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Determination of mass-loss rates from early-type stars on the basis of "$\log(W_1)-\log(W_1^0)$" diagrams

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Summary. Using realistic expressions for the velocity $v(r)$ and opacity $\tau^r_{12}(X')$ distributions in rapidly expanding atmospheres, we present numerical results for the first order moment $W_1 \propto \int (E(\lambda)/E_c - 1)(\lambda - \lambda_{12}) d\lambda$ of a P Cygni line profile calculated as a function of the parameter $W_1^0 \propto \dot{M}\bar{n}$(level), where $\dot{M}$ represents the mass-loss rate and $\bar{n}$(level) the average fractional abundance of the relevant ion. Our calculations clearly show that the resulting "$\log(W_1)-\log(W_1^0)$" curves depend almost essentially on $\tau^r_{12}(X')$ and that for unsaturated P Cygni line profiles the relation $W_1 \propto \dot{M}\bar{n}$(level) holds irrespective of $v(r)$ and $\tau^r_{12}(X')$. For $W_1 \gtrsim 0.24$, the line profiles become saturated and the first order moment W_1 does not provide anymore accurate information on the mass-loss rate. This technique of mass-loss determination and that which consists in fitting observed line profiles with theoretical ones are compared.

Key words: radiative transfer – Sobolev approximation – mass-loss – P Cygni profile – first order moment

1. Introduction

Castor et al. (1981, referred to below as CLS) have first established that, in the framework of the Sobolev approximation (Sobolev, 1947, 1957, 1958) and for the case of optically thin lines, the first order moment $W_1 \propto \int (E(\lambda)/E_c - 1)(\lambda - \lambda_{12}) d\lambda$ of a P Cygni profile provides a good mean for deriving the mass-loss rate $\dot{M}$ of a central object. Although CLS have applied this technique for specific distributions of the velocity and line opacity, Surdej (1982) has shown that a unique relation does actually exist between W_1 and the quantity $\dot{M}\bar{n}$(level) – $\bar{n}$(level) representing the average fractional abundance of an ion in the lower atomic level associated with the given line transition – irrespective of various physical (collisions, limb darkening, etc.) and geometrical (velocity law $v(r)$, rotation, etc.) conditions prevailing in the expanding atmosphere. Furthermore, it was subsequently demonstrated (Surdej, 1983) that the relation $W_1 \propto \dot{M}\bar{n}$(level) is also independent of any Sobolev-type approximations used for the transfer of line radiation.

Adopting realistic expressions for the velocity and opacity distributions, we illustrate for the case of supersonic flows the results of various numerical applications showing the dependence of W_1 vs. the quantity $W_1^0 \propto \dot{M}\bar{n}$(level). The goal of these calculations is at least three-fold:

(i) to provide star observers the possibility of deriving the quantity $\dot{M}\bar{n}$(level) from a "$\log(W_1)-\log(W_1^0)$" diagram on the basis of an observed value for the first order moment W_1 of a P Cygni line profile. This method is much more flexible than the tedious comparison of an observed line profile with theoretical ones (Castor and Lamers, 1979);

(ii) to assign an error estimate to the mass-loss rate determination;

(iii) to determine the critical value W_1^* such that for $W_1 > W_1^*$ the first order moment ceases to be a sensitive mass-loss indicator, i.e. the P Cygni line profile is saturated.

2. Models for the expanding atmosphere

Estimates of mass-loss rates for early-type stars are usually derived from a comparison between observed and theoretical P Cygni line profiles. The latter are calculated under the assumption of resonance scattering and make use of the Sobolev approximation for the transfer of line radiation (cf. "An atlas of theoretical P Cygni profiles" by Castor and Lamers, 1979). In this context, the model for the expanding envelope is essentially characterized by two functions: the velocity distribution $v(r)$ and the radial optical depth $\tau^r_{12}(v)$. Empirical models based on this formulation clearly indicate that preferential velocity and opacity distributions (see the compilation in Table 1) lead to the best model fittings of observed P Cygni profiles, specially those associated with resonance lines in the ultraviolet spectrum of early-type stars (Castor and Lamers, 1979; Olson and Castor, 1981; Garmany et al., 1981). Therefore, in the remainder of this paper we shall adopt these same velocity and opacity distributions in order to calculate realistic values for the first order moment W_1 of P Cygni profiles.

As usually (see Table 1), R^* denotes the radius of the stellar core and v_0, v_∞ represent the initial and terminal velocities of the flow. $k_\alpha, \ldots k_\eta$ are constants related to atomic, physical and stellar parameters. Let us note that among the eighteen possible models inferred from Table 1, case $(C \cdot \alpha)$ refers to a radiation-driven atmosphere (cf. Castor et al., 1975) in which there is mass conservation of the relevant species.

In the calculation of W_1, we assume that the stellar core emits a continuum that is flat over the frequency range of the line profile, with no photospheric absorption line and no limb darkening. The effects of photospheric absorption and limb darkening have been discussed by Surdej (1982).

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Table 1. Velocity and opacity distributions adopted in the present work for calculating the first order moment W_1 of P Cygni line profiles

(A)	$v(r) = v_o + (v_\infty - v_o)(1-\sqrt{R^*/r})$
(B)	$v(r) = v_o + (v_\infty - v_o)(1-R^*/r)$
(C)	$v(r) = v_\infty\sqrt{1-(1-(v_o/v_\infty)^2)R^*/r}$
(α)	$\tau^r_{12}(v) = k_\alpha/(r^2 v \frac{dv}{dr})$
(β)	$\tau^r_{12}(v) = k_\beta(1-v(r)/v_\infty)$
(γ)	$\tau^r_{12}(v) = k_\gamma$
(δ)	$\tau^r_{12}(v) = k_\delta\sqrt{1-v(r)/v_\infty}$
(ε)	$\tau^r_{12}(v) = k_\varepsilon(1-v(r)/v_\infty)^2$
(η)	$\tau^r_{12}(v) = k_\eta/v(r)$

3. The first order moment W_1

Following CLS, the first order moment of a P Cygni line profile is defined as

$$W_1 = \int_{-1}^{1}\left(\frac{E(X)}{E_c}-1\right)x\,dx\,, \tag{1}$$

where

$$X = X'\cos(\theta)\,, \tag{2}$$

with

$$X' = -v(r)/v_\infty\,, \tag{3}$$

refers to the dimensionless frequency of a line photon, emitted at a radial distance r along a direction $\mu=\cos(\theta)$, with respect to a distant observer. $E(X)/E_c$ represents the line profile function in the frequency interval $X\in[-1,1]$.

Surdej (1982, hereafter Paper I) has shown that Eq. (1) can be conveniently reduced to the form

$$W_1 = \int_{-X_{\min}}^{1} 4L^2(X')\tau^r_{12}(X')\gamma_{12}(X')\left[1-\beta^3_{12}(X')/\beta^1_{12}(X')\right]x'dx'\,, \tag{4}$$

where $L(X')$ expresses the radial distance r in stellar radii R^* units. For the velocity distributions in Table 1, we have

$$X_{\min} = -v_0/v_\infty \tag{5}$$

and let us recall [cf. Eqs. (I.11), (I.60), and (I.73)[1]] that the escape probabilities β^1_{12}, β^3_{12} and γ_{12} are averages of quantities essentially depending on the fictive opacity

$$\tau_{12}(X',\mu) = \tau^r_{12}(X')\Big/\left(\mu^2\left(1-\frac{d\ln L(X')}{d\ln X'}\right)+\frac{d\ln L(X')}{d\ln X'}\right). \tag{6}$$

1 I.e. Eqs. (11), (60), and (73) in Paper I

For the case of optically thin lines (see Surdej, 1983, hereafter Paper III), Eq. (4) reduces to

$$W_1^0 = \frac{\pi e^2}{mc} f_{12}\lambda_{12}\frac{A(\text{element})\dot{M}\bar{n}(\text{level})}{4\pi\bar{\mu}M_{\text{amu}}v_\infty^2 R^*}q^c(L_{\max})\,, \tag{7}$$

with

$$q^c(L_{\max}) = \int_1^{L_{\max}}(W(L)-1)\frac{dL}{L^2}\,, \tag{8}$$

and

$$\bar{n}(\text{level}) = \int_1^{L_{\max}}(W(L)-1)\frac{n(\text{level})}{L^2}dL/q^c(L_{\max})\,, \tag{9}$$

where $W(L)$ is the geometrical dilution factor, $L_{\max}$ the maximum size of the expanding atmosphere and A(element) the abundance of the relevant element. All other symbols in Eq. (7) have their usual meaning.

Noticing [see Eq. (I.19)] that in terms of atomic and stellar parameters we have

$$\tau^r_{12}(X') = \frac{\pi e^2}{mc}f_{12}\lambda_{12}\frac{A(\text{element})\dot{M}n(\text{level})}{2\pi\bar{\mu}M_{\text{amu}}v_\infty^2 R^*}\frac{d(1/L)}{d(X'^2)}\,, \tag{10}$$

it is easy to rewrite the general expression of the fictive radial opacity as

$$\tau^r_{12}(X') = \frac{2W_1^0 n(\text{level})}{q^c(L_{\max})\bar{n}(\text{level})}\frac{d(1/L)}{d(X'^2)}\,. \tag{11}$$

From this result and Eq. (4) we conclude that for a given model in Table 1, the first order moment W_1 of a P Cygni line profile is a function of the unique parameter W_1^0, assuming of course that the values of $X_{\min}$ and $L_{\max}$ are known.

4. "Log(W_1)−log(W_1^0)" diagrams

Since the best theoretical fittings of observed P Cygni line profiles suggest no truncation of the expanding atmosphere, i.e. $L_{\max}\gg 1$, and a ratio $v_0/v_\infty \sim 0.01$ (Castor and Lamers, 1979; Garmany et al., 1981), we have performed extensive calculations of the first order moment W_1 [see Eq. (4)] assuming that $L_{\max}=1000$ and $X_{\min}=-0.01$. Considering separately the opacity distributions (α), (β), ..., (η) in Table 1, we have illustrated in Figs. 1–6 the results of these numerical applications for each of the three velocity distributions. The resulting diagrams represent the quantity $\log_{10}(W_1)$ as a function of $\log_{10}(W_1^0)$. 50 values of $\log(W_1^0)$ equally spaced in the range $[-3,3]$ have been used for constructing a single curve in Figs. 1–6. In order to assess an error estimate to the quantity $\dot{M}\bar{n}$(level) derived from a "$\log(W_1)-\log(W_1^0)$" diagram, we have superposed in Fig. 7 the results of the 18 model calculations.

5. Discussion and conclusions

A look at Figs. 2–6 first suggests that the "$\log(W_1)-\log(W_1^0)$" curves calculated for a same opacity distribution $\tau^r_{12}(X')$ are very little dependent on the type used for the velocity field $v(r)$. Recalling (see Paper I) that the relative error, expressed in percent, which results between the two determinations of W_1 when taking and not taking into account the finite size of the central source is

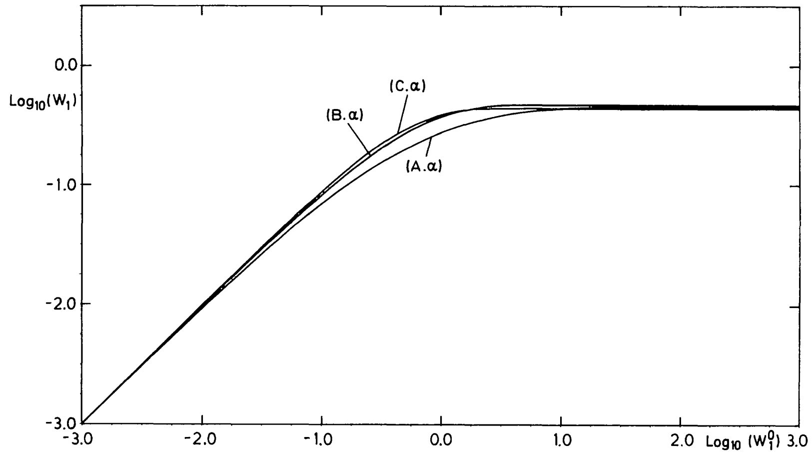

Fig. 1. "$\log(W_1) - \log(W_1^0)$" curves for the three models $(A\cdot\alpha)$, $(B\cdot\alpha)$, and $(C\cdot\alpha)$ (see Table 1)

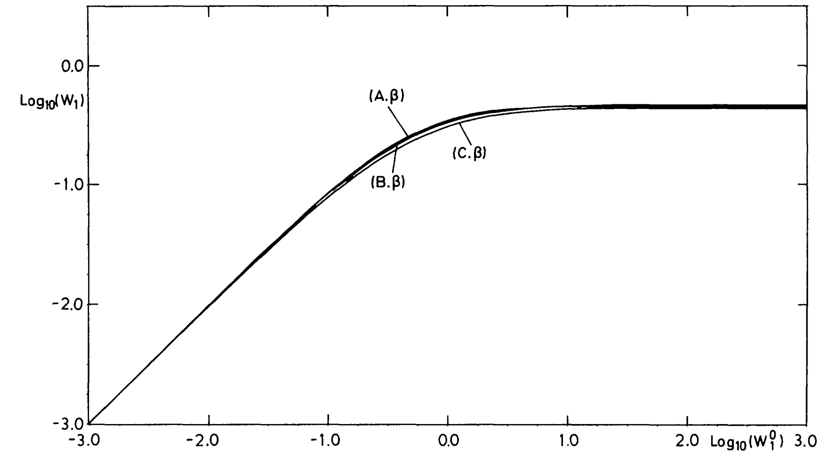

Fig. 2. "$\log(W_1) - \log(W_1^0)$" curves for the three models $(A\cdot\beta)$, $(B\cdot\beta)$, and $(C\cdot\beta)$ (see Table 1)

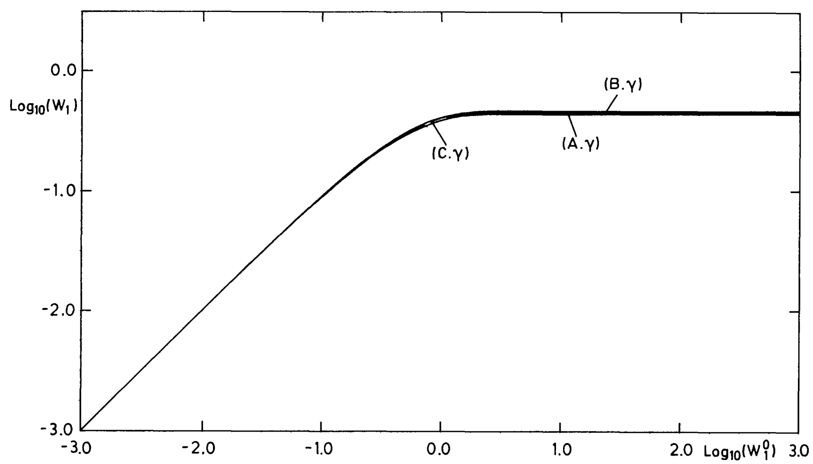

Fig. 3. "$\log(W_1) - \log(W_1^0)$" curves for the three models $(A\cdot\gamma)$, $(B\cdot\gamma)$, and $(C\cdot\gamma)$ (see Table 1)

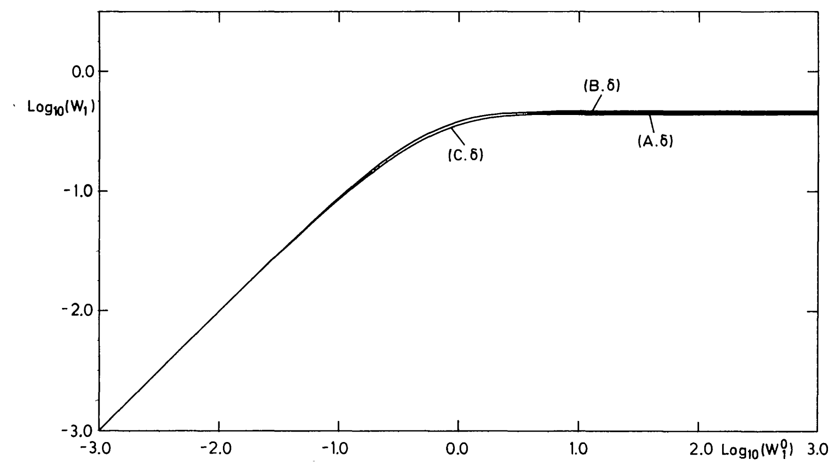

Fig. 4. "$\log(W_1) - \log(W_1^0)$" curves for the three models $(A\cdot\delta)$, $(B\cdot\delta)$, and $(C\cdot\delta)$ (see Table 1)

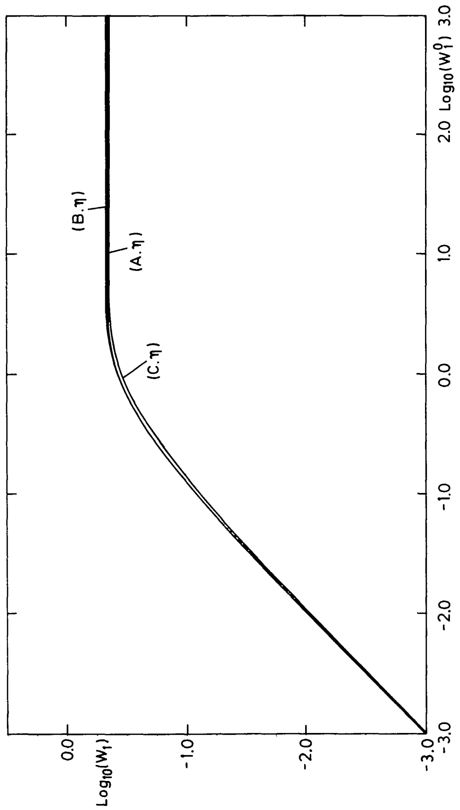

Fig. 6. "$\log(W_1)-\log(W_1^0)$" curves for the three models $(A\cdot\eta)$, $(B\cdot\eta)$, and $(C\cdot\eta)$ (see Table 1)

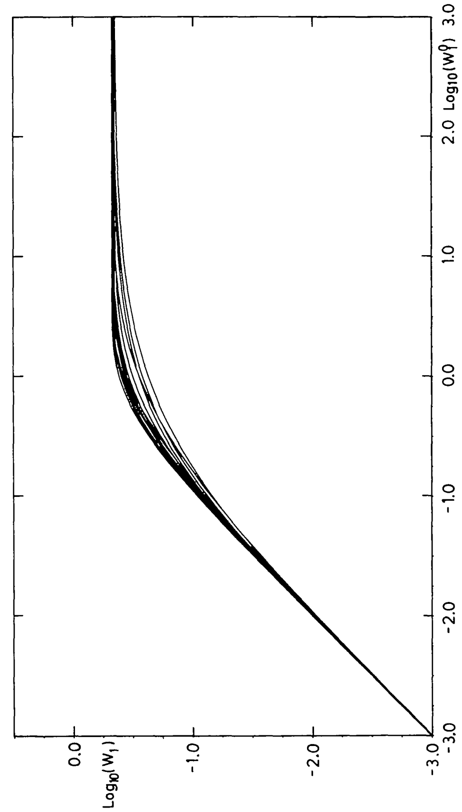

Fig. 7. Superposition of the eighteen "$\log(W_1)-\log(W_1^0)$" curves shown in Figs. 1–6

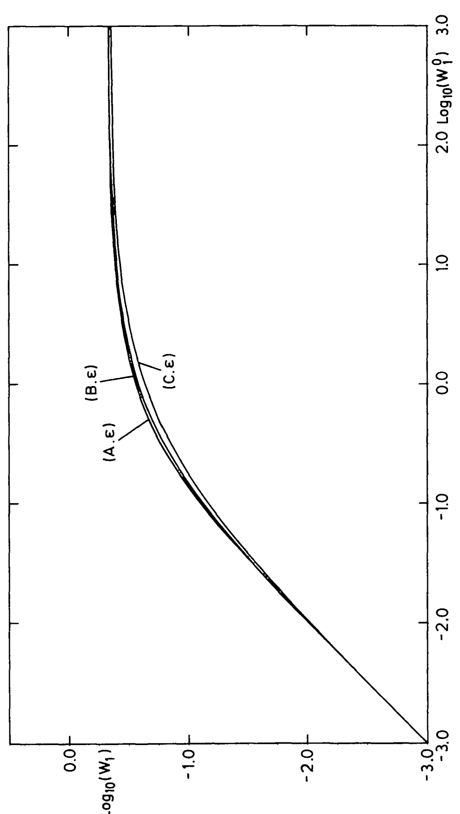

Fig. 5. "$\log(W_1)-\log(W_1^0)$" curves for the three models $(A\cdot\varepsilon)$, $(B\cdot\varepsilon)$, and $(C\cdot\varepsilon)$ (see Table 1)

of the order of 10% and that, within the "point-like star" approximation, we have

$$W_1=\int_{-X_{\min}}^{1}(1-\exp(-\tau^r_{12}(X')))X'dX', \tag{12}$$

i.e., the first order moment W_1 does only depend on the opacity distribution $\tau^r_{12}(X')$, the very slight deviations observed in Figs. 2–6 between each of the three "$\log(W_1)-\log(W_1^0)$" curves are directly interpreted. For the calculations presented in Fig. 1, the assumption of mass conservation in the flow implies distinct opacity distributions $\tau^r_{12}(X')$ for each of the velocity fields $v(r)$. It results that the "$\log(W_1)-\log(W_1^0)$" curves differ appreciably between each other. Let us still insist that for a known or given opacity distribution $\tau^r_{12}(X')$, Figs. 2–6 offer the possibility of deriving the quantity $\dot{M}\bar{n}$(level), almost irrespectively of the velocity distribution $v(r)$.

As expected, Fig. 7 clearly shows that all "$\log(W_1)-\log(W_1^0)$" curves are characterized by a common linear branch – i.e. $W_1=W_1^0$ – for small values of $\log(W_1^0)$. It is this trend of variation that was predicted in Paper I. Furthermore, the possibility of deriving the quantity $\dot{M}\bar{n}$(level) from the analysis of an unsaturated P Cygni line profile was generalized to the case of arbitrary (e.g. non-Sobolev type) velocity fields. Let us recall the useful relation

$$\dot{M}(M_\odot/\mathrm{yr})\bar{n}(\mathrm{level}) = -1.19\,10^{-21}\frac{v^2_\infty(\mathrm{km\,s^{-1}})R^*(R_\odot)W_1^0}{f_{12}\lambda_{12}(10^3\,\text{Å})A(\mathrm{element})}, \tag{13}$$

derived in Paper II.

For increasing values of $\log(W_1^0)$, the P Cygni profiles become saturated. Using "$\log(W_1)-\log(W_1^0)$"curves, it is then only possible to infer a lower limit for the $\dot{M}\bar{n}$(level) quantity. The first order moment W_1 is anymore a sensitive mass-loss indicator. As $\log(W_1^0)\rightarrow\infty$, there results an asymptotic value $\log(W_1)\sim-0.32$.

Finally, assuming that the models compiled in Table 1 encompass most of realistic cases, we can estimate from the dispersion of the "$\log(W_1)-\log(W_1^0)$" curves in Fig. 7 the lower value W_1^* such that, due to the uncertainty in the choice of a model, the relative error affecting a mass-loss rate determination is about equal to 100%. We easily find that $W_1^*=0.24$.

6. Comparison between the "line profile fitting" and "first order moment" techniques for the determination of mass-loss rates

We think that it is very appropriate at this stage to compare the two principal techniques used for the determination of mass-loss rates on the basis of line profile analysis.

In the "line profile fitting" technique, the types of velocity and opacity distributions are obtained by matching observed profiles with theoretical ones, using for instance the atlas of Castor and Lamers (1979). By means of the following relation [cf. Eq. (10)]

$$\dot{M}n(\text{level})=\frac{2\pi\bar{\mu}M_{\text{amu}}v_\infty^2R^*}{\frac{\pi e^2}{mc}f_{12}\lambda_{12}A(\text{element})}\left(\tau_{12}^r(X')\frac{d(X'^2)}{d(1/L)}\right), \qquad (14)$$

one can then calculate the quantity $\dot{M}n$(level). When evaluating the right hand side of Eq. (14), the value of $L(X')$ corresponding to $X'=-0.5$ is generally chosen such that the quantity in parentheses depends very little on the accurate type of velocity field $v(r)$. Contrary to our results [see Eq. (13)], this implies that the fractional abundance n(level) in Eq. (14) does not represent an average value across the envelope but that calculated at $L(X'=-0.5)$. Consequently, in order to derive $\dot{M}$, n(level) should be evaluated at $X=-0.5$, a requirement that is never fulfilled.

Before concluding, let us point out some similarities between the two techniques. Indeed, Castor and Lamers (1979) have first reported that the absorption component of a P Cygni profile depends very strongly on the opacity distribution $\tau_{12}^r(X')$, but only very little on the velocity law $v(r)$ and that in the limit of very thick lines, the profiles are independent of $\tau_{12}^r(X')$. The reader will notice at once the striking analogy between these remarks and those discussed in the previous section for the case of the first order moment W_1.

In conclusion, it is clear that a compromise between the two mentioned techniques will lead to the best determination of a mass-loss rate. For the case of underresolved line profiles, the "first order moment" will be the only useful technique. For the case of unsaturated – but sufficiently resolved – P Cygni profiles, the "line profile fitting" technique will always provide a good estimate for the opacity and velocity distributions as well as for the mass-loss rate [n(level) being evaluated at $X=-0.5$!] characterizing the flow. In addition, the measurement of W_1 will then allow a straight determination of the quantity $\dot{M}\bar{n}$(level) from the relevant "$\log(W_1)-\log(W_1^0)$" curve in Figs. 2–6.

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