}$$

where $\boldsymbol{F}_\nu$ represents the vectorial form of the astrophysical flux

$$\boldsymbol{F}_\nu=\frac{1}{\Pi}\int_{\Omega=4\Pi}I_\nu\boldsymbol{l}d\omega. \tag{7}$$

Combining both last expressions, we find

$$\boldsymbol{F}_{ij}=\frac{k_{ij}}{c}\int_{\Omega=4\Pi}\boldsymbol{l}d\omega\int_{\nu_{ij}-\Delta\nu/2}^{\nu_{ij}+\Delta\nu/2}\Phi(\nu-\nu_{ij})I_\nu d\nu. \tag{8}$$

In an expanding envelope, the expression

$$\int_{\nu_{ij}-\Delta\nu/2}^{\nu_{ij}+\Delta\nu/2}\Phi(\nu-\nu_{ij})I_\nu d\nu$$

calculated at $C_0$ along a fixed direction $\boldsymbol{l}$ represents the intensity of spectral line radiation which can interact with a moving atom along the direction $\boldsymbol{l}$ at $C_0$. Because the emission of radiation by atoms is isotropic in our model, this process will not contribute to the radiative force.

### a) *Local Contribution to Radiative Forces in* A.E. *and* D.E. *Envelopes*

In A.E. and D.E. envelopes neighbouring atoms interact with each other through the spectral line radiation they emit (see Paper I). If we fix a point and a direction in those media, the so-called "neighbourhood" lies within a distance across which the Doppler displacement at the line frequency $\nu_{ij}$ changes by an amount equal to $\Delta\nu$.

In the following, we evaluate at a given point the local contribution $\boldsymbol{F}^L_{ij}$ of the radiative force acting on an atom due to spectral line radiation emitted by its neighbourhood. In Paper I, we obtained [see Eqs. (3.3) and (6.3)]

$$\int_{\nu_{ij}-\Delta\nu/2}^{\nu_{ij}+\Delta\nu/2}\Phi(\nu-\nu_{ij})I_\nu d\nu=S_{ij}(1-(1-\exp(-|\tau_{ij}|))/|\tau_{ij}|). \tag{9}$$

Because of the isotropic property of $S_{ij}$, expression (8) for $\boldsymbol{F}_{ij}$ then reduces to

$$\boldsymbol{F}^L_{ij}=\frac{k_{ij}}{c}\,S_{ij}\int_{\Omega=4\Pi}\boldsymbol{l}d\omega(1-\exp(-|\tau_{ij}|))/|\tau_{ij}| \tag{10}$$

or

$$\boldsymbol{F}^L_{ij}=\frac{4\Pi}{c}\,k_{ij}S_{ij}\boldsymbol{\beta}^1_{ij}, \tag{11}$$

where we define the escape probability vector $\boldsymbol{\beta}^1_{ij}$

$$\boldsymbol{\beta}^1_{ij}=\int_{\Omega=4\Pi}(1-\exp(-|\tau_{ij}|))/|\tau_{ij}|\,\boldsymbol{l}\,\frac{d\omega}{4\Pi}. \tag{12}$$

$\boldsymbol{\beta}^1_{ij}$ gives the preferential direction for a photon created in the line transition $j\rightarrow i$ to escape the medium locally. Under the hypotheses assumed in this work we have rigorously

$$\boldsymbol{\beta}^1_{ij}=\boldsymbol{0} \tag{13}$$

and thus

$$\boldsymbol{F}^L_{ij}=\boldsymbol{0}. \tag{14}$$

Indeed, neglecting the transfer of photons in space (local approximation) we assume (see Paper I) that in the neighbourhood of every point in the envelope the volume absorption coefficient $\alpha_{ij}$, the volume emission coefficient $\varepsilon_{ij}$ and the velocity gradient $\partial V_s/\partial s$ (along a fixed direction) are constant and that the line profile function remains the same. Therefore the fictive optical depth $|\tau_{ij}|$ being identical in both directions $\boldsymbol{l}$ and $-\boldsymbol{l}$, the escape probability vector $\boldsymbol{\beta}^1_{ij}$ is null. Supposing that diffusions of photons occur around the place they were created (spatial transfer of photons) Sobolev (1957) and Castor (1974) evaluated $\boldsymbol{F}^L_{ij}$ in the case of outward-

[1] The local frequency of a photon emitted or passing at a given point is the frequency seen by an observer moving with the medium at that point

accelerating flows. This non local approximation consists of taking into account the effects of anisotropy due to radial gradients of the source function $S_{ij}$, of the absorbing atom density $n_i$ and to the curvature in the velocity law. Let us remark that in this new approach, the radiative force $\boldsymbol{F}_{ij}^L$ would still reduce to zero under the hypotheses we assumed here, namely the presence of high velocity gradients throughout the envelope.

*b) Stellar Core Contribution to Radiative Forces in* A.E. *and* D.E. *Envelopes*

Assuming the stellar core to radiate continuously, like a black body at temperature $T$, without limb darkening and with an intensity $I_c$ constant over the line frequency we derived in paper I expressions for the specific intensity $I_\nu$, defined at the local frequency $\nu$, due to the core radiation at every point of an expanding envelope. Furthermore, we found in the case of A.E. envelopes that [see Eqs. (10.3) and (11.3) in Paper I]

$$\int_{\nu_{ij}-\Delta\nu/2}^{\nu_{ij}+\Delta\nu/2} \Phi(\nu-\nu_{ij})I_\nu d\nu = I_c(1-\exp(-|\tau_{ij}|))/|\tau_{ij}|. \tag{15}$$

Similarly in the case of D.E. envelopes [see Eq. (15.4) in Paper I]

$$\int_{\nu_{ij}-\Delta\nu/2}^{\nu_{ij}+\Delta\nu/2} \Phi(\nu-\nu_{ij})I_\nu d\nu = I_c(1-\exp(-|\tau_{ij}|))\exp(-|\tau'_{ij}|)/|\tau_{ij}|. \tag{16}$$

In the last expression the factor $\exp(-|\tau'_{ij}|)$ expresses the probability for a stellar photon not to be absorbed in the neighbourhood of a distant point $C'_0$ whose relative velocity to the point $C_0$ at which we evaluate the radiative force lies within the chaotic velocities of the atoms. Of course, the physical quantity $|\tau_{ij}|$ appearing in expressions (15) and (16) is calculated at $C_0$. If we denote by $\boldsymbol{F}_{ij}^{CA}$ and $\boldsymbol{F}_{ij}^{CD}$ the radiative forces due to the stellar core radiation acting on atoms respectively in A.E. and D.E. envelopes, combinations of Equations (8), (15) and (16) lead to

$$\boldsymbol{F}_{ij}^{CA} = \frac{4\Pi}{c} k_{ij} I_c \boldsymbol{\beta}_{ij}^3 \tag{17}$$

and

$$\boldsymbol{F}_{ij}^{CD} = \frac{4\Pi}{c} k_{ij} I_c \boldsymbol{\beta}_{ij}^5 \tag{18}$$

with respectively

$$\boldsymbol{\beta}_{ij}^3 = \int_{\Omega=4\Pi W} (1-\exp(-|\tau_{ij}|))/|\tau_{ij}| \boldsymbol{l} \frac{d\omega}{4\Pi} \tag{19}$$

and

$$\boldsymbol{\beta}_{ij}^5 = \int_{\Omega=4\Pi W} (1-\exp(-|\tau_{ij}|))\exp(-|\tau'_{ij}|)/|\tau_{ij}| \boldsymbol{l} \frac{d\omega}{4\Pi}. \tag{20}$$

The vector $\boldsymbol{\beta}_{ij}^3$, evaluated at a point $C_0$ in an A.E. envelope, gives the preferential direction for a photon emitted at $C_0$ in the line transition $j\rightarrow i$ to reach unhindered the stellar core. The vector $\boldsymbol{\beta}_{ij}^5$ represents the same quantity in the case of D.E. envelopes. Integrations in expressions (19) and (20) are performed over every direction joining the stellar core to the point $C_0$, i.e. within the solid angle $\Omega = 4\Pi W$ where $W$ is the dilution factor.

*c) Contribution to Radiative Forces from Distant Atoms in* D.E. *Envelopes*

When evaluating radiative forces acting on an atom at a point $C_0$ in a D.E. envelope we must consider the spectral line radiation emitted by distant atoms at $C'_0$ towards $C_0$.

In Paper I, we obtained [see Eqs. (12.4) and (13.4)]

$$\int_{\nu_{ij}-\Delta\nu/2}^{\nu_{ij}+\Delta\nu/2} \Phi(\nu-\nu_{ij})I_\nu d\nu = S'_{ij}(1-\exp(-|\tau'_{ij}|))(1-\exp(-|\tau_{ij}|))/|\tau_{ij}|, \tag{21}$$

where $S'_{ij}$, $\tau'_{ij}$ are evaluated at the distant point $C'_0$.

Inserting Equation (21) in (8) and designing by $\boldsymbol{F}_{ij}^{DD}$ the radiative force per atom exerted by line radiation from distant atoms, we have

$$\boldsymbol{F}_{ij}^{DD} = \frac{4\Pi}{c} k_{ij} \int_{\Omega(C_0,C'_0)} S'_{ij}(1-\exp(-|\tau'_{ij}|))(1-\exp(-|\tau_{ij}|))/|\tau_{ij}| \boldsymbol{l} \frac{d\omega}{4\Pi}, \tag{22}$$

the integration being extended over all directions from $C_0$, for which a couple of points $(C_0, C'_0)$ exists.

## 3. Numerical Applications to the Two-level Atom Model

In the following we will often refer the reader to the second section of Paper II: "Numerical Applications to Radiative Transfer in Expanding Envelopes: The Two-level Atom Model".

*a) Model*

Let us first remark that because of the spherical symmetry existing in the expanding envelopes, all radiative

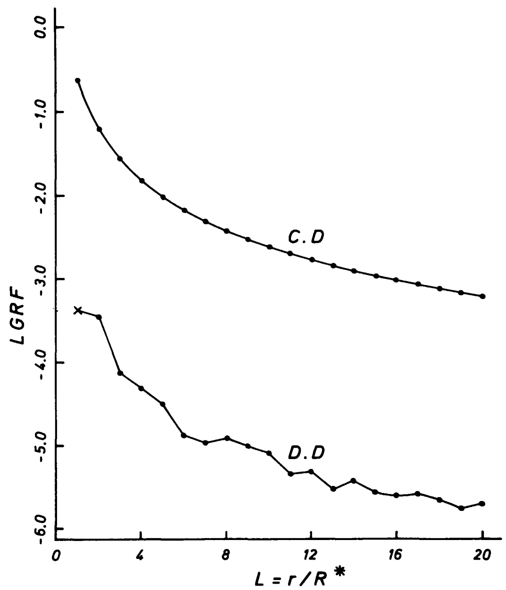


**Fig. 1.** Plot of the quantities LGRF "$C.D$" and LGRF "$D.D$" as a function of the variable $L$ in the case of a D.E. envelope. The following parameters were used: $l=0.5$, $g_1/g_2=1$, $n_0=10^2$ atoms/cm$^3$, $V_0=10^9$ cm/s, $R^*=6.96\,10^{10}$ cm, $f_{12}=0.5$ and $\lambda_{12}=10^{-5}$ cm with $\lambda_{12}T=0.289\,79$ cm·deg

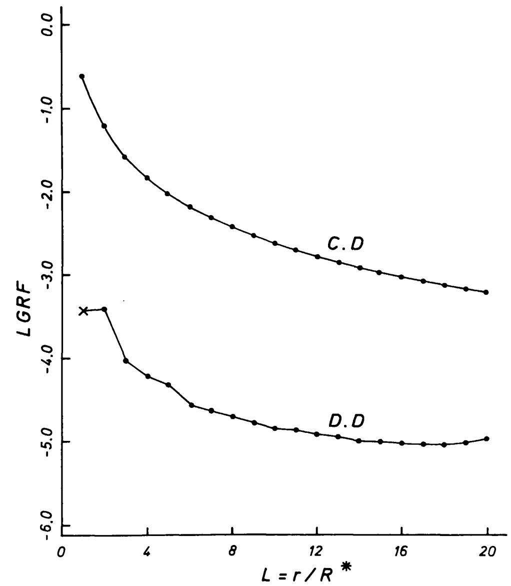


**Fig. 2.** Plot of the quantities LGRF "$C.D$" and LGRF "$D.D$" as a function of the variable $L$ in the case of a D.E. envelope. The parameters used are the same as those in Figure 1 with the exception that $l=1$

forces are directed radially. Considering an atom model with two levels between which occur only radiative exchanges we calculate the expressions $\boldsymbol{F}_{12}^{CA}\cdot\boldsymbol{n}$ and $\boldsymbol{F}_{12}^{CD}\cdot\boldsymbol{n}$, $\boldsymbol{F}_{12}^{DD}\cdot\boldsymbol{n}$ respectively in the cases of A.E. and D.E. envelopes under different physical conditions. $\boldsymbol{n}$ represents a unit vector directed radially outward the envelope at the point we evaluate the preceding expressions. Numerical determination of these requires the simultaneous resolution of the statistical equilibrium equation. This last problem was entirely discussed in Paper II. Similarly, we adopt here velocity distributions of the type

$$V(r)=V_0L^{-l}, \tag{23}$$

$V_0$ being the velocity at the stellar surface and $l$ a decelerating (resp. accelerating) parameter when positive (resp. negative). The variable $L$ is defined by

$$L=r/R^*, \tag{24}$$

with $R^*$ being the stellar radius and $r$ the distance from the center of the star to the point we evaluate $V(r)$.

If $n_0$ denotes the density, in numbers per unit volume, at the stellar surface, the density distribution $n(r)$ at the distance $r$ is given by

$$n(r)=n_0L^{l-2}. \tag{25}$$

### *b) Results*

The results of our calculations are illustrated in Figures 1–9. It is there assumed that the envelope extends up to 20 stellar radii ($L=20$), the determination of the radiative force expressions being performed at subsequent points distant from each other by one stellar radius (cf. in Paper II).

For D.E. envelopes we define the following relations

$$\text{LGRF "}C.D\text{"}=\log_{10}(\boldsymbol{F}_{12}^{CD}\cdot\boldsymbol{n}/K_{12}), \tag{26}$$

$$\text{LGRF "}D.D\text{"}=\log_{10}(\boldsymbol{F}_{12}^{DD}\cdot\boldsymbol{n}/K_{12}), \tag{27}$$

where the constant $K_{12}$ is given by

$$K_{12}=\frac{4\Pi}{c}I_c\frac{\Pi e^2}{mc}f_{12}, \tag{28}$$

$f_{12}$ being the oscillator strength for the transition $1\rightarrow2$.

In Figures 1, 2 and 3 are plotted the quantities LGRF as a function of the variable $L$ respectively for the values of the parameter $l=0.5$, 1 and 2 assuming that $g_1/g_2=1$, $n_0=10^2$ atoms/cm$^3$, $R^*=6.96\,10^{10}$ cm or $R^*=R_\odot$, $V_0=10^9$ cm/s, $f_{12}=0.5$ and $\lambda_{12}=10^{-5}$ cm with $\lambda_{12}\cdot T=0.28979$ cm·deg ($\lambda_{12}$ is the wavelength of the line transition $1\rightarrow2$). In Figures 4–6 are plotted the quantities LGRF as a function of the variable $L$ respectively for the values of the parameter $l=0.5$, 1 and 2 assuming that $g_1/g_2=1$, $n_0=10^5$ atoms/cm$^3$, $R^*=6.96\,10^{11}$ cm or $R^*=10\,R_\odot$, $V_0=10^7$ cm/s, $f_{12}=0.5$ and $\lambda_{12}=10^{-5}$ cm with $\lambda_{12}\cdot T=0.28979$ cm·deg.

For A.E. envelopes the following relation is used

$$\text{LGRF}=\log_{10}(\boldsymbol{F}_{12}^{CA}\cdot\boldsymbol{n}/K_{12}) \tag{29}$$

and plotted in the Figures 7–9 as a function of the

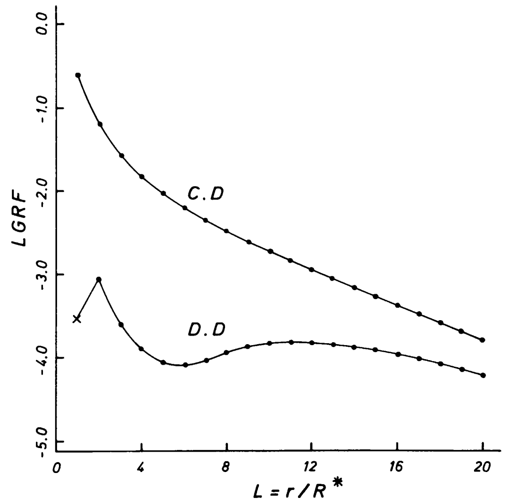


**Fig. 3.** Plot of the quantities LGRF "*C.D*" and LGRF "*D.D*" as a function of the variable $L$ in the case of a D.E. envelope. The parameters used are the same as those in Figure 1 with the exception that $l=2$

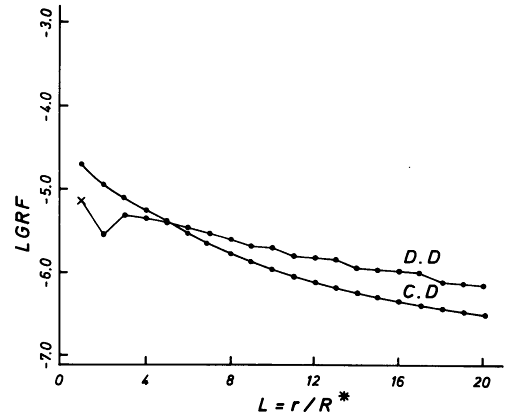


**Fig. 4.** Plot of the quantities LGRF "*C.D*" and LGRF "*D.D*" as a function of the variable $L$ in the case of a D.E. envelope. The following parameters were used: $l=0.5$, $g_1/g_2=1$, $n_0=10^5$ atoms/cm$^3$, $V_0=10^7$ cm/s, $R^*=6.96\ 10^{11}$ cm, $f_{12}=0.5$ and $\lambda_{12}=10^{-5}$ cm with $\lambda_{12}T=0.28979$ cm·deg

variable $L$. The three curves appearing in each of these figures refer to different values of the accelerating parameter, i.e. $l=-0.5$, $-1$ and $-2$. Furthermore we have in Figure 7 $n_0=10^2$ atoms/cm$^3$, $V_0=10^9$ cm/s; in Figure 8 $n_0=10^5$ atoms/cm$^3$, $V_0=10^7$ cm/s and in Figure 9 $n_0=10^{15}$ atoms/cm$^3$, $V_0=10^7$ cm/s. All the remaining parameters are $g_1/g_2=1$, $R^*=6.96\ 10^{11}$ cm or $R^*=10\,R_\odot$, $f_{12}=0.5$ and $\lambda_{12}=10^{-5}$ cm with $\lambda_{12}T=0.28979$ cm · deg. In computing all radiative forces LGRF and solving simultaneously the statistical equilibrium equations to determine the excitation degree $x=n_1/n_2$ (see paper II) throughout the envelope we carried our calculations with a precision $\Delta x/x$ better than 0.005. All integrals $(\beta^1_{12}, \beta^3_{12}, \ldots, \boldsymbol{F}^{CA}_{12}\cdot\boldsymbol{n}, \boldsymbol{F}^{CD}_{12}\cdot\boldsymbol{n}, \ldots)$ were computed with 150 steps of integration.

In Figures 1–6 the quantities LGRF "*D.D*" are plotted with points (·) when the corresponding radiative force $\boldsymbol{F}^{DD}_{12}$ is directed radially outward the envelope and with crosses (×) otherwise.

## 4. Discussion

The following discussion is based on the results presented in Figures 1–9:

(i) The radiative force due to spectral line radiation emitted by distant atoms in D.E. envelopes has the general effect of accelerating outward the atoms on which it acts. Under normal physical conditions (see Figs. 4–6) the amplitude of this radiative force can reach and overreach the one due to the stellar core continuum.

(ii) If the medium becomes very transparent, i.e. if the fictive optical depth averaged over directions is such as $|\bar{\tau}_{12}|\ll 1$, the intensity of spectral line radiation due to distant atoms $I'(C_0)$ that a moving atom at a point $C_0$ can interact with along a direction $\boldsymbol{l}$ is $I'(C_0)\simeq S'_{12}|\bar{\tau}_{12}|$. The contribution from the stellar core is similarly given by $I(C_0)\simeq I_c$. It follows naturally that $S'_{12}|\bar{\tau}_{12}|\ll I_c$ and the departures between the curves in Figures 1–3 can be easily understood qualitatively. Furthermore if $|\bar{\tau}_{12}|\ll 1$, Equation (20) reduces to

$$\beta^5_{12}\simeq\frac{1}{4}\left(\frac{1}{L}\right)^2\boldsymbol{n}$$

and thus

$$\boldsymbol{F}^{CD}_{12}\simeq\frac{\Pi}{c}I_c\frac{\Pi e^2}{mc}f_{12}\left(1-\frac{1}{x}\right)\left(\frac{1}{L}\right)^2\boldsymbol{n}.$$

The general behaviour of the curves in Figures 1–3 is very well approximated by this last relation.

(iii) When the fictive optical depth is such as $|\bar{\tau}_{12}|\gg 1$, we can similarly find the following approximation

$$\boldsymbol{F}^{CD}_{12}\simeq\frac{\Pi}{c}I_c\frac{\Pi e^2}{mc}f_{12}\left(1-\frac{1}{x}\right)\left(\frac{1}{L}\right)^2\frac{1}{|\bar{\tau}_{12}|}\boldsymbol{n},\quad \text{or}$$

$$\boldsymbol{F}^{CD}_{12}\simeq\frac{\Pi}{c}I_c\frac{v_{12}\,\overline{\dfrac{\partial v}{\partial s}}^{\,s}}{n_1 c}\,\frac{(x-1)}{\left(x-\dfrac{g_1}{g_2}\right)}\left(\frac{1}{L}\right)^2\boldsymbol{n}$$

when using the definition of $\overline{\tau_{12}}$ and the relation between the oscillator strength $f_{12}$ and the probability $B_{12}$ (cf. Paper II). We reach the same conclusion as stated by Lucy and Solomon (1970) and Castor (1974) in the case of A.E. envelopes, namely that the force due to an optically thick line is independent of the line strength.

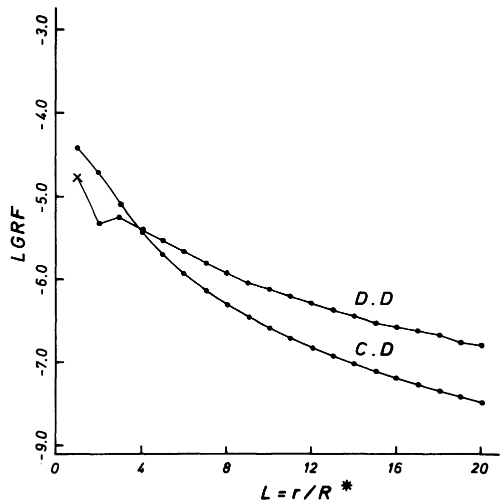


**Fig. 5.** Plot of the quantities LGRF "*C.D*" and LGRF "*D.D*" as a function of the variable $L$ in the case of a D.E. envelope. The parameters used are the same as those in Figure 4 with the exception that $l=1$

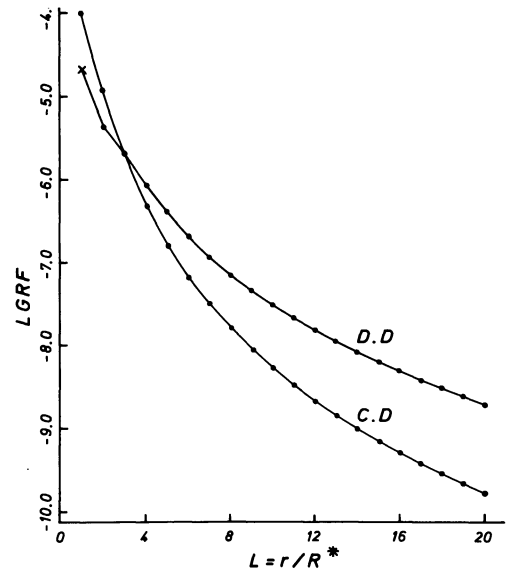


**Fig. 6.** Plot of the quantities LGRF "*C.D*" and LGRF "*D.D*" as a function of the variable $L$ in the case of a D.E. envelope. The parameters used are the same as those in Figure 4 with the exception that $l=2$

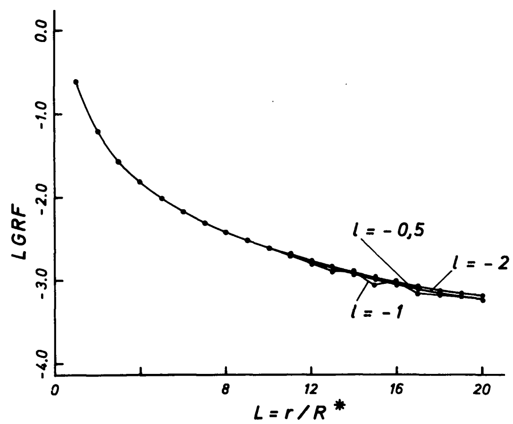


**Fig. 7.** Plot of the quantities LGRF as a function of the variable $L$ in the case of an A.E. envelope for different values of the accelerating parameter $l=-0.5$, $-1$ and $-2$, supposing that $g_1/g_2=1$, $n_0=10^2$ atoms/cm$^3$, $V_0=10^9$ cm/s, $R^*=6.96\ 10^{11}$ cm, $f_{12}=0.5$ and $\lambda_{12}=10^{-5}$ cm with $\lambda_{12}T=0.28979$ cm·deg

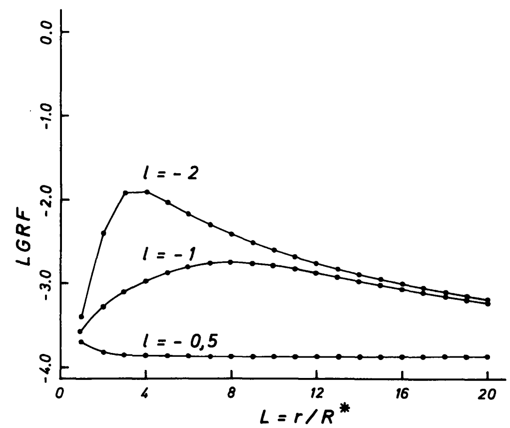


**Fig. 8.** Plot of the quantities LGRF as a function of the variable $L$ in the case of an A.E. envelope for the different values of $l=-0.5$, $-1$ and $-2$. The parameters used are the same as those in Figure 7 with the exception that $n_0=10^5$ atoms/cm$^3$ and $V_0=10^7$ cm/s

In the same way it can be easily shown that the radiative force $F_{12}^{DD}$ becomes independent of the line strength, in the optical thick case.

(iv) Between one and two stellar radii in D.E. envelopes, the radiative force due to distant atoms is directed radially towards the stellar core. Within the same distances the radiative force due to the absorption of stellar continuum is directly opposite and always greater in amplitude (see Figs. 1–6). The overall effect would remain that of a radial outward-acceleration. However, in that region of transition between the star and the envelope additional physical processes, namely entering the complex ejection phenomena, would play a more important role in the evaluation of the radiative force. We do not intend here to discuss this very difficult point.

(v) In A.E. envelopes, the "radiative force" function depends very much too on the physical and geometrical conditions existing in the medium. In Figure 7, because of the high transparency of the envelope ($n_0=10^2$ atoms/cm$^3$, $V_0=10^9$ cm/s...) the radiative forces do not differ, within the precision of computation, from the ones existing in an hypothetical transparent medium at rest. The variations of the "radiative force" function appearing in Figures 8 and 9 are entirely determined by both geometrical and physical dilutions of the stellar core radiation. The geometrical dilution factor $W$ [see Eq. (19)]

$$W=\frac{1}{2}\left(1-\left(1-\left(\frac{R^*}{r}\right)^2\right)^{1/2}\right),$$

is a decreasing function with the distance to the stellar center rather than the physical dilution factor $P_{ij}$ [see Eq. (19)] which accounts for the probability that a stellar photon will arrive unhindered at the point we evaluate its expression

$$P_{ij}=(1-\exp(-|\tau_{ij}|))/|\tau_{ij}|,$$

is an increasing function of $r$ in all A.E. envelopes. In Figure 7 the variations due to the geometrical dilution factor are much more important than those due to the physical one $P_{ij}$. In Figure 9 we have the reversed situation. Referring to the curve $l=-2$, the factor $P_{ij}$ varies between $L=1$ and $L=20$ by a ratio more than $5\ 10^3$ rather than the $W$ ratio is only about $8\ 10^2$. Finally, in Figure 8 we have the intermediate case where both factors $W$ and $P_{ij}$ show independently some predominance between $L=1$ and $L=20$.

(vi) When considering a $n$-level atom model the total radiative force acting on an atom would be of course given by the summation of radiative forces due to each line transition.

## 5. Conclusions

Lucy and Solomon (1970) proposed a mechanism to explain the mass loss observed for luminous, hot stars. They showed that the ultraviolet resonance lines of abundant ions can give strongly negative effective gravities in the outer parts of the reversing layers of hot stars. Castor et al. (1975) have also discussed the force exerted on the stellar material as a result of absorption and scattering of line radiation. They found that the fact of taking into account the large number of subordinate lines of a representative ion has a dominant effect on the force of radiation acting on material in O star atmospheres. All these theories require outward-accelerating envelopes, the effective gravities being negative in those expanding media.

It is very likely to encounter outward-decelerating envelopes around some objects, stars, etc. The physical picture can be that of a star ejecting matter by some adequate mechanism and later slowed down under the law of gravity. Hydrodynamics of that problem could be now entirely solved by taking into account the transfer of line radiation and the radiative force expressions in D.E. envelopes developed in this work and in Paper I. This would be of the greatest interest in order to answer some questions such as: Would the radiative forces created by different D.E. envelopes let always the decelerating flow stable? Would the line profiles formed in such media be distinct from those formed in A.E. flows? Etc.

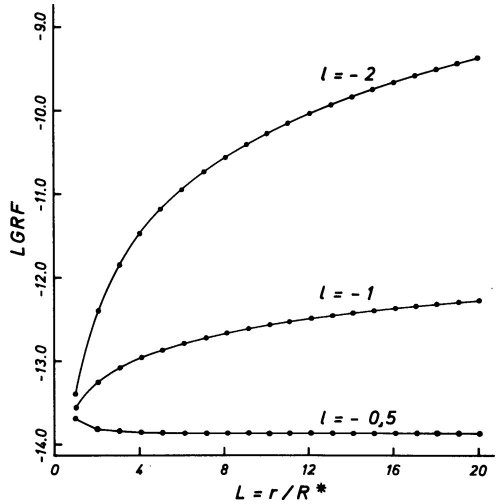


**Fig. 9.** Plot of the quantities LGRF as a function of the variable $L$ in the case of an A.E. envelope for the different values of $l=-0.5$, $-1$ and $-2$. The parameters used are the same as those in Figure 7 with the exception that $n_0=10^{15}$ atoms/cm$^3$ and $V_0=10^7$ cm/s

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