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Toward modelization of quark and gluon transversity generalized parton distributions

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Quark and gluon helicity flip generalized parton distributions (GPDs) encode the information on the nucleon structure in the transversity sector. In order to build a theoretically consistent phenomenological parametrization for these hadronic matrix element within the framework of the dual parametrization of GPDs (or with the equivalent approach of the $SO(3)$ partial waves (PW) expansion with the Mellin-Barnes integral techniques) we establish the set of combinations of parton helicity flip GPDs suitable for the expansion in the cross channel $SO(3)$ PWs.

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1. Introduction and preliminaries

The transversity partonic structure of hadrons constitutes a longstanding challenge for both theoretical and experimental studies [1]. The appealing feature of the transversity dependent sector is that due to the chiral-odd property of transversity quark distributions it turns out to be possible to clearly separate quark and gluonic contents. In view of the notorious experimental difficulties with accessing quark transversity distributions directly through inclusive dilepton production with transversely polarized beam and target [2], several detour approaches were proposed in the present day literature. Our strategy deals with the description of hard exclusive reactions within the collinear factorization approach in terms of transversity dependent generalized parton distributions (GPDs). The chiral-oddity of the quark helicity flip operator prevents the corresponding GPDs for contributing into photon or meson leptoproduction amplitudes at the leading twist. However, recently there were some attempts to circumvent this restriction [3, 4]. In particular, the transverse target spin asymmetries measured by COMPASS in vector meson exclusive leptoproduction [5] have been interpreted [4] as a signal for transversity quark contributions. On the contrary, the gluon sector does not suffer from any selection rules and gluon helicity flip GPDs appear at the leading twist level in amplitudes of various hard exclusive reactions, for instance in the deeply virtual Compton scattering (DVCS) $O(\alpha_s)$ contribution to the leptoproduction of a real photon. This contribution can be separated through a harmonic analysis [6]. Particularly, the (3ϕ) modulation of the interference contribution to the unpolarized beam-longitudinally polarized target asymmetry seen in the HERMES data [7] for the DVCS on a nucleon may call for a significant gluon transversity contribution.

The practical applications of quark and gluon helicity flip GPD formalism require the construction of flexible phenomenological parametrizations of these non-perturbative objects. Below we review the main points of Ref. [8] in which we established the crossed channel properties of parton helicity flip GPDs that are of direct importance for a theoretically consistent model building in the spirit of the double partial wave expansion of GPDs. The phenomenological side of this study is postponed for future publications.

Let us briefly specify our set of conventions. Throughout this paper we employ the usual GPD kinematical notations for the average momentum $P=\frac{1}{2}(p+p')$, t-channel momentum transfer $\Delta = p'-p$ and the skewness variable ξ. To the leading twist accuracy, the form factor decomposition of the non-forward nucleon matrix element of the quark tensor operator $\hat{O}_T^q$ [1] contracted with the appropriate projector involves 4 invariant functions [9]:

$$\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ixP^+\lambda}\langle N(p')|\bar{\Psi}(-\lambda n/2)i\sigma^{+i}\Psi(\lambda n/2)|N(p)\rangle = \frac{1}{2P^+}\bar{U}(p')\left[H_T^q i\sigma^{+i}+\tilde{H}_T^q\frac{P^+\Delta^i-\Delta^+P^i}{m^2}\right.$$
$$\left.+E_T^q\frac{\gamma^+\Delta^i-\Delta^+\gamma^i}{2m}+\tilde{E}_T^q\frac{\gamma^+P^i-P^+\gamma^i}{m}\right]U(p), \tag{1.1}$$

where n denotes the light-cone vector and the Latin index $i=1,2$ refers to the transverse spatial directions. Each of the four invariant functions H_T^q, $\tilde{H}_T^q$, E_T^q and $\tilde{E}_T^q$ depend on the usual GPD variables. Due to hermiticity and time reversal invariance, the four invariant functions are real

[1] Here and in eq. (1.2) we omit the Wilson gauge links by sticking to the light-cone gauge $A^+=0$.

valued. Moreover, one may check [9] that H_T^q, $\tilde{H}_T^q$, E_T^q are even functions of ξ while $\tilde{E}_T^q$ is an odd function of ξ.

Similarly, the parametrization of the nucleon matrix element of the appropriately projected gluon tensor operator $\hat{O}_T^g$ to the leading twist accuracy involves four invariant functions [9]:

$$\frac{1}{P^+}\int \frac{d\lambda}{2\pi} e^{ixP^+\lambda} \langle p'|\mathbb{S} G^{+i}(-\lambda n/2) G^{j+}(\lambda n/2)|p\rangle = \mathbb{S}\, \frac{1}{2P^+} \frac{P^+\Delta^j - \Delta^+ P^j}{2mP^+} \bar{U}(p') \Big[H_T^g i\sigma^{+i} + \tilde{H}_T^g \frac{P^+\Delta^i - \Delta^+ P^i}{m^2} + E_T^g \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^g \frac{\gamma^+ P^i - P^+\gamma^i}{m} \Big] U(p), \quad (1.2)$$

where the $\mathbb{S}$ symbol stands for the symmetrization in the two transverse spatial indices and removal of the corresponding trace. From the combination of hermiticity and T-invariance they are real valued. H_T^g, E_T^g, $\tilde{H}_T^g$ are even functions of ξ while $\tilde{E}_T^g$ is an odd function of ξ. Moreover, the C-invariance demands that H_T^g, E_T^g, $\tilde{H}_T^g$ and $\tilde{E}_T^g$ are even functions of x.

2. SO(3) partial wave expansion of quark and gluon GPDs with helicity flip

The realistic strategy for extracting GPDs from the data relies on employing of phenomenologically motivated GPD representations and simultaneous fitting procedures for the complete set of observable quantities. The clue for building up a valid phenomenological representation for GPDs is provided by implementation of the non-trivial requirements following from the fundamental properties of the underlying quantum field theory.

Historically, one of the first parametrizations for GPDs suitable for phenomenological applications - the famous Radyushkin double distribution Ansatz - was based on the double distribution representation for GPDs. It is employed within the extremely popular Vanderhaeghen-Guichon-Guidal (VGG) code [10] for the DVCS observables and saw some success in the description of the available data. The alternative way for building up of a GPD representation resides on the expansion of GPDs over a suitable orthogonal polynomial basis in order to achieve the factorization of certain variable dependence. The appealing possibility is to perform the expansion of GPDs over the conformal PW basis in order to achieve the diagonalization of the leading order evolution operator. Nowadays two main versions of such GPD representations are utilized in phenomenology: the one based on the Mellin-Barnes integral techniques [11] and the one using the idea of the Shuvaev-Noritzsch transformation [12]. It turns out extremely instructive to further expand the conformal moments over the basis of the t-channel SO(3) rotation group partial waves. In the context of the Shuvaev transform techniques the resulting GPD representation is known as the dual parametrization of GPDs [13]. Within the Mellin-Barnes integral techniques [11] this version of the conformal partial wave expansion is referred in the literature as the SO(3) partial wave expansion. Each version of the formalism employs a rather intricate mathematical apparatus, however, as argued in [14], these two approaches turn out to be completely equivalent.

Finding out the combinations of GPDs suitable for the t-channel SO(3) partial waves and the choice of the appropriate basis of the orthogonal polynomials represents an important task. For example, for the case of the unpolarized quark and gluon nucleon GPDs this kind of analysis gives rise to the so-called electric and magnetic combinations of GPDs [15]: $H^{E\,\{q,g\}} =$

$H^{\{q,g\}} + \tau E^{\{q,g\}}$; $\quad H^{M\{q,g\}} = H^{\{q,g\}} + E^{\{q,g\}}$, where $\tau \equiv \frac{\Delta^2}{4m^2}$. These combinations are to be expanded respectively in terms of $P_J(\cos\theta)$ and $P'_J(\cos\theta)$, with $P_J(\cos\theta)$ standing for the Legendre polynomials and θ referring to the t-channel scattering angle in the $N\bar{N}$ center-of-mass frame.

Following the receipt of sect. 4.2 of ref. [15], in order to identify the combinations of quark helicity flip GPDs suitable for the partial wave expansion in the t-channel partial waves, we consider the form factor decomposition of the N-th Mellin moments of quark and gluon helicity flip GPDs analytically continued to the cross channel ($t > 0$). Thus we are dealing with the form factor decomposition of the N-th Mellin moments of quark helicity flip $N\bar{N}$ generalized distribution amplitudes (GDAs). To find which partial waves contribute into the corresponding matrix elements, we compute the spin-tensor structures for spinors of definite (usual) helicity in the $N\bar{N}$ center-of-mass (CMS) frame using the explicit expressions for the nucleon spinors with definite ordinary helicity. Let us briefly review the main stages of the calculation for the case of quark helicity flip GPDs. We project out the combination of the matrix elements with definite helicity $J_3 = \pm 1$ of the corresponding operator:

$$\begin{aligned}&\langle N(p',\lambda')\bar{N}(-p,\lambda)|\hat{O}_T^{q+1,++\ldots+}|0\rangle \pm i\langle N(p',\lambda')\bar{N}(-p,\lambda)|\hat{O}_T^{q+2,++\ldots+}|0\rangle \\ &\equiv \langle N(p',\lambda')\bar{N}(-p,\lambda)|\hat{O}_T^{q+(1\pm i2),++\ldots+}|0\rangle. \end{aligned} \tag{2.1}$$

The combinations (2.1) possess definite phases depending on the azimuthal angle ϕ. Now one can decompose (2.1) in the partial waves with total angular momentum J. The θ dependence is governed by the Wigner "small-d" rotation functions $d^J_{J_3,|\lambda'-\lambda|}$. For the case $|\lambda'-\lambda| = 0$ (*i.e.* the aligned configuration of nucleon and antinucleon helicities[2] ($N^\uparrow\bar{N}^\uparrow$ or $N^\downarrow\bar{N}^\downarrow$)) and $J_3 = \pm 1$ one has to use

$$d^J_{\pm 1,0}(\theta) = (\pm 1)\frac{1}{\sqrt{J(J+1)}}\sin\theta\, P'_J(\cos\theta). \tag{2.2}$$

For the case when $|\lambda'-\lambda| = 1$ (*i.e.* the opposite helicity configuration of nucleon and antinucleon: $N^\uparrow\bar{N}^\downarrow$ or $N^\downarrow\bar{N}^\uparrow$) depending on the operator helicity $J_3 = \pm 1$ (2.1) is to be expanded in

$$d^J_{\pm 1,1}(\theta) = \frac{1}{J(J+1)}(1\pm\cos\theta)\left[P'_J(\cos\theta) + \cos\theta P''_J(\cos\theta) \mp P''_J(\cos\theta)\right]. \tag{2.3}$$

After the inverse crossing (2.1) back to the s-channel, within the DVCS kinematics $\cos\theta$ up to higher twist corrections becomes $\cos\theta \to \frac{1}{\xi\beta} + O(1/Q^2)$, where $\beta \equiv \sqrt{1-\frac{4m^2}{t}}$. At this stage we switch to massless hadrons so that we could consider hadron helicities as true quantum numbers thus making simple the crossing relation between the corresponding partial amplitudes (in particular excluding mixing). This implies setting $\beta = 1$ (which means systematically neglecting the threshold corrections $\sim \sqrt{1-\frac{4m^2}{t}}$). However, up to the very end we keep the non-zero mass within the Dirac spinors to keep the counting of independent tensor structures. Finally, we conclude that the following combinations of quark helicity flip GPDs are to be expanded in $P'_J(1/\xi)$:

$$\begin{aligned}&\tau\tilde{H}^q_T(x,\xi,\Delta^2) - \frac{1}{2}E^q_T(x,\xi,\Delta^2); \\ &-H^q_T(x,\xi,\Delta^2) + \tau\tilde{H}^q_T(x,\xi,\Delta^2) - \frac{1}{2}E^q_T(x,\xi,\Delta^2).\end{aligned} \tag{2.4}$$

[2]Note, that our hadron helicity labeling refers to the t-channel. Obviously, when crossing back to the direct channel the helicity λ is reversed.

while the combinations

$$H_T^q(x,\xi,\Delta^2)+\tau\tilde{H}_T^q(x,\xi,\Delta^2)\pm\tau\tilde{E}_T^q(x,\xi,\Delta^2) \tag{2.5}$$

are to be expanded in $P'_J(1/\xi)+\frac{1\mp\xi}{\xi}P''_J(1/\xi)$.

The gluon case can be considered according to the same pattern (see Sec.3.2 of Ref. [8]) giving rise to the combinations of matrix elements expanded in terms of the Wigner functions $d^J_{\pm2,0}(\theta)$, $d^J_{\pm2,1}(\theta)$ and $d^J_{0,0}(\theta)$.

The method also allows to work out the set of selection rules for the J^{PC} quantum numbers for the t-channel resonance exchanges contributing into the N-th Mellin moments of quark helicity flip GPDs. Due to the CPT invariance, this kind of J^{PC} matching in the cross channel automatically ensures the T invariance and the correct counting of the independent generalized form factors of the operator matrix element in the direct channel. The selection rules we establish coincide with those worked out with the general method of X. Ji and R. Lebed [16].

The alternative method to work out the set of quark and gluon helicity flip GPDs suitable for the partial wave expansion in the cross-channel partial waves consists in the explicit calculation of the cross channel spin-J resonance contributions into corresponding GPD. The advantage of this method is that it is fully covariant and allows to determine the net resonance exchange contributions into scalar invariant functions $H_T^{q,g}$, $E_T^{q,g}$, $\tilde{H}_T^{q,g}$, $\tilde{E}_T^{q,g}$. Within this approach the nucleon matrix element of the light-cone operator $\hat{O}$ is represented as an infinite sum of t-channel resonance exchange contributions. Symbolically it can be written in the following form:

$$\langle N(p')|\,\hat{O}\,|N(p)\rangle \sim \sum_{R_J}\sum_{\substack{\text{polarizations}\\ \text{of } R_J}} \frac{1}{\Delta^2-M^2_{R_J}}\times\underbrace{\langle N(p')R_J(\Delta)|N(p)\rangle}_{V_{R_JNN}\text{ eff. vertex}}\otimes\underbrace{\langle 0|\hat{O}|R_J(\Delta)\rangle}_{\text{Fourier Transform of DA of } R_J},$$

where M_{R_J} stand for the resonance masses and $\otimes$ denotes the convolution in the appropriate Lorentz indices. The resulting on-shell polarization sums for spin-J resonances can be performed with the contracted projectors method (see *e.g.* Chapter I of [17]). This calculation allows to recover the same combinations of quark and gluon helicity flip GPDs suitable for SO(3) PW expansion in the t-channel. Moreover, as a byproduct, we build up a simple $f_2(1270)$ meson exchange model for gluon helicity flip GPDs that is similar to b_1 meson exchange model for quark helicity flip GPDs suggested in [18]. The relevant $f_2N\bar{N}$ coupling constants can be obtained from low energy NN scattering studies and the gluon helicity flip distribution amplitude normalization constant can be estimated as suggested in [19]. This model allows for the first time to work out the physical normalization of gluon helicity flip GPDs.

3. Conclusions

In order to construct a theoretically consistent parametrization of these hadronic matrix elements, we work out the set of combinations of the transversity GPDs suitable for the SO(3) PW expansion in the cross-channel. This universal result will help us to build up a flexible parametrization of these important hadronic non-perturbative quantities, using for instance the approaches based on the conformal PW expansion of GPDs such as the Mellin-Barnes integral or the dual parametrization techniques. We also propose a simple $f_2(1270)$ meson exchange model for gluon helicity flip

GPDs that allows to estimate the physical normalization of gluon transversity effects. This work is partly supported by the Polish Grant NCN No DEC-2011/01/B/ST2/03915, the Joint Research Activity "Study of Strongly Interacting Matter" (acronym HadronPhysics3, Grant 283286) under the Seventh Framework Programme of the European Community, by the COPIN-IN2P3 Agreement, by the French grant ANR PARTONS (ANR-12-MONU-0008-01) and by the Tournesol 2014 Wallonia-Brussels-France Cooperation Programme.

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