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Measuring the effect of stochastic perturbation component in cellular automata urban growth model

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Abstract

Urban environments are complex dynamic systems whose prediction of the future states cannot exclusively rely on deterministic rules. Although several studies on urban growth were carried out using different modelling approaches, the measurement of uncertainties was commonly neglected in these studies. This paper investigates the effect of uncertainty in urban growth models by introducing a stochastic perturbation method. A cellular automaton is used to simulate predicted urban growth. The effect of stochastic perturbation is addressed by comparing series of urban growth simulations based on different degree of stochastic perturbation randomness with the original urban growth simulation, obtained with the sole cellular automata neighbouring effects. These simulations are evaluated using cell-to-cell location agreement and a number of spatial metrics. The model framework has been applied to the Ourthe river basin in Belgium. The results show that the accuracy of the model is increased by introducing a stochastic perturbation component with a limited degree of randomness, in the cellular automata urban growth model.

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Keywords: Uncertainty; Stochastic perturbation; Cellular Automata; Urban growth; Validation

1. Introduction

Urban environments are complex dynamic systems influenced by various driving forces. To take into account the complexity of such systems, cellular automata (CA) provide a promising direction of thought for simulating changes

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of dynamic urban systems[1]. CA explicitly take into account neighbouring effects and therefore, they can be used to model spatial auto-correlated landscape patterns. Subsequently, CA are widely implemented to model urban growth through time due to their ability to fit such complex urban environment using simple and effective rules. Gutowitz[2] defined CA system as: *"dynamical systems in which space and time are discrete. The states of cells in a regular lattice are updated synchronously according to a deterministic local interaction rule"*. Traditional CA models are based on a purely microscopic approach, i.e., they are originally built upon a basic unit of behaviour. Land development reflects the behaviour of a developer rather than a ward or district[3].

In a complex urban environment, urban growth is generally affected by a number of dynamic forces. In the literature, many explanatory drivers have been analysed[4,5,6,7,8]. These drivers could be categorized into four factors: socioeconomic factors, geo-physical factors, land-use policies and accessibility factors. The complex interaction of these driving forces propagates uncertainties and errors in the model, and thus leads to a high level of parameter sensitivity in CA urban models[9]. A number of authors focused on this issue by means of analysing the variation on the model results by adjusting various parameters such as neighbour effects, neighbour windows dimensions, time steps, multiple spatial resolutions[11,12,13].

Urban CA models, like other land-use modelling approaches, are always subject to uncertainties due to limited human knowledge, complexity of urban environment and limitation of technology. Therefore, uncertainties are unavoidable and can affect the simulation accuracy of CA[15]. Uncertainty in the model could be a result of[10]:

- Input uncertainty: the future values of the exogenous variables are not well predicted.
- Model errors: errors in the model's input data and/or parameters introduced in the model's equations.

Input uncertainty represents all kind of variables which could not be predictable well. Indeed, the prediction of geo-physical and accessibility factors could be measured based on analyzing of the current situation. It is far from easy to predict socio-economic factors and policies in the future. On the one hand, a series of factors are related to micro decisions on local effects that typically fall outside the scope of CA model, for instance, the availability of land at the parcel level and administrative decisions regarding land use plans. On the other hand, global factors like population and household size forecasts will affect the general output of the system. These forecasts are very difficult to predict as they rely on complicated and interrelated factors such as population lifecycle, migration movements, societal values and standards, gender relationships and the relationships between parents and children. These kinds of factors depend always on something subject to change and therefore, it is difficult to forecast for decades to come. Furthermore, urban growth models are biased by the model's developer view so each developer defines the urban growth drivers based on his own analysis and view.

A huge volume of Geographical Information System (GIS) data is usually used in urban growth models. It is well known that most GIS data are affected by a series of errors. Changes in scale, digitizing, conversion from raster to vector, etc. are all examples of possible sources of errors in model's input data.

This paper will typically focus on the local effects of model's input uncertainty and model errors due to input data and equations introduced in the model, whose influence will be modelled through stochastic perturbation.

Uncertainties can be considered as a component of fuzziness and randomness[1]. García et al.[9] analyzed the effect of two of randomness methods in urban-CA models: Monte Carlo and stochastic perturbation methods. Wang et al.[1] analyzed uncertainties in urban growth models based on fuzziness.

1.1. Objectives

The motivations of this paper include a methodological perspective and an empirical perspective as well. At a methodological level, the paper focuses on one method to consider uncertainties in urban growth models, stochastic perturbation method. It further pays attention to a series of validation methods of urban growth models.

At an empirical level, the paper introduced urban CA model that applied to Ourthe river basin area in Belgium. The model runs once with only cell's neighbouring effects and several times with cell's neighbouring effects and stochastic perturbation. The results of different model's runs were compared in order to measure the effect of stochastic perturbation component introduced in the model by using different validation techniques.

1.2. Stochastic perturbation

The stochastic perturbation proposed by White and Engelen[14] is calculated with the following formula:

$$R = 1 + (-\ln(rand))^{\alpha} \tag{1}$$

where R is the scalable random perturbation term of cell (i, j) at time t, $rand$ is a uniform random variable varying between 0 and 1, and α is a parameter that controls the size of the perturbation introduced in the model. R will always be greater or equal to 1 since the smallest value that *ln* function can take is *0*. The '-' sign is added to allow *ln* function taking only positive values. Fig.1 shows the variation of the stochastic component according to different values of α. High values of α means that extreme values of *rand* are given more weight[9]. One of the strengths of this approach is the controlling of randomness size contrary to a number of other approaches, for instance, the Monte Carlo approach does not allow for the control of the degree of randomness[9].

2. Method

2.1. Study area

This study was conducted for the Ourthe river basin located in Wallonia, in the southern part of Belgium. It occupies an area of 2,140 km^2 and consists of 37 administrative communes. It has 664,744 inhabitants in 2013[28]. The geography of the area goes from flat to hilly with altitude ranges from +47 to +618 m above the sea level. The largest metropolitan area is Liège city with population of 195,931 in 2013[28]. According to Corine Land Cover (CLC) raster datasets 2000, built-up lands covered 19.34%, arable lands covered 44.10%, grasslands covered 0.13% and forests covered 35.99% of the total area. The Ourthe River is a 165 km long river in the Ardennes in Wallonia.

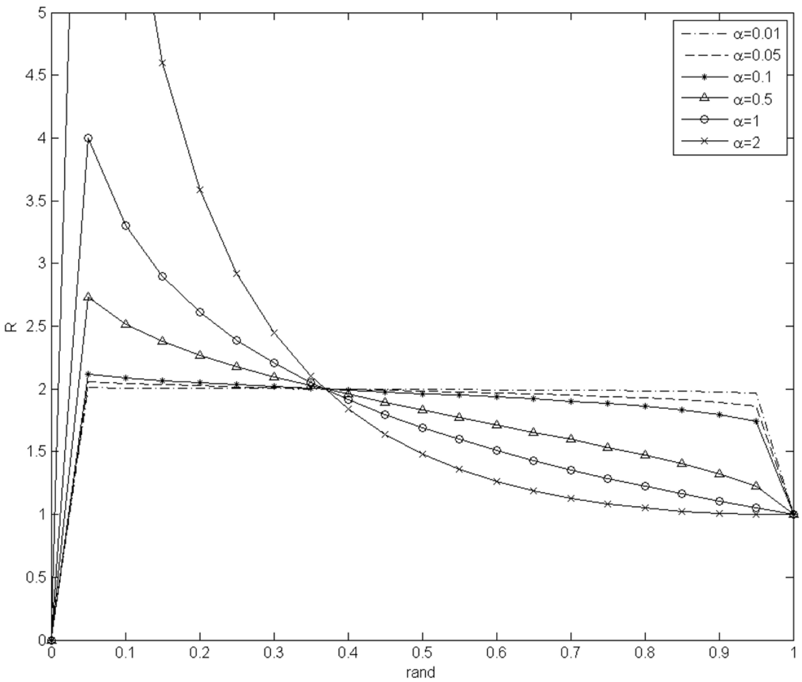

Fig. 1. Variation of the stochastic perturbation according to different size of α.

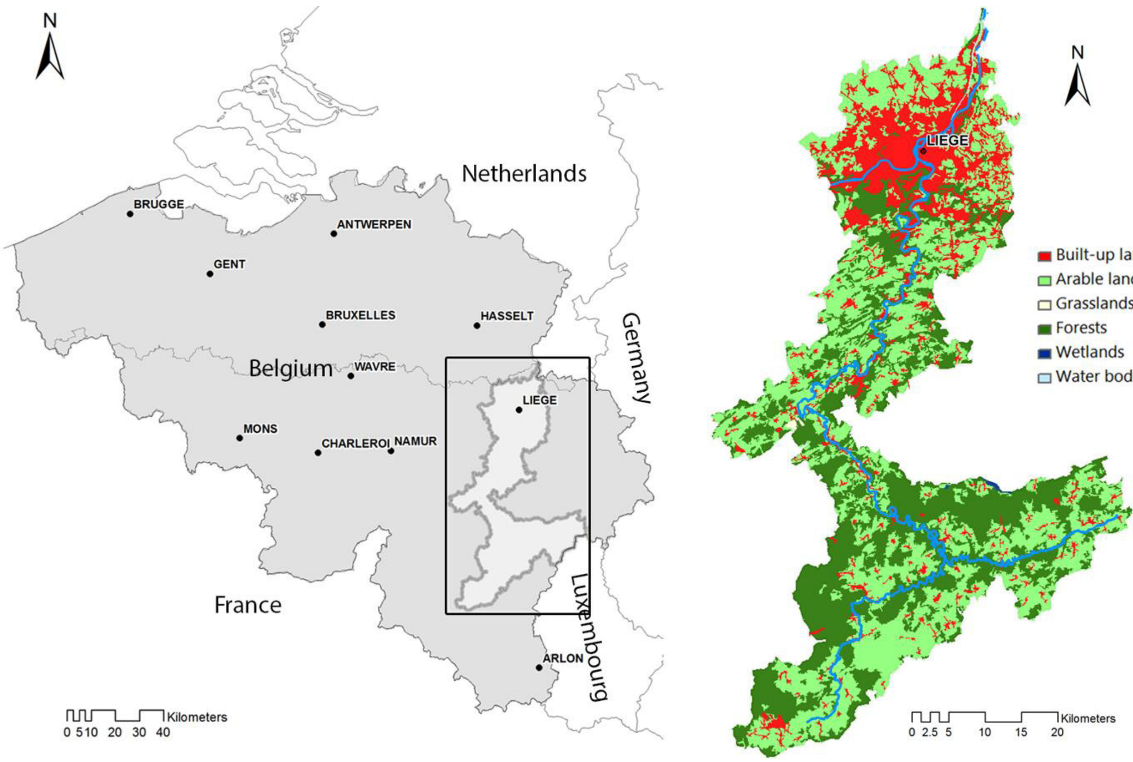

Fig. 2. Study area.

It is formed at the confluence of the Ourthe Occidentale (Western Ourthe) and the Ourthe Orientale (Eastern Ourthe), west of Houffalize. After the confluence of the two Ourthes, the Ourthe flows in north direction. It flows into the river Meuse in the city of Liège. Study area is highly influenced by city of Liège and Luxembourg, Fig. 2.

2.2. Cellular automata urban growth model

The proposed urban growth model is based on a CA modelling approach. The model simulates urban growth over time based on the impacts of neighbour effects in a CA model framework. The initial state of simulation starts from land-use in the year 1990 and proceeds to simulate an urban growth of 2000. The analysis of land-use change is based on the CLC with resolution of 100*100 m for the years 1990 and 2000. CLC has been selected because it is an excellent dataset consistent with European standards[30]. The 44 classes of CLC datasets have been reclassified into six classes. The accuracy of CLC has been assessed and reported in Feranec et al.[16] with a total accuracy of 87.0 ± 0.8%, therefore the accuracy of 85% specified by EEA is fulfilled. A core part in CA model is to define realistic transition rule-sets (pulls and pushes). In this context, model considers built-up change probabilities based on a search window of 9x9 cells Moore neighbourhood. The neighbouring factors are based on calibrations reported on another study regarding urban growth modelling done for the northern part of Belgium[11]. These results mainly rely on the expert knowledge and calibration-based rules, based on spatial metrics. Each neighboring cell C is identified by its relative location on the map by i and j (row and column) and has one state s ranging from 1 to 6 (1 built-up land, 2 arable land, 3 pastureland, 4 forest, 5 wetland and 6 water respectively) at time t with distance d of 1, 2 or 3 (0 for the cell under evaluation, 100 or 200 meters respectively) form the cell under evaluation C_e. According to that, the neighboring factor for each cell at time t can be calculated with the following formula:

$$C_e^t = \sum_{d}^{i=NN} \sum_{i=1}^{i=NN} W_{c^s_{(i,j)}}^d \tag{2}$$

where NN is a number of neighbors and $W_{C^s_{(i,j)}}^d$ is a weight factor for each neighbor based on cell state and distance from the cell under evaluation. If a cell state is built-up in 1990, it automatically remains the same in 2000.

The built-up area demands of 2000 were extracted from observed CLC maps. The urban growth between 1990 and 2000 is 47.24 km^2 (2.21%). As a result, 4,724 of the 173,926 non-built-up cells should be converted into built-up cells in order to simulate the actual quantity of 2000 urban growth.

Along with transition probability according to neighbouring constraint, a built-up suitability map has been produced. A binomial logistic regression model was used to compute the suitability map in which the input dependent variable (Y) is a binary map of real built-up changes between 1990 and 2000 (1 if the cell is being developed into built-up state and 0 if the cell is remaining in the current state) and independent variables are the most significant urban growth driving forces in the study area. The model considers distance to national roads, distance to local roads, distance to main cities, slope, job potential within 20 km and zoning as independent variables (X_n) for the logistic regression model. This type of regression analysis is usually employed in estimating a model that defines the relationship between one or more independent variable(s) to the binary dependent variable. The suitability value of each cell can be estimated with the following formula[7]:

$$S_{LU_X} = \frac{e^{a+b_1X_1+b_2X_2+b_3X_3+...+b_nX_n}}{1+e^{a+b_1X_1+b_2X_2+b_3X_3+...+b_nX_n}} \tag{3}$$

where S_{LUx} is suitablity of land-use x; a is the intercept representing the value of Y when the values of the independent variables are zero; b_1, b_2, …, b_n are regression coefficients; X_1, X_2, …, X_n are independent variables. The coefficients are based on a maximum likelihood estimation procedure. In order to minimise the spatial autocorrelation that violates the logistic regression results[11], the model is calibrated using a stratified random sample of 3000 cells, 1.4% of the study area, with an equal sampling to 1 and 0 observations of the Y variable. The outcome of logistic regression model is a suitability map in which each cell in the study area presents transition suitability value ranging from 0 (the lowest transition suitability) to 1.

2.3. Introducing stochastic perturbation in cellular automata model

The stochastic perturbation has been introduced in the model by multiplying stochastic component, equation 1, by the suitability of each cell in the study area to be converted into built-up cell (built-up suitability map) then multiplying the result by traditional CA transition probability (neighbouring effects) cell by cell.

In general terms, introducing stochastic component in CA models could violate the deterministic rules of CA model which are purely based on neighbouring factors. In the study area, 173,926 cells could be converted into built-up land-use between 1990 and 2000. According to traditional CA model, the top ranked 4,724 cells were converted into built-up land-use. To analyse the effect of introducing stochastic perturbation in the model, the variability of change (from non-built-up to built-up) for each cell was tested through 500 runs of the model for different α values (*0.01, 0.05, 0.1, 0.5, 1* and *2*). Fig. 3 shows that number of change (from non-built-up to built-up) for the top 15,000 ranked cells (based on traditional CA). The variability of change decreases for increasing α value as the model creates more randomness.

3. Model validation and comparisons

The validation of the model is the process of measuring the accuracy of the simulated result against real world observations. In this paper, the validation process includes a validation of the simulated suitability map and different model's simulations.

In regards with suitability map, the fitness of the model was assessed for both the selected samples (3000 cells) and for the whole suitability map. For the samples, the fit of the model was evaluated using classification table of the samples, Pseudo R^2, Relative Operating Characteristic (ROC) procedure and for the suitability map of the whole area ROC procedure was used. Table 1 shows the classification table of the samples in which the predicted probability with 0.5 is employed as a dividing point, i.e., classifying all samples as 0 if the predicted probability is less than 0.5 or else as 1. Pseudo R^2 is calculated with the following formula:

$$P\_R^2 = 1 - \left(\frac{\ln(\tilde{\theta})}{\ln(\hat{\theta})} \right) \tag{4}$$

where $\ln(\tilde{\theta})$ is the likelihood for the full model as fitted and $\ln(\hat{\theta})$ is the likelihood for the model if the value of all coefficients except the intercept are 0[18]. The P_R^2 of the samples is 0.959. It indicates an almost perfect fit where the perfect fit should be 1. ROC, proposed by Pontius and Schneider[19], is considered as an excellent method to evaluate validity of the models that predict the location of the occurrence of change by comparing the predicted probability map to a map with the observed changes of built-up cells between 1990 and 2000 in order to validate the model's ability to specify the location of change. The procedure first calculates the proportion true positives and false-positives for a range of specified threshold values and relates them to each other in a graph. The ROC statistic measures the area under the curve in the graph and should range between 0.5 (random fit) and 1 (perfect fit)[11]. The ROC result of the samples with 100 thresholds equals 0.998.

The fit of the suitability map, not only samples, is assessed using ROC procedure and equals 0.742 with 50 thresholds (Fig. 4).

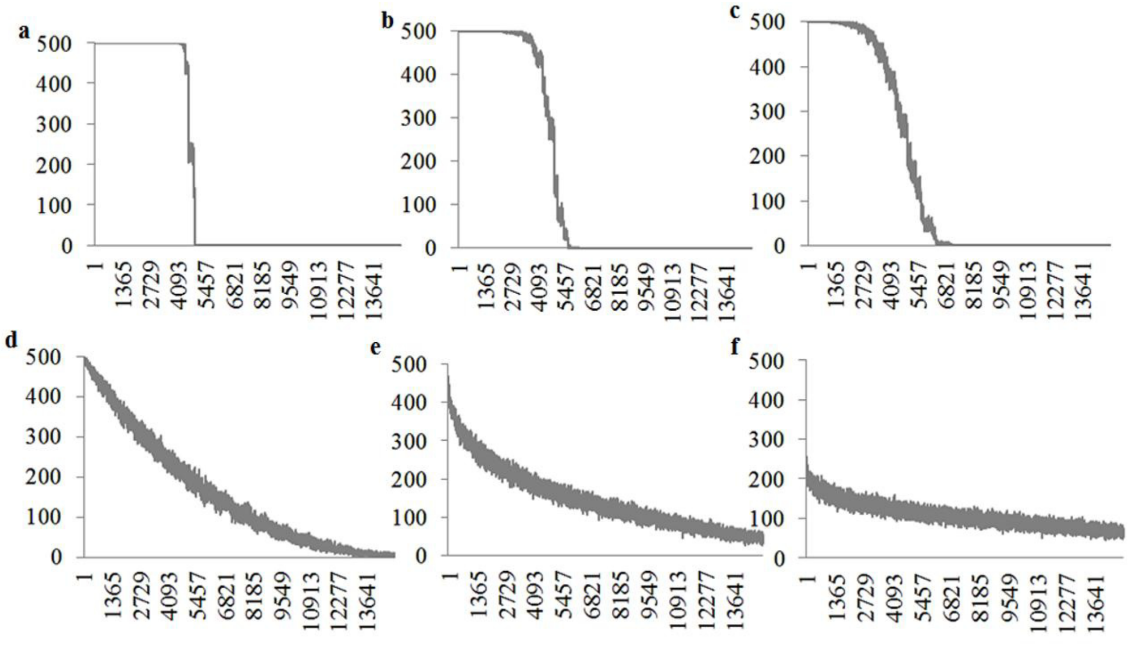

Fig. 3. Number of changes from 0 to 500 times (y axis) of the top ranked 15,000 cells (x axis). a:α=0.01, b:α=0.05, c:α=0.1, d:α=0.5, e:α=1, f:α=2

Table 1. Classification table of the selected 3000 samples with equal observations of 0 and 1.

Observed	Fitted-0	Fitted-1	Percent correct
0	1491	9	99.40
1	18	1482	98.80

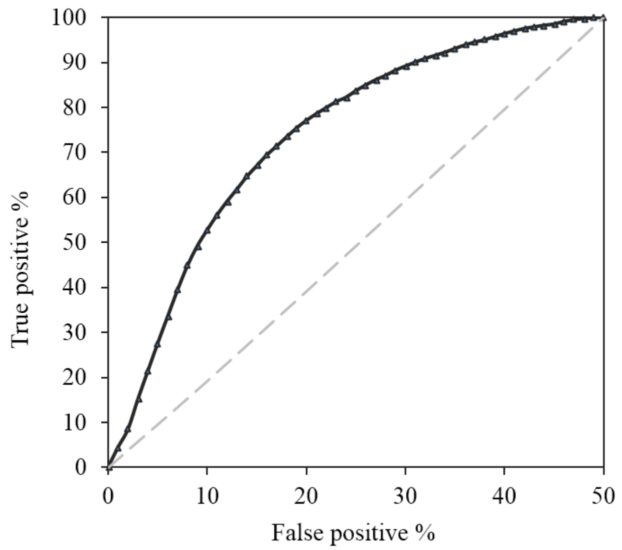

Fig. 4. ROC curve for the suitability map of the study area with 50 thresholds.

The different runs of the model performance without stochastic perturbation and with different values of *α* in stochastic perturbation (*α=0.01, 0.05, 0.1, 0.5, 1* and *2*) are compared and assessed in order to analyse the effect of introducing stochastic perturbation in the traditional CA model.

The validation of different model runs has been evaluated by several means. Pontius et al.[29] proposed a validation method to measure agreement between two categorical maps through cell-to-cell comparison. This method separates agreement and disagreement between the two maps into main three components due to chance, quantity and location. The agreement of chance is the agreement that could be obtained with no information of location and no information of quantity. The agreement of quantity is the quantity accuracy of each category. The agreement due to location is the accuracy in terms of cell's location of each category. In this paper, the scope of analysis pertains to the investigation of accuracy of different runs in terms of cell location. One of the shortcomings of this method is that it cannot discriminate between "near-miss" and "far-miss" errors and therefore fails to detect spatial patterns[11]. Pontius et al.[29] suggested to evaluate the model runs at different multiple resolutions to solve such a shortcoming. Poelmans[11]; Akin et al.[22] and Wanga et al.[23] used this method to evaluate their urban growth models. The agreement due to location is measured for all built-up cells, e.g. Wu[3], and for the only newly built-up cells changed between two time steps, e.g. Poelmans[11] and Wanga et al.[23]. The overall agreement due to location for a number of previous studies are ranging between 92% and 69% for all built-up cells while for newly built-up cells ranging between 38% and 4.5%[3,11,23]. Table 2 shows the cell-to-cell agreement due to location for all built-up cells and for only newly built-up cells. Furthermore, Table 2 shows also Kappa Index of Agreement (KIA) which reported as a commonly used index for accuracy assessment of results of spatial models[24,25]. KIA indicates a perfect agreement of 1 and chance agreement of 0. The results show that the cell-to-cell agreement due to location for all built-up cells is over 90% in all model simulations. However, this high cell-to-cell location agreement for the whole built-up cells between the two time steps is caused by the persistence of built-up cells between the two time steps. By considering the accuracy percentage or KIA of allocation of newly built-up cells, it is clear that the model performed much better with *α=0.01* and *0.05* than other simulations.

Urban growth model validation might also rely on evaluating how a model simulates certain spatial properties, e.g., a real change in spatial patterns. In this respect, a number of authors validated the performance of their models by means of spatial metrics[9,11,20]. Generally, spatial pattern matrices are grouped into four main levels corresponding to the level of heterogeneity: cell-level, patch-level, class-level and landscape-level[21]. Class-level matrices are used to evaluate categorical map patterns such as land-use maps[9,21].

Table 2. Cell-to-cell agreement due to location for different model's runs.

Model	Built-up overall agreement (%)	Built-up overall KIA	Newly built-up agreement (%)	Newly built-up KIA
No Stoc.	91.878	0.899	28.831	0.272
α=0.01	91.900	0.899	29.022	0.274
α=0.05	91.902	0.899	29.043	0.274
α=0.1	91.871	0.899	28.768	0.271
α=0.5	91.576	0.895	26.185	0.245
α=1	91.259	0.891	23.412	0.216
α=2	90.561	0.883	17.295	0.154

Before selecting spatial matrices, landscape pattern should be defined well including map contents and the purpose of analysis. The purpose here is to quantify the complexity and compactness (fragmentation and dispersion) of model's simulations and real urban change pattern of 2000. It is difficult to capture these complex spatial properties using a single matrix[9,21]. García et al.[9] selected, number of patches, mean patch area, area weighted mean patch fractal dimension and euclidean mean nearest neighbour distance spatial matrices to validate their model's results while Parker and Meretsky[20] selected, number of patches, mean patch area, area weighted mean shape index, edge density, total edge, euclidean mean nearest neighbour distance and class area concentration matrices to evaluate their model's outcomes.

In this study, two matrices measuring fragmentation (number of patches, mean patch area), one matrix measuring the complexity (area-weighted mean shape index), one matrix measuring dispersion (patch cohesion index) one matrix measuring dominance (largest patch index) and one matrix measuring isolation (Euclidean mean nearest neighbour distance) have been selected and calculated for newly simulated and observed built-up cells based on Moore neighbourhood method. Further explanation on the meaning of these matrices is stated below:

- Number of Patches (NP): A patch is defined as a group of continues cells belong to the same land-use class. This matrix counts the total number of patches in the landscape. This matrix is a basic index for computing other metrics and also used to evaluate fragmentation index. As a number of patches decreases, the fragmentation index also decreases[20,26].

- Mean Patch Area (MPA): This spatial matrix is widely used to define fragmentation index where lower MPA index produces more fragmented pattern. The MPA index can be calculated based on the following formula[26]:

$$MPA = \frac{\sum_{i=1}^{i=NP} ai}{NP} \tag{5}$$

where ai is the area of the patch.

- Area-Weighted Mean Shape Index (AWMSI): AWMSI is the most straightforward measure of shape complexity and irregularity. This matrix measures the weighted average deviation of the patch shape which minimizes edge/area ratio. Holding class area and the class NP constant, the edge/area ratio will fall as patch shape become more complex[20]. This index can be calculated based on the following formula[20,26]:

$$AWMSI = \sum_{i=1}^{i=NP} \left\{ \frac{0.25pi}{\sqrt{ai}} \right\} \left\{ \frac{ai}{A} \right\} \tag{6}$$

where p_i is the patch perimeter and A is the total area of the class.

- Patch Cohesion Index (PCI): This matrix indicates how the land-uses class under analysis in dispersion. PCI value is measured in percentage with 0% if all patches are confined to single isolated cells, and with 100% if each cell is included in a patch and calculated according to the following formula[26,27]:

$$PCI = \left[1 - \frac{\sum_{i=1}^{i=NP} pi}{\sum_{i=1}^{i=NP} pi\sqrt{ai}} \right] \left[1 - \frac{1}{\sqrt{A_T}} \right]^{-1} \times 100 \tag{7}$$

where A_T is the total landscape area.

- Largest Patch Index (LPI): This matrix expresses the dominance of largest patch size by percent of the total landscape and can be calculated based on the following formula:

$$LPI = \frac{A_{LP}}{\sqrt{A_T}} \times 100 \tag{8}$$

where A_{LP} is the area of largest patch.

- Euclidean Mean Nearest Neighbour Distance (EMNND): This spatial index measures accessibility which is an important factor of urban sprawl[20] and also it is the simplest way to quantify patch isolation. However, EMNND is used here to measure patch isolation. This index equals the mean distance between each patch to the nearest neighboring patch in meters based on shortest edge-to-edge distance (the edge-to-edge distance is from cell center to cell center).

Table 3 lists the values of the analysed spatial patterns, Fig. 5, for the different runs of the model. When analysing the result, it confirms that the model simulations with a stochastic perturbation with α of *0.01, 0.05* and *0.1* yield values much better than the model simulation without stochastic perturbations (no stoc.) and were nearest to real changes of 2000 (obs. 2000) for NP, MPA, AWMSI, PCI and LPI and with α of *2* for EMNND.

Table 3. Analysing of model's simulations spatial patterns.

Model	NP	MPA	AWMSI	PCI	LPI	EMNND
Obs.2000	2063	22899	1.3414	49.1165	0.0161	311.5345
No Stoc.	2574	18353	1.1999	35.1906	0.003	271.3698
α=0.01	2496	18926	1.219	36.724	0.0032	271.1663
α=0.05	2529	18679	1.2147	36.2139	0.003	271.4315
α=0.1	2527	18694	1.2114	36.1165	0.003	272.6755
α=0.5	2733	17285	1.1709	31.7663	0.0029	272.9275
α=1	3020	15642	1.1385	26.7767	0.0018	275.9213
α=2	3395	13915	1.1082	20.8915	0.0015	305.8424

Comparing to model simulation without stochastic components, the results clearly show that the model simulation is affected by introducing stochastic perturbation in terms of spatial properties. Fragmentation in analysed patterns is decreased by introducing stochastic perturbation with very small values of α and sharply increased with high values of *α*. The fragmentation index is always decreased by decreasing NP and increasing MPA. Consequently, model simulations with *α=0.01, 0.05* and *0.1* seem proper than other simulations. Model simulations with *α=0.01, 0.05* and *0.1* give the result better than other simulation in terms of complexity. Model simulation with *α=2* tends to be a perfect square where AWMSI value of 1 represents a perfect regular shape. According to PCI, model simulation with *α=2* around 21% of the modelled cells are confined in patches which results highly dispersal pattern. On the other hand, model simulations with *α=0.01*, *0.05* and *0.1* give more cohesive pattern. The largest patch (LPI) tends to increase its size in case of the model simulations with *α=0.01*. Regarding the analysis of patches isolation in the entire landscape, EMNND index for model simulations with *α=2* yield the values that nearest to real changes of 2000. Fig. 6 shows the real changes of 2000 and the different simulations of the model within a clipped part of the study area.

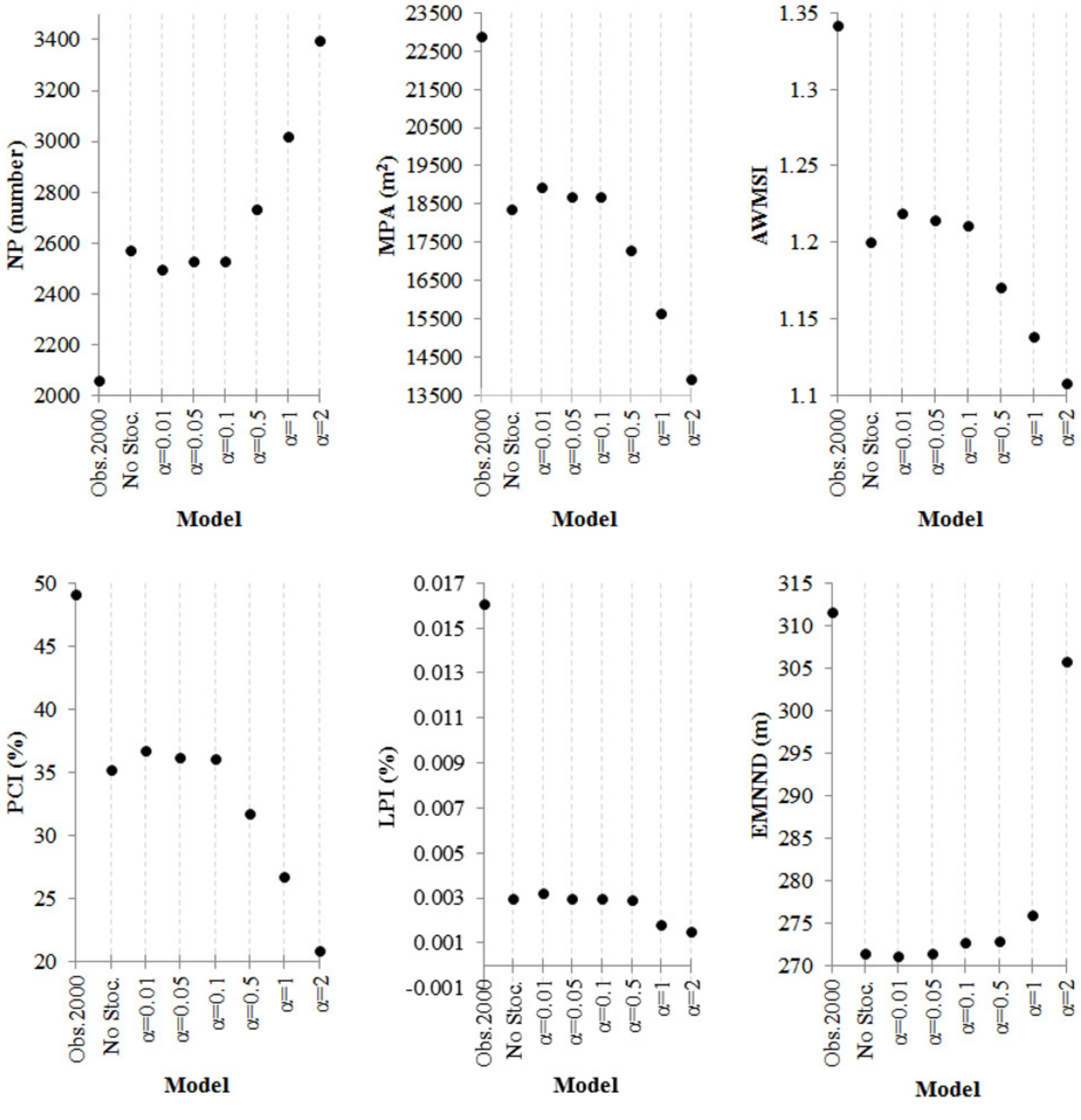

Fig. 5. Spatial matrices outcomes.

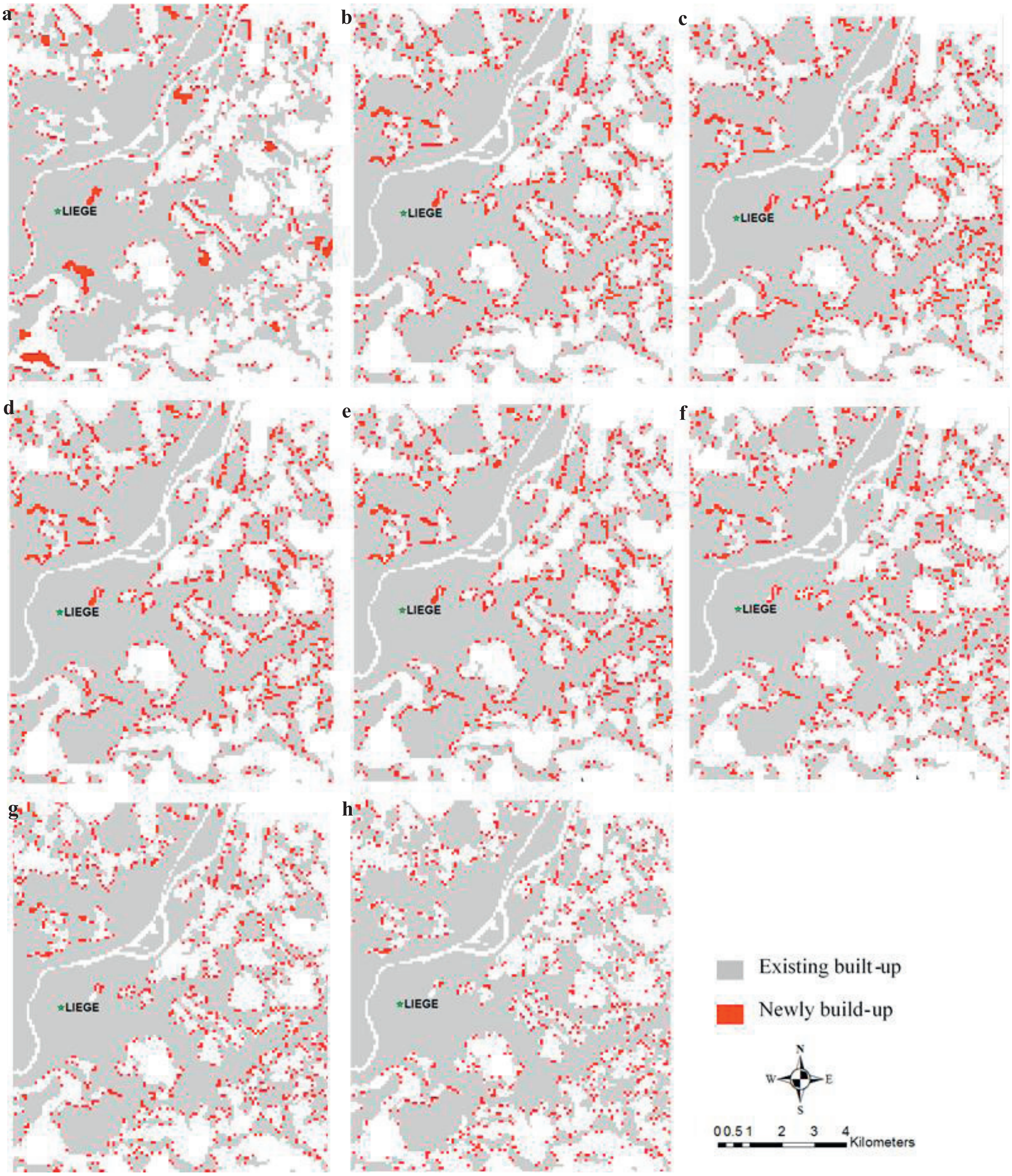

Fig. 6. Simulated built-up changes (b: no stoc., c:α=0.01, d:α=0.05, e:α=0.1, f: α=0.5, g:α=1, h:α=2) compared with real changes of 2000 (a).

4. Conclusions

Land-use change models usually deal with two kinds of time directions: the forward direction and the inverse direction. For the inverse direction, land-use change models try to simulate a situation in the past regarding a set of transition rules based on the best available description of some phenomena. This kind of analysis usually depends on the deterministic analysis approach. For the forward direction, the model simulates a situation in the future. Bearing in mind the high level of complexity of urban environments, such models should be built on robust transition rules. However, this kind of analysis must be able to deal with uncertainties.

This paper introduced and tested one of the widely used stochastic components in land-use change models. The model presented in this paper is a CA model used to simulate urban growth between 1990 and 2000. A stochastic perturbation component was introduced in the CA model to address how randomness propagates in urban growth model and how uncertainties affect simulation results. A cell-to-cell location validation technique has been used to evaluate the model results. It provides statistical information of how well allocation procedures succeeded. Since spatial matrices can potentially analyse landscape pattern under analysis, for instance, complexity and fragmentation, these analysis provide further information about how the model output patterns match real-world landscape patterns. Thus, a number of widely used spatial matrices have been used also to evaluate the model results.

The results revealed that the model accuracy increases with very small size of stochastic perturbation and then decreases when stochastic perturbation size increases. Consequently, stochastic perturbation produces high changes in the degree of randomness for very low variations in the coefficient that controls the size of randomness. In this light, it is difficult to control the degree of randomness using stochastic perturbation and therefore, the calibration of the model to match a specific growth pattern using this technique is hard. This result is data-dependent relies on the study area, datasets and type of analysis.

Finally, much work remains to define the best stochastic component and the degree of randomness in urban growth models. This work might include testing of other randomness techniques such as Monte Carlo method and fuzziness.

References

1. Wang H, He S, Liu X, Dai L, Pan P, Hong S, Zhang W. Simulating built-up growth using a cloud-based cellular automata model: A case study of Jiangxia, Wuhan, China. *Landscape and Urban Planning* 2013;**110**:99-112.
2. Gutowitz H. *Cellular Automata: Theory and Experiment*. Cambridge, MA: MIT Press; 1999. p. vii.
3. Wu F. Calibration of stochastic cellular automata: the application to rural-urban land conversions. *International Journal of Geographical Information Science* 2002;**16(8)**:795-818.
4. Verburg P, Schot P, Dijst J, Veldkamp A. Land use change modelling: current practice and research priorities. *Geojournal* 2004;**61**:309-324.
5. Beckers A, Dewals B, Erpicum S, Dujardin S, Detrembleur S, Teller J, Pirotton M, Archambeau P. Contribution of land use changes to future flood damage along the river Meuse in the Walloon region. *Nat. Hazards Earth Syst. Sci.* 2013;**13**:2301-18.
6. Cammerer H, Thieken A, Verburg P. Spatio-temporal dynamics in the flood exposure due to land use changes in the Alpine Lech Valley in Tyrol (Austria). *Natural Hazards* 2013;**68**:1243-1270.
7. Serneels S, Lambin E. Proximate causes of land-use change in Narok District, Kenya: a spatial statistical model. *Agriculture, Ecosystems & Environment* 2001;**85**:65-81.
8. Teixeira Z, Teixeira H, Marques J C. Systematic processes of land use/land cover change to identify relevant driving forces: Implications on water quality. *Science of the Total Environment* 2014;**(470-471)**:1320-1335.
9. García A, Santé I, Crecente R, Miranda D. An analysis of the effect of the stochastic component of built-up cellular automata models. *Computers, Environment and Urban Systems* 2011;**35(4)**:289-296.
10. Jong G, Daly A, Pieters M, Miller S, Plasmeijer R, Hofman F. Uncertainty in traffic forecasts: literature review and new results for The Netherlands. *Transportation* 2007;**34**:375-395.
11. Poelmans L. Modelling built-up growth and its hydrological impacts. Unpublished PhD dissertation, Katholieke Universiteit Leuven; 2010.
12. Dietzel C, Clarke K. Toward Optimal Calibration of the SLEUTH Land Use Change Model. *Transactions in GIS* 2007;**11(1)**:29-45.
13. Jantz C, Goetz S. Analysis of scale dependencies in an urban land-use-change model. *International Journal of Geographical Information Science* 2005;**19(2)**:217-241.
14. White R, Engelen G. Cellular-automata and fractal urban form: A cellular modelling approach to the evolution of urban land-use patterns. *Environment and Planning A* 1993;**25(8)**:1175-1199.
15. Yeh A, Li X. Errors and uncertainties in urban cellular automata. *Computers, Environment and Urban Systems* 2006;**30(1)**:10-28.

16. Feranec J, Jaffrain G, Soukup T, Hazeu G. Determining changes and flows in European landscapes 1990-2000 using CORINE land cover data. *Applied Geography* 2010;**30(1)**:19-35.
17. Monteiro A, Fava F, Hiltbrunner E, Marianna G, Bocchi S. Assessment of land cover changes and spatial drivers behind loss of permanent meadows in the lowlands of Italian Alps. *Landscape and Urban Planning* 2011;**100(3)**:287-294.
18. Hu B, Shao J, Palta M. Pseudo-R2 in logistic regression model. *Statistica Sinica* 2006;**16**:847-860.
19. Pontius Jr R G, Schneider L C. Land-cover change model validation by an ROC method for the Ipswich watershed, Massachusetts, USA. *Agriculture, Ecosystems & Environment* 2001;**85(1-3)**:239-248.
20. Parker D, Meretsky V. Measuring pattern outcomes in an agent-based model of edge-effect externalities using spatial metrics. *Agriculture, Ecosystems and Environment* 2004;**101(2-3)**:233-250.
21. McGarigal K. Landscape pattern metrics. *Encyclopedia of Environmetrics* 2012;**3**.
22. Akin A, Clarke K, Berberoglu S. The impact of historical exclusion on the calibration of the SLEUTH urban growth model. *International Journal of Applied Earth Observation and Geoinformation* 2014;**27(B)**:156-168.
23. Wang H, He S, Liu X, Dai L, Pan P, Hong S, Zhang W. Simulating urban growth using a cloud-based cellular automata model: A case study of Jiangxia, Wuhan, China. *Landscape and Urban Planning* 2013;**110**:99-112.
24. Monserud R, Leemans R. Comparing global vegetation maps with the Kappa statistic. *Ecological modelling* 1992;**62(4)**:275-293.
25. Pontius Jr R G, Cornell J, Hall C. Modelling the spatial pattern of land-use change with GEOMOD2: application and validation for Costa Rica. *Agriculture, Ecosystems and Environment* 2001;**85(1-3)**:191-203.
26. Saura S. Effects of minimum mapping unit on land cover data spatial configuration and composition. *International Journal of Remote Sensing* 2002;**23(22)**:4853-4880.
27. Schumaker N. Using landscape indices to predict habitat connectivity. *Ecology* 1996;**77(4)**:1210-1225.
28. Institut wallon de l'évaluation, de la prospective et de la statistique. *Population*. Accessed on February 08, 2014, from *http://www.iweps.be/evolution-de-la-population/*.
29. Pontius Jr R G, Huffaker D, Denman K. Useful techniques of validation for spatially explicit land-change models. *Ecological Modelling* 2004;**179(4)**:445-461.
30. EEA-European Environment Agency. *CORINE Land Cover*. Accessed on February 28, 2014, from *http://www.eea.europa.eu/data-and-maps2014/*.