MODELS OF THE MULTIPERSON CHOICE PROCESS WITH APPLICATION TO THE ADOPTION OF INDUSTRIAL PRODUCTS

Jean-Marie Choffray
Gary L. Lilien

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ABSTRACT

Considerable progress has been made recently in the use of quantitative techniques to aid in marketing of consumer products. A comparable development has not occurred on the industrial side. In this paper, a model of industrial adoption of capital equipment is proposed. This model explicitly considers the process of group decision-making involved in the adoption of such products. The literature dealing with multi-person choice processes is reviewed. A set of criteria to be satisfied by descriptive models of this process are identified and four models of group choice are proposed: a weighted probability model, a proportionality model, a unanimity model, and an acceptability model. Finally, the general structure of an industrial marketing decision support system which incorporates such models is outlined, and major steps involved in its implementation are discussed.

I. INTRODUCTION

The last few years have witnessed considerable progress in the use of quantitative techniques in the marketing of consumer products. Urban's [27] PERCEPTOR model helps managers in creating and positioning new frequently purchased consumer products. Silk and Urban [26] propose a pretest market evaluation model along with measurement procedures for new packaged goods. New methods and structures to incorporate consumer perceptions and preferences into the design stage of product development have also been proposed by Hauser and Urban [10]. Later in the product life cycle, Little's [13] [14] BRANDAID model helps managers make marketing mix decisions in consumer markets. Other research in this field is referenced in the work noted above.

A comparable development has not occurred on the industrial side. Management science methods have been used to treat sales-force related problems (see Lodish [15]). Lilien and Little [12] have also done some early work in the industrial advertising area. But by and large, the same level of quantitative development has not been brought to problems of industrial marketing.

Why this lack of development?

Industrial marketing, the marketing of goods and services to organizations, involves the purchase of goods and services as varied as sulfuric acid, computer software, and nuclear power plants by organizations as varied as one-man businesses and AT&T. The range of products is much wider than in consumer markets. Perhaps more important, buying behavior is far more complex.

For capital equipment, a multi-person decision process is the normal mode of behavior. Several individuals, with different levels of knowledge, and different product-supplier perceptions, play the roles of users, influencers, buyers, deciders and gatekeepers in the organizational buying process (Webster and Wind [28], p. 75-87). We do not, now, have a methodology capable of characterizing who is involved in the various stages of this process and to what extent in a particular organization. Nor do we have a method of taking the perceptions and preferences of a number of individual decision participants and combining them into organizational purchase probabilities.

In this paper, we briefly discuss the general nature of organizational buying behavior for capital equipment. Next we review the literature surrounding the multiperson choice process. We then develop models which might be appropriate for describing it. Finally, we indicate how these models could be used within the context of an industrial marketing decision support system.

2. ORGANIZATIONAL ADOPTION PROCESS

The organizational adoption process differs from the consumer adoption process in many respects. First, organizational buying decisions usually involve several people representing different functions in the organization and who can play different roles. Second, industrial purchasing decisions, especially for capital equipment, tend to involve more technical complexities relating to the specific product being purchased. Third, the organizational adoption process can be disaggregated into basic phases more easily than the consumer adoption process, as in the

former case different organizational functions are usually associated with these decision stages. Finally, organizational decisions typically take longer to make. Consequently, there tend to be significant lags between the application of a marketing strategy and the buying response (see Webster and Wind [28], p. 5-8).

Figure 1 describes a conceptual model of the organizational adoption process for capital equipment. This model is based on the following two assumptions:

- organizations have actually recognized the need to buy a product from the generic class we are interested in, and
- the adoption decision, for this type of product, is the result of a systematic decision-making process.

Sheth [25] indicates that this last assumption might not be valid in all organizational purchasing situations, and stresses the influence that situational factors may have on the final decision. In the case of capital equipment, however, this assumption appears reasonable.

INSERT FIGURE 1 HERE

According to the model, environmental and organizational constraints influence the purchasing decision process by limiting the number of product alternatives of which decision participants are aware, and that can potentially satisfy the organizational needs. A recent example of an environmental constraint is the energy crisis. As to the second type of influence, it is well illustrated by organizational selection criteria such as: "eliminate suppliers who cannot deliver within x days or whose product's expected life falls below y years".

The resulting set of feasible alternatives constitutes the choice set of the organization, and specifies the domain over which individual perceptions and preferences are defined. It has been suggested that decision participants, who differ in their background and function in the organization, have different expectations and tend to develop their own product evaluation criteria [25] [28]. In this paper, we use the expression "decision participant category" to refer to those individuals who are involved in the adoption process and have essentially the same responsibility in their respective organizations.

The last element of this model links individual preferences to group preferences through some group interaction procedure. This accounts for the existence of group "decision rules" that organizations use, either explicitly or implicitly, to reach decisions.

In this paper, we concentrate on the last element of this model: the group selection of a specific alternative from the organization's feasible set. We will thus be concerned with models of multiperson decision-making with application to the adoption of industrial products. Our goal is not to explain, but rather to predict organizational responses to the industrial marketing mix by making formal assumptions about the kind of interaction that occurs among the different participants involved in this process. These models can be used to support product design and positioning, to help entry strategies and aid in marketing mix decisions.

3. PREVIOUS WORK

Research concerning models of multiperson decision-making has involved a variety of academic disciplines. We will distinguish three main schools of thought: Economics, Decision Analysis and Social Psychology. The assumptions that underlie the nature of individual and group choice behavior are different in these three schools.

3.1 Economics

Economists have long studied the preferences and choice behavior of economic "units" such as individuals, households and nations. Typically, however, they have been more concerned with individual preferences and choices than with groups.

Economists generally assume that an individual's preferences consist of a weak order relation defined over his choice set. Individuals are supposed to have complete information about all alternatives available to them. Their preferences are static over time and are uniquely determined by the observable and measurable characteristics of these alternatives. Moreover, it is assumed that individuals behave rationally; that is, they always choose the alternative they prefer most (see Newman [21] for a comprehensive discussion of these assumptions).

As to group preferences and choices, Economists essentially investigated the existence of social welfare functions. This problem can be stated as follows:

given a set of individual preference orderings and a group preference ordering does there exist any function that relates them while satisfying some reasonable assumptions about group functioning?

An important result, in this area, is Arrow's impossibility theorem [1]. He shows that the only aggregation procedures to pass from individual preference structures to a group preference structure which satisfies five reasonable conditions are either imposed or dictatorial.

A related question is investigated by Fishburn [8], who states conditions for a group ordinal preference structure to be written as the sum of monotone transformations of the ordinal individual references. More recently, Pattanaik [22] studied the existence of a socially best alternative under various group decision rules when individual preference orderings are lexicographic.

The study of individual and group choice by Economists is essentially axiomatic. For this reason it has been sometimes referred to as an "algebraic" theory of preference and choice behavior (Luce and Suppes [18]). As Newman writes, "rational behavior is neither good nor bad, foolish nor wise, beautiful nor monstrous, it is simply behavior which obeys the axioms," ([21], p. 17]. Different axiom systems of rational behavior could be proposed and would inevitably lead to different preference structure for both individuals and groups. For this reason, Economists' models of individual and groups choice are of little help to describe the actual choice behavior of individuals and groups.

3.2 <u>Decision Analysis</u>

Decision Analysis is concerned with the systematic analysis of decision-making under uncertainty. It provides decision makers with an analytical structure to make choices, when the consequences of the available alternatives are affected by uncertain events. The procedure consists

in independently assessing the utility of each of the possible consequences and quantifying the decision maker's feelings about the likelihood that these uncertain events will occur. A rational decision maker then chooses that specific alternative which maximizes his expected utility.

Attempts have been made to extend this approach to problems involving several decision participants (Raiffa [23], p. 220-38). The "group bayesians" consider that the behavioral assumptions implied by the decision analysis axioms of individual preferences and choice behavior are equally compelling when applied to a group acting as the decision making unit. So, a decision group should develop its own utility function, combine the individual subjective probability estimates of his members into group probability estimates, and choose the alternative which maximizes its expected utility.

As of now, neither of these two problems -- aggregation of individual utility functions and aggregation of individual probability estimates -- has been solved satisfactorily. Bacharach [2] proves that under a reasonable set of axioms, the group ranking of the alternatives cannot be arrived at by any rules for separately combining the probabilities and the utilities. Studying the problem of aggregation of probability estimates in the case of panels of experts, Wrinkler [30] suggests that several aggregation procedures should be used and their impact on the final decision be studied.

Keeney and Kirkwood [11] address the problem of constructing a group cardinal utility function whose arguments are the individual utility functions of group members. They show that under two reasonable conditions about group functioning, the group utility function has a multiplicative form. The assessment of the function parameters, however, itself calls

for a group decision whose solution is not evident.

The use of Decision Analysis to study group choice requires that a solution be found to these two important problems. The aggregation of individual utility functions is affected by interpersonal comparison of utilities. Although the axioms of modern utility theory, as developed by von Neumann and Morgenstern [20] do imply the existence of interval-scaled individual utility functions, these last do not possess a unique zero point and are expressed in terms of arbitrary units of measurement (Luce and Raiffa [17], p. 33-34).

Moreover the validity of some of the Decision Analysis axioms may be questioned when applied to group decision problems. This is specially true for the substitution principle which assumes independence between the assessment of utilities and the quantification of judgments about uncertain events (See Raiffa [23], p. 233-37).

In general, the interpretation of the Decision Analysis axioms raises problems even at the level of descriptive individual decision-making. This leads Coombs, Dawes and Tversky ([5], p. 124), to emphasize that the use of utility theory as a behavioral model "has to be supplemented by a psychological theory that accounts for situational variables that affect risky choices." In marketing, however, prescriptive utility theory has been used recently with some success to describe individual choice behavior (Hauser and Urban [9], [10]). From the previous discussion, it is not clear how this methodology could be extended to groups.

3.3 Social Psychology

The uncertainty with which individuals report preferences, and the inconsistency they exhibit when faced with choices involving complex alternatives led psychologists to develop models of choice where the traditional concept of preference is replaced by the notion of choice probabilities.

Probabilistic theories of preferences and choice differ with respect to the locus of the random element in the individual decision process (see Luce and Suppes [18] for a review). In random utility models, the utility values themselves undergo random fluctuations and the choice mechanism is completely deterministic. An individual is assumed to choose the alternative which has the highest momentary utility.

Constant utility models, on the other hand, consider that the decision rule itself is subject to randomness, while individuals' subjective evaluations of the alternatives are constant over time. Selection probabilities are then defined as functions of the utility associated with the choice alternatives. Luce's model of individual choice is an example of this approach [16].

Social psychologists have also developed descriptive models of group choice behavior by making formal assumptions about the type of interaction which takes place among its members. The theory of social decision schemes (Davis [6]) is concerned with the development and testing of such models. Its aim is to account for the distribution of group decisions when decision participants' choice probabilities are known. Up to now, it has centered mainly on task-oriented groups.

A social decision scheme may be viewed as an <u>approximation</u> to the actual interaction process which takes place within a group. It can be regarded as the way in which the group deals with all internal distributions of choices across its members.

The social decision scheme approach is concerned with modeling the group interaction as a combinatorial process. Assuming a set of mutually exclusive alternatives and letting the choice process of all decision participants be described by a probability mass function across these alternatives, one can compute the probability that the group members average themselves according to any particular internal distribution of choices. For any such internal distribution of choices the group decision scheme postulated is then used to yield the probability distribution of group choice.

This theory provides an intuitively appealing way to model group choice behavior. We will discuss later extensions of two simple decision schemes that have received empirical support (Davis [6]; Davis, Cohen, Hornik and Rissman [7]).

As it stands now, the social decision scheme theory is not a general theory of group decision-making. It makes no provisions for different types of tasks or stages in the decision process. Moreover, from a computational viewpoint, when the number of decision participants is larger than two, and they have different initial choice probabilities, it becomes increasingly difficult to compute group choice probabilities.

To summarize, Economists, Decision Analysts, and Social Psychologists have all faced the problem of modeling the group choice process. The approach used by Economists leads to models that are neither measurable nor descriptive of behavior, and thus are of little use in this context. Decision

analysts have made some progress. Important questions are left unanswered, however, and their work tends to be more of a prescriptive nature. Some of the approaches of Social Psychologists, suitably refined, might hold promise for modeling the organizational adoption process.

4. CRITERIA FOR MODELS CF ORGANIZATIONAL CHOICE

Definitive, useable models of group choice have not been developed so far. Our objective in the remainder of the paper is to first suggest some criteria that useable descriptive models of group choice should satisfy, suggest several models which meet those criteria and then outline a structure for their use in the context of an industrial marketing decision support system.

We do not intend to develop a model of organizational choice that would be applicable in all product and market situations. Rather, we suggest alternative ways to structure that process, based on different assumptions about the nature of group interaction. When faced with a particular problem, the industrial marketing manager will be asked to make use of his own experience in selecting a model whose structure best corresponds to the understanding he has of the organizational choice process, in specific market segments.

The following criteria were used to develop the models presented next:

- <u>Understandable</u>: The general nature of the model must be easy to grasp by those involved in industrial marketing activities.
- Extendable: The model structure must be broad enough to handle a varying number of decision participants as well as a varying number of choice alternatives.

- <u>Probabilistic</u>: Even if the preferences of all decision participants were completely deterministic, our current knowledge about group decision processes is so limited that at best we can expect to reproduce group choice probabilistically.
- Heterogeneous at the Individual Level: Decision participants have different backgrounds, expectations and product evaluation criteria. Consequently they can have different choice probabilities for product alternatives. Models of organizational choice must recognize this and allow for heterogeneous individual choice probabilities.

To develop models of the organizational choice process for use in industrial marketing decision making, we consider the adoption process at the individual level first, and then postulate the existence of a group interaction procedure at the organizational level. We introduce the following assumptions. Some of them may appear restrictive, given the complexity of this process, but we are confident that in future work the model structure will be expanded and some assumptions relaxed.

Our assumptions are:

- Organizations recognize the need to purchase a product from the class to which the alternative we are interested in belong. Moreover, we assume for simplicity that this product falls in each organization's feasible set of alternatives.
- 2. The organizational adoption of a specific product from this class is the result of a systematic decision process. This assumption is reasonable for most capital goods but might be simplistic in cases where the organizational choice is driven by situational factors [25].

- 3. Decision participants in the organizational adoption process can be identified and grouped on the basis of their responsibility in their respective organization. Such "decision participant categories" might include for instance: production engineers, plant managers, financial controllers, etc. Moreover, we assume for simplicity that choice probabilities are the same for all individuals who belong to a specific category.
- 4. Microsegements can be formed in the potential market by grouping organizations homogeneous in the structure of their adoption process. This assumption implies that within any microsegment the same categories of decision participants are involved in the choice process for all companies.

Assumptions 3 and 4 are required for estimation purposes and for measurement of the aggregate response at the market level. Choffray [4] is currently investigating them and developing methods for explicitly segmenting industrial markets in this way.

In the next pages, we make use of the following notation:

- D_i, i=1,...r refer to the various categories of decision participants who are involved in the organizational choice process for the product under investigation.
- d_i refer to any individual belonging to category D_i .
- A = $\{a_j, j=0, \dots n\}$ stands for the feasible set of alternatives, and defines the domain of individual and organizational preferences.

- An individual's preference structure is denoted θ_k . It is a weak ordering of the choice alternatives of the type $(a_0 \ge a_1 \ge \dots \ge a_n)$ which reflects his personal preferences for the alternatives. Given (n+1) alternatives, there are exactly (n+1)! possible preference structures.
- $P_i(\theta|A)$ is the distribution of preferences for decision participant d_i . It assigns a probability $P_i(\theta_k|A)$ to each preference structure θ_k .

Hence we have:
$$0 \le P_{\mathbf{i}}(\theta_k|A) \le 1 \qquad \text{for all } \theta_k, \quad \text{and}$$

$$\sum_{\forall \theta_k} P_{\mathbf{i}}(\theta_k|A) = 1 \qquad \text{for any d}_{\mathbf{i}}$$

- We then define the probability that d_i actually chooses a_j from A, denoted $P_i(a_j;A)$, as equal to the probability that a_j is his most preferred alternative. Hence we get

$$P_{\mathbf{i}}(a_{\mathbf{j}}; A) = \sum_{\theta_{\mathbf{k}}} P_{\mathbf{i}}(\theta_{\mathbf{k}} | A)$$

where the summation is taken over all preference structures $\theta_{\boldsymbol{k}}$ in which a_j is most preferred. Hence, we have:

$$0 \le P_i(a_j; A) \le 1$$
 for all a_j and
$$\sum_{j=0}^{n} P_i(a_j; A) = 1$$
 for any d_i .

5. DESCRIPTIVE MODELS OF THE GROUP CHOICE PROCESS

Four models are discussed. Each corresponds to different assumptions

about the nature of the choice process. We distinguish a Weighted Probability Model,
a Proportionality Model, a Unanimity Model and an Acceptability Model.

All of them are proposed for a typical organization of an unspecified

microsegment of the potential market.

5.1 Weighted Probability Model

The Weighted Probability model assumes that the group, as a whole, is likely to adopt a given alternative, say $a_0 \in A$, proportionally to the relative importance of those members who choose it.

To keep the notation simple,

1et

$$P_G(a_j;A)$$
 = probability that the group chooses a_j w_i = relative importance, on the average, of decision participant d_i , i=1, ...r in the choice process. So,
$$\sum_{i=1}^r w_i = 1 \cdot \sum_{i=1}^r w_i$$

Then the weighted probability model postulates that

[1]
$$P_G(a_0; A) = \sum_{i=1}^{r} w_i P_i(a_0; A).$$

We can interpret [1] as a two-step sampling process where in step one, the organization samples a decision maker from the set of decision participants proportionally to each participant's relative importance in the choice process. In step two, the sampled decision maker selects an alternative according to

his own choice probabilities.

There are two interesting special cases of the weighted probability model:

- (a) Autocracy: If $w_e = 1$, then all other $w_{i\neq e} = 0$, then a single decision participant, d_e is responsible for the group choice.
- (b) Equiprobability: If w_i =1/r, for all i, then every decision participant has equal weight in the process. This is an appealing model as it is sort of a zero-information or naive model. The industrial marketing manager need only identify the decision participants and does not have to measure or provide subjective estimates of the importance coefficients.

The equiprobability form of the weighted probability model has received some empirical support, both in dyadic decision making (Davis, Cohen, Hornik and Rissman [7]) and in group decisions involving more than two participants (Davis [6]). Moreover, the model was found to accurately describe group risk shifts (Davis [6], p. 108).

One must be careful, however, in interpreting these results. Indeed, although the cumulative frequencies of actual group decisions were reproduced accurately by the equiprobability model, these experiments mainly involved ad-hoc groups whose members had little experience in working together. The equiprobability model might then be a reasonable approximation to organizational choice behavior in situations that involve decision participants from different departments who are not accustomed to working together.

5.2 A Numerical Example

It may be easier to understand these models through the use of a numerical example.

Consider an organization with three decision participants d_1 , d_2 , d_3 and three alternatives $A = \{a_0, a_1, a_2\}$. Assume their respective distribution of preferences are given by:

TABLE 1

vilaneti vino Pre	eference Dist	tributions I	$\frac{\theta_k A}{\theta_k}$
θ _k mane som og s	$P_1(\theta_k A)$	$P_2(\theta_k A)$	$P_3(\theta_k A)$
$\theta_1 = a_0 \ge a_1 \ge a_2$.13	.06	. 24
$\theta_2 = a_0 \ge a_2 \ge a_1$	dedoi9 bean	. 24	.46
$\theta_3 = a_1 \ge a_0 \ge a_2$	alba.22 bayl	.13	. 04
$\theta_4 = a_{1-2-0}^{>a}$.28	.07	.16
$\theta_5 = a_2 \stackrel{>}{_{-}} a_0 \stackrel{>}{_{-}} a_1$.18	.19	.08
$\theta_6 = a_2 \ge a_1 \ge a_0$.12	.31	.02

Following our definition in section four, we compute individual choice probabilities:

TABLE 2

Individual Choice Probabilities P_i(a_j;A)

a j	$P_{1}(a_{j};A)$	P ₂ (a _j ;A)	P ₃ (a _j ;A)
a ₀	. 2	: 3	. 7
a	:5	. 2	. 2
a 2	.3	. 5	d. 0629 • 16

Then $P_G(a_0;A)=.2w_1+.3w_2+.7w_3$. An equiprobability model with $w_1=1/3$ i=1,2,3 will yield $P_G(a_0;A)=.4$. An autocratic model with $w_1=1$ will yield $P_G(a_0;A)=.2$; with $w_3=1$ will yield $P_G(a_0;A)=.7$. These are upper and lower bounds on $P_G(a_0;A)$ for the weighted probability model. In terms of our example: $.7 \geq P_G(a_0;A) \geq 2$.

5.3 The Proportionality Model

The Proportionality Model attributes the same weight to all individuals involved in the group decision process. It states, however, that the probability $P_G(a_0;A)$ that the group will choose alternative a_0 is proportional to the <u>number</u> of decision participants who individually choose it.

Let
$$x_{ij} = 1$$
 if d_1 chooses a_j ,

so,
$$Pr(x_{ij}=1) = P_i(a_j;A)$$

The number of decision participants who choose a is then given by:

$$z = \sum_{j=1}^{r} x_{ij}$$

Also, let Ψ denote any internal distribution of choices of the type z_0, z_1, \ldots, z_n in which exactly z_n individuals choose alternative z_n . Hence, the proportionality model states that

[2]
$$P_G(a_0; A) = \frac{1}{r} \left\{ \sum_{z_0=0}^{r} Z_0 Pr[\Psi_{z_0}] \right\}$$

where $\Pr[\Psi]$ stands for the probability of getting an internal distribution of choices Ψ_{z_0} , in which exactly z_0 decision participants chose alternative a_0 , given their individual choice probabilities $\Pr(a_j;A)$. For any value of z_0 , there exists exactly $\binom{n+r-z_0}{r-z_0}$ such internal distributions of choices, corresponding to the number of $(r-z_0)$ -selections of the (n+1)-set of alternative A. (See Ryser [24], p. 9).

The proportionality model states that whenever group decision participants disagree as to the alternative to be adopted, the group adopts a given alternative with a probability proportional to the number of its members who support that alternative. This model has received empirical support in conditions similar to those used to test the equiprobability model (Davis [6]). Although the equiprobability model was found to be a little more accurate than the proportionality model in accounting for the risk shift phenomenon, Davis concludes that, "Clearly, larger groups, and/or tasks with more alternatives are required in order to distinguish between the equiprobability and proportionality models for risk-taking tasks " ([6], p. 108).

As an example of the Proportionality Model consider the choice probabilities in Table 1 again. Here we get:

TABLE 3

Internal Distribution of Choices Ψ .		ibution	Probability of Occurrence Pr(Y.)		Number of Decision Participants Supporting a.		
^d 1	$\frac{d}{2}$	<u>d</u> ₃		<u>a</u> 0	<u>a</u> 1	$\frac{a}{2}$	
a ₀	a ₀	^a 0	.042	3	0	0	
a ₀	a ₀	a ₁	.012	2	1	0	
a ₀	^a 0	a ₂	.006	2	0	1 .	
a ₀	a ₁	a ₀	.028	2	1	0	
a ₀	a ₁	a ₁	.008	1	2,	0	
a ₀	a ₁	a ₂	.004 - BAG.	1	1	1	
^a 0	a ₂	a ₀	.070	2	0	1	
a ₀	a ₂	a ₁	.020	1	1	1	
^a 0	a ₂	a ₂	.010	1	0	2 300	
^a 1	^a 0	^a 0	.105	2	1	0	
^a 1	^a 0	a ₁	.030	1 + 800	2	0	
а 1	^a 0	a ₂	.015	1	1	1	
a ₁	^a 1	^a 0	.070	1	2	0	
a ₁	^a 1	a ₁	.020	0	3	0	
а ₁	a ₁	a ₂	.010	.0	2	arrest Light	
a ₁	a ₂	^a 0	.175	1 8 . =	$(\Lambda,\boldsymbol{1},)$	1	
a ₁	a ₂	^a 1	.050	0	2	1	
^a 1	a ₂	a ₂	.025	0	1	2	

TABLE 3 (Continued)

Internal Distribution of Choices $\boldsymbol{\Psi}$		Probability of Occurrence $Pr(\Psi.)$				Decision Par- Supporting a.		
<u>d</u> 1	<u>d</u> 2	<u>d</u> ₃		<u>a</u> 0		<u>a</u> 1		<u>a</u> 2
a ₂	a ₀	a ₀	.063	2		0		1
a ₂	a ₀	a ₁	.018	1		1		1
a ₂	a ₀	a ₂	.009	1		0		2
a ₂	a ₁	^a 0	.042	1		1	*	1
a 2	а ₁	a ₁	.012	0		2		1
a ₂	a ₁	a ₂	.006	0		1		2
a ₂	a ₂	^a 0	.105	1		0		2
a ₂	a ₂	a ₁	.030	0		1		2
a ₂	a ₂	a ₂	.015	0		0		3

Hence, we get:

$$P_{G}(a_{0};A) = \frac{1}{3} \{ [3 \times (.042)] + [2 \times (.012 + .006 + .028 + .070 + .105 + .063)] + [1 \times (.008 + .004 + .020 + .010 + .030 + .015 + .070 + .175 + .018 + .009 + .042 + .105)] \}$$

$$= .4$$

In the same way,

$$P_{G}(a_{1};A) = .3$$

 $P_{G}(a_{2};A) = .3$

An interesting extension of this model is the "voting" model which states that the probability $P_G(a_0;A)$ that the group will choose alternative a_0 is equal to the probability that a_0 is selected by the <u>largest number</u> of decision participants. In terms of our previous notation, this means:

[3]
$$P_G(a_0;A) = Probability [z_0 = \max_{j} (z_j)].$$

If ties occur, in which case several alternatives are supported by the same number of decision participants, i.e.

$$z_e = z_k = \max_j (z_j)$$
, for some $e \neq k$,

we assume that these alternatives are equally likely to be the group choice.

Considering the internal distributions of choices appearing in Table 3, we get

$$P_{G}(a_{0};A) = .326 + (\frac{1}{3} \times .274) = .417$$

$$P_{G}(a_{1};A) = .200 + (\frac{1}{3} \times .274) = .291$$

$$P_{G}(a_{2};A) = .200 + (\frac{1}{3} \times .274) = .291$$

A special case of the voting model occurs when we require $z_0 \ge (\frac{r}{2} + 1)$ or $z_0 \ge (r+1)/2$ according to whether the number of decision participants r is even or odd respectively. In this case, let $\Pr(z_j \ge z_{majority})$ represent the probability that the <u>majority</u> of decision participants choose a_0 . In terms of our previous example,

$$Pr(z_0 \ge 2) = .326$$

 $Pr(z_1 \ge 2) = .200$
 $Pr(z_2 \ge 2) = .200$

The group choice probabilities, conditional to the use of the majority rule are then given by:

$$P_{G}(a_{0};A) = .450$$

 $P_{G}(a_{1};A) = .275$

$$P_{C}(a_{2};A) = .275$$

Note that the proportionality model led to estimates of group choice probabilities $P_G(a_j;A)$ close to those obtained with the voting and majority models. This result is due to our specific example, however, which involves only three decision participants and three alternatives. For larger values of r and n, these three models can lead to quite different results.

5.4 The Unanimity Model

This model assumes that, in order to be accepted by the group, an alternative, say a_0 , has to be the actual choice of all decision participants involved in the choice process. Thus a group might, in theory, "vote" over and over again until unanimity is reached. Empirical studies of the industrial adoption process indicate that this model does capture some of the essence of the multi-person choice involved in this process. (Buckner [3], p. 18).

Formally, the unanimity model implies that

$$P_{G}(a_{0};A) = \frac{\prod_{i=1}^{r} P_{i}(a_{0};A)}{\prod_{j=0}^{r} \prod_{i=1}^{r} P_{j}(a_{j};A)}$$

assuming that individual preferences distributions are mutually independent. This is the $\underline{\text{conditional}}$ probability that the product \mathbf{a}_0 is selected, $\underline{\text{given}}$ that the group reached unanimity.

The unanimity model has an interesting property: The addition of a completely undecided decision participant to the group, does not affect the conditional probability $P_G(a_0;A)$ that the group will unanimously choose alternative a_0 .

Indeed, assume that the new decision participant is the (r+1)st. So, we get

[5]
$$P_{G}'(a_{0};A) = \frac{\prod_{i=1}^{r+1} P_{i}(a_{0};A)}{\prod_{j=0}^{n} \prod_{i=1}^{r+1} P_{i}(a_{j};A)}$$

As $P_{r+1}(a_0;A) = \frac{1}{n+1}$ for all $a_j \in A$, by assumption, we get

$$P_{G}'(a_{0};A) = \frac{\frac{1}{(n+1)} \prod_{i=1}^{r} P_{i}(a_{0};A)}{\frac{1}{(n+1)} \prod_{j=0}^{n} \prod_{i=1}^{r} P_{i}(a_{j};A)}$$
$$= P_{G}(a_{0};A)$$

As an example, consider again the probabilities in Table 2. Here, we get:

$$P_G(a_0; A) = \frac{.042}{.042 + .020 + .015}$$

In the same way:

$$P_{G}(a_{1};A) = .259$$

$$P_{G}(a_{2};A) = .195$$
.

It should be noted that the nature of the unaninimity model, i.e. requiring complete agreement, makes it a model of choice conditional to agreement. If agreement is not immediate, group members might update their probabilities. The unanimity model as structured here assumes stationary individual choice probabilities. Various models of updating such probabilities on the basis of past behavior have been suggested (see Massy, Montgomery and Morrison [19] for a review).

5.5 Acceptability Model

This model assumes that if a group does not reach unanimous agreement, it is more likely to choose the alternative which "perturbs" individual preference structures least. Suppose the following pattern of individual preferences holds in a group of two.

TABLE 4

Decision Participant	Preference Pattern	Probability of getting Pattern $\theta_{\mathbf{k}}$
d ₁	θ _{1e} :a ₀ >a ₁ >a ₂	P ₁ (θ _{1e} /A)
d ₂	$\theta_{2h}: a_2 \ge a_0 \ge a_1$	$P_2(\theta_{2h}/A)$

where θ_{ik} means that individual i exhibits a preference structure θ_{k} . Given the pattern of preference structures $\gamma_{eh} = \{\theta_{ie}, \theta_{2h}\}$, we define the perturbation $Q(a_{ij}|\gamma_{eh})$ associated with the choice of alternative a_{ij} as

the total number of preference shifts necessary for a to be everyone's first choice. In the above example, we get:

$$Q(a_o|\gamma_{eh}) = 1$$

$$Q(a_1|\gamma_{eh}) = 3$$

$$Q(a_2|\gamma_{eh}) = 2$$

Assuming that all preference shifts are strictly comparable, we have:

$$P_{G}(a_{0}|\gamma_{eh}) = 3 P_{G}(a_{1}|\gamma_{eh})$$
$$P_{G}(a_{0}|\gamma_{eh}) = 2 P_{G}(a_{2}|\gamma_{eh})$$

As
$$\sum_{j=0}^{n} P_{G}(a_{j}|\gamma_{eh}) = 1, \text{ we get}$$

$$P_{G}(a_{0}|\gamma_{eh}) = 6/11$$

$$P_{G}(a_{1}|\gamma_{eh}) = 2/11$$

$$P_{G}(a_{2}|\gamma_{eh}) = 3/11$$

Formally, given the distribution of preferences P_i ($\theta \mid A$) for each decision participant, we can compute the probability that a specific pattern of preference structures γ_w will occur across individuals. Using our previously defined notation, we get

$$\begin{aligned} \Pr[\gamma_w] &= & \Pr[\theta_{1k}, \theta_{2k}, \dots \theta_{rk}] \\ &= & \prod_{i=1}^r P_i(\theta_{ik}|A) & \text{where } w = 1, \dots k^r \\ &\text{and} & k = 1, \dots (n+1)! \end{aligned}$$

assuming that individual distributions preference are mutually independent.

Letting Q(a $_j^{}|\gamma_w^{})$ be the perturbation associated with alternative a $_j^{}$ in the pattern $\gamma_w^{},$ we postulate that

[6]
$$\frac{P_{G}(a_{j}|\gamma_{w})}{P_{G}(a_{e}|\gamma_{w})} = \frac{Q(a_{e}|\gamma_{w})}{Q(a_{j}|\gamma_{w})}$$

[7]
$$if \ Q(a_e | \gamma_w) = 0, \text{ then}$$

$$P_G(a_e | \gamma_w) = 1 \text{ and}$$

$$P_G(a_j | \gamma_w) = 0 \text{ for } j \neq e$$

As the total number of possible preference shifts -- r $\begin{bmatrix} \sum_{j=1}^{n} (j) \end{bmatrix}$ -- j=1

is fixed, these conditional probabilities are uniquely determined. Hence, the unconditional probabilities of group choice are given by:

[8]
$$P_{G}(a_{j};A) = \sum_{w} P_{G}(a_{j}|\gamma_{w}) \cdot Pr[\gamma_{w}]$$

Although conceptually simple, the acceptability model entails combinatorial difficulties. Its justification follows from the observation that many groups seems to choose "everybody's second choice," or more precisely, the alternative which perturbs individual preferences least.

As a numerical example, consider the data in Table 1. In order to reduce computation, assume that the group is composed only of decision participant d_1 and d_2 . Assuming three alternatives $A = \{a_0, a_1, a_2\}$, there are $(3!)^2$ possible patterns of preference structures. Table 5 enumerates these patterns, gives their probability of occurrence and the perturbation associated with each alternative respectively.

TABLE 5

_							
	of Individual ce Structure	$Pr(\gamma_{w})$	$Q(a_0 \gamma_w)$	Q(a	$\frac{1}{1} \gamma_{w} $	$Q(a_2 \gamma_w)$	
d ₁	d ₂						
θ_1	θ_1	.0078	0		_	a -	
01	θ_2	.0312	0		_	<i>2</i> ⁹ =	
θ_1^-	θ3	.0169	1		1	4	
θ_1	θ_4	.0091	2		1	3	
θ_1	θ ₅	.0247	1		3	2	
θ_{1}^{1}	θ_6	.0403	2		2	2	
θ_2	θ_1	.0042	0		-	- a = -	
θ2	θ_2	.0168	0		_		
θ_{2}^{2}	θ3	.0091	1		2	3	
θ_2^2	θ_{4}	.0049	2		2	39718 2 410	
θ_2^2	θ ₅	.0133			4	elied Jose	
θ_2^2	θ6	.0217	2		3	has [d] eso	
θ_3^2	θ_1	.0132	1		1	4	
θ_3	θ_2	.0528	1		2	3	
θ_3	θ_3	.0286	9 THEY		0	_	
θ_3	94	.0154	_		0	-	
θ_3	9 5	.0418	2		2	utlyib21 lo	
	9 6	.0682	3		1	2	
⁰ 3	6 ⊕	.0168	2		1	3	
94	$egin{pmatrix} heta & 1 & \ heta & 2 & \ \end{pmatrix}$.0672	0 2		2	2	
94	93	.0364	66. <u> </u>		0	-	
94	94	.0196	, i		0	_	
04	4	.0532	3		2	1	
94	0 5				1	1	
04	0 6	.0108	° ' 1		3	2	
⊕ ₅	01	.0432	1 1		4	1	
⊕ ₅	0 2	.0234	2		2	2	
⁰ 5	⊕ ₃	.0126	3		2	1	
95	04	022	-				

TABLE 5 (Continued)

Pattern of Preference		$\Pr(\gamma_{W})$	$\frac{Q(a_0 \gamma_w)}{}$	$Q(a_1 \gamma_w)$	$Q(a_2 \gamma_w)$
d ₁	d ₂		Carle of the		
θ,	θ ₅	.0342	- -	-	0
θ_{5}	θ ₆	.0558	_	arven ==	0
θ6	θ_1	.0072	2	2	2
θ ₆	θ_2	.0288	2	3	1
θ ₆	θ_3	.0156	3	1	2
θ6	θ_4	.0084	4	1	1
θ6	θ ₅	.0228	_	5000	0
θ6	θ6	.0372	-	5 (48)	0

Table 6 gives the probabilities of group choice conditional to the occurrence of each pattern γ_w . These probabilities are obtained through the use of relations [6] and [7].

TABLE 6

Pattern of Individual Preference Structure	$P_{G}(a_0 \gamma_w)$	P	$G^{(a_1 \gamma_v)}$) P	_G (a ₂ γ _w)	
γ_1	1		0		0	
Y ₂	1		0		0	
γ_3	. 444		.444		.111	
Υ ₄	.272		.545		.181	
γ_5^{-1}	.545		.181		.272	
γ_6	.333		.333		. 333	
Υ ₇	1		0		0	
Υ8	1		0		0	
$\gamma_{\mathbf{q}}$.545		.272		.181	
Υ10	.333		.333		.333	

TABLE 6 (Continued)

			120105	
Pattern of Individual Preference Structure	$\frac{P_{G}(a_{0} \gamma_{w})}{Q}$	$P_{G}(a_{1} \gamma_{w})$	$\frac{P_{G}(a_{2} \gamma_{w})}{a_{1}}$	
γ_{11}	.444	.111	.444	
γ_{12}	.272	.181	.545	
γ_{13}	. 444	. 444	.111	
γ ₁₄	.545	.272	.181	
γ ₁₅	0	. 1	0	
γ ₁₆	DECISION OFFICE	DATE BEAT DATA	CAROL MA 10 HELLTHO	
Υ ₁₇	.333	.333	.333	
γ_{18}	.181	.545		
γ_{19}	.272	.545	.181	
Y ₂₀	.333	.333	.333	
Υ ₂₁	0	ial markering	used by indestination	
	ilieOmi na da s	s the stricter	Figure 2 pre0ent	
Y ₂₃	.181	.272	.545	
Y ₂₄	.111	. 444	.444	
Υ ₂₅	.545	.181	.272	
γ ₂₆	.444	.111	.444	
Y ₂₇	.333	.333	.333	
Υ ₂₈	.181	.272	.545	
Reting support for the	mam 0 (makil Is	OsasmensA	eliyat element, t ${f l}_{ m e}$	
Y ₃₀ hospin to amount	0	0	rabhu isuboro istrania	
Υ ₃₁	.333	.333	.333	
Y ₃₂	.272	.181	.545	
	bak a.181 da vali	.545	.272 antelmavCA b	
Y ₃₄ maddown sales	ntage fonds purch	.444	louisa ishinatog a as	
Υ ₃₅	0	0	1	
Υ ₃₆	0	P (a, a 2001)	1	

Using relation [8], we get the unconditional probabilities of group choice:

$$P_{G}(a_{0};A) = .271$$

$$P_{G}(a_{1};A) = .334$$

$$P_{G}(a_{2};A) = .394$$

6. OUTLINE OF AN INDUSTRIAL MARKETING DECISION SUPPORT SYSTEM

Thus far we have developed four models of group decision making which satisfy the criteria developed in Section 4. Now, we discuss how they could be used by industrial marketing managers.

Figure 2 presents the structure of an industrial market response model which parallels our conceptual model discussed earlier.

Insert Figure 2 Here

The industrial adoption process is formalized as a four element model. The first element, the <u>Awareness</u> model links the marketing support for the industrial product under investigation — measured in terms of spending rates for such activities as <u>Personal Selling (PS)</u>, <u>Technical Service (TS)</u>, and <u>AD</u>vertising (AD) — to the probability that an individual d_i will evoke it as a potential solution to the organizational purchasing problem. Let

$$P_i(a_0 = EVOKED)$$
 $i=1,...r$

denote this probability. We postulate that

$$P_i(a_0 = EVOKED) = f(PS, TS, AD)$$

where $f(\cdot)$ is empirically estimated or provided by the industrial marketing manager on the basis of his personal experience with this market.

When several decision participants are involved in the adoption decision, we assert

$$P_G(a_0 = EVOKED) = \max_{i} [P_i(a_0 = EVOKED)]$$

The second element of the response model is the <u>Acceptance</u> sub-model which relates the design characteristics \underline{X}_0 of product a_0 to the probability that it will fall in the feasible set A of any organization. This sub-model accounts for the process by which organizations in the potential market screen out "impossibles" by setting product selection criteria (e.g. limits on price, reliability, payback period, number of successful installation, etc...). This probability is denoted

$$P(a_0 = FEASIBLE \mid EVOKED) = g(\underline{X}_0)$$

The specification of $g(\cdot)$ will usually require a survey of companies in the potential market. An alternative would be to use salesmen's experience. The authors are currently working on the development of new methods to collect this information.

The third element, called <u>Individual Choice</u> models relate individual perceptions of product characteristics to preferences, for each category of decision participant involved in the adoption process. These models are essential when managers want to perform a sensitivity analysis on industrial market response to changes in product design or positioning. Indeed, in some early results, Choffray [4] reports that both product evaluation spaces and individual's preferences vary across categories of decision participants.

In other cases, however, individual choice models can be overlooked, as one can directly use the observed preference structures to empirically specify the preference distribution $P_i(\theta|A)$ i=1,...r of the various categories of decision participants. A naive approach, when samples are large, is simply to set $P_i(\theta_k|A)$ equal to the frequency $f_i(\theta_k)$ with which individuals belonging to category D_i exhibit the preference structure θ_k . The choice probabilities are then obtained, by computing:

$$P_{i}(a_{j};A) = \sum_{\theta_{k}} P_{i}(\theta_{k}|A)$$

where the summation is taken over all $\boldsymbol{\theta}_k$ in which a, is the most preferred alternative.

The last element of the model is the <u>Group Decision</u> model that maps individual choice probabilities into an estimate of the group probability of choice:

$$P_{G}(a_{0};A) = h \{P_{i}(a_{0};A), i = 1,...r\}$$

Section five of this paper was devoted to the development of such models.

Combining these sub-models, we get the expression for the unconditional probability of <u>organizational</u> choice:

6.1 Use of the Model

Suppose a new industrial product has been developed and management wants to refine its design. Several steps are involved in the use of the proposed models.

1. Macrosegmentation

The first step, called Macrosegmentation following the terminology proposed by Wind and Cardozo [29], consists in specifying the target market for the new product. Bases for segmentation, at this level, might be as general as S.I.C. code, geographic location, etc. The main purpose of this segmentation is to reduce the cost and effort involved in subsequent stages, and narrow the scope of the analysis to organizations more likely to purchase the new product.

2. Measurement of organization selection criteria

"established policies" to select products from the class we are interested in. This information is the main input to calibrate the acceptance submodel. Measurement of the selection criteria, however, raises interesting methodological issues. For new industrial products specially, these criteria tend to develop when companies are actually faced with such purchasing problems. So, any attempt to measure them with "not-buying" companies can bias results. Methods are currently being developed to collect this information.

3. Disaggregation of the Industrial Adoption Process

Step three is concerned with an analysis of the structure of the adoption process for the product under investigation. Microsegments of organizations homogeneous as to the categories of decision participants that are involved in their adoption process are identified and their relative importance in the potential market is assessed. Let these microsegments be denoted by:

and the percentage of companies in the potential market that fall in each of them by

Each of these microsegments is characterized by the different categories of decision participants that are usually involved in the adoption process for the companies it comprises. For instance, in segment S_1 , Top Management along with Purchasing Officers might be the main categories of decision participants involved. In S_2 , Production Engineers are also involved, etc. The average relative importance of these categories of participant, denoted by w_i , is also assessed either judgementally or empirically.

Hence, each microsegment S_q is characterized by a set of decision participant categories, denoted $DEC_q = \{D_i, i=1, \dots, r_q\}$ and a set of importance coefficients $IMP_q = \{w_i, i=1, \dots, r_q\}$.

4. Measurement of product perceptions and preferences for each category of decision participant

For each category of decision participant, individual product preferences and perceptions are measured using standard methods of marketing research. Perceptual evaluation spaces are derived for each category of decision participant, if desired, and models of individual choice are calibrated. The preference distribution $P_i(\theta|A)$ is estimated for each of these categories as well as the choice probabilities $P_i(a_i;A)$.

5. Managerial Input

At this level, the industrial marketing manager is asked to specify for each microsegment what models of multiperson interaction best reproduce his understanding of the adoption process for the companies that belong to the microsegment.

For segment S_q his estimates might be:

Weighted Probability Model:α_{la}

is a supportionality $\mathtt{Model}_{\mathtt{Cap}}: lpha_{\mathtt{2q}}$ does see supportionality.

Unanimity Model $:\alpha_{39}$

Acceptability Model $:\alpha_{4q}$

with $\sum_{e} \alpha_{eq} = 1$ for each microsegment q.

6. Construction of Microsegment Response Model

Let ${\rm M}_{\rm q}({\rm a}_0)$ denote the estimated share of microsegment S that finally adopt product ${\rm a}_0$. Hence,

$$M_q(a_0) = \sum_{e} \alpha_{eq} Pr[a_0; A | MOD_e, DEC_q]$$

where $\Pr[a_0;A|MOD_e, DEC_q]$ is the probability that a_0 is the <u>organizational</u> choice given the involvement of the decision categories DEC_q and an interaction model MOD_e .

7. Estimation of Total Sales

Let the Potential Sales for new product a_0 be denoted by Q. Hence, we can estimate actual sales of a_0 by computing:

SALES(
$$a_0$$
) = $Q\left[\sum_{q=1}^{s} V_q M_q(a_0)\right]$.

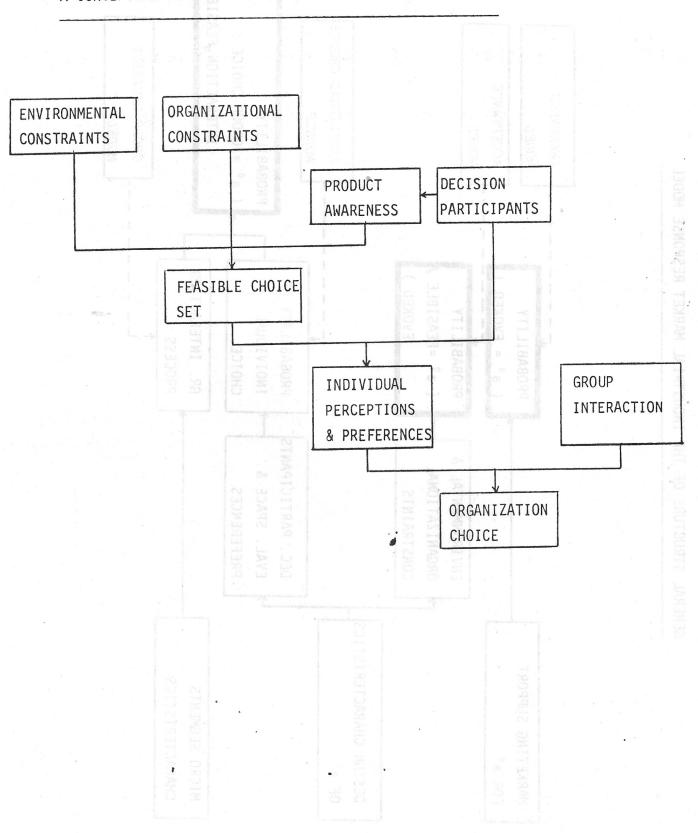
7. DISCUSSION

The models presented here and their use for industrial marketing decision making are still in an early stage of development. Work is currently under way to extend them and apply them in the context of the design of solar powered industrial air conditioning systems.

Due to the lack of research aimed at modeling the industrial adoption process, unique research opportunities exist in this area. Our hope is that this preliminary work will help serve as an incentive for such future research.

FIGURE 1

A CONCEPTUAL MODEL OF THE INDUSTRIAL ADOPTION PROCESS



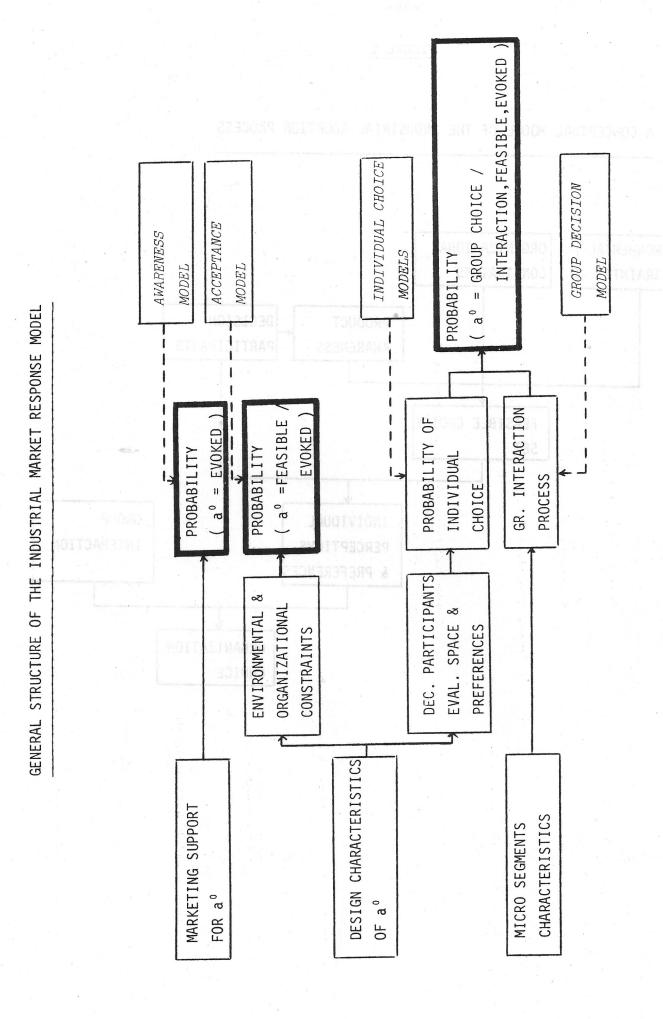


FIGURE 2

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