Using POD and DMD for comparing CFD and experimental results in unsteady aerodynamics

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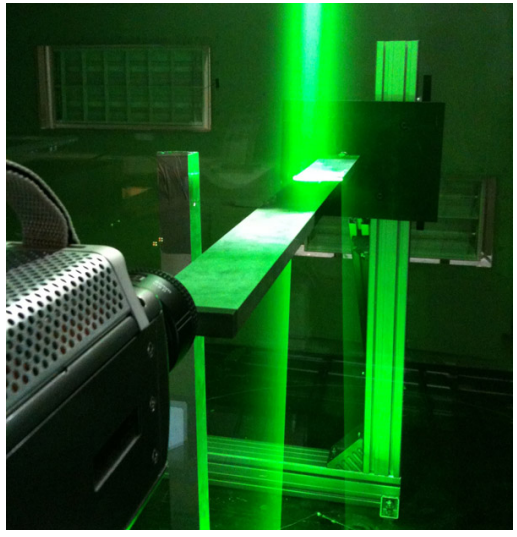
November 25, 2014

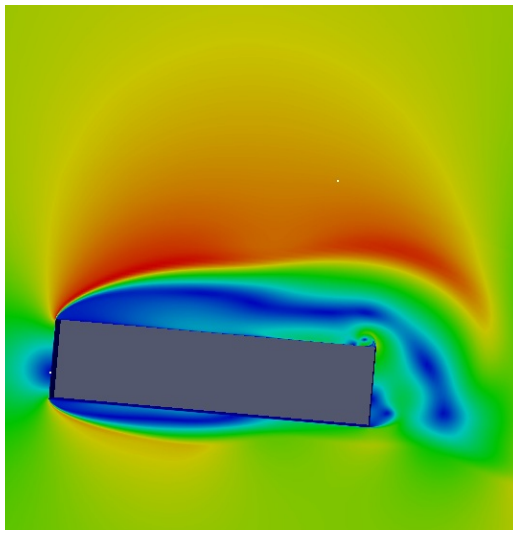
APS-DFD

Motivation

Experiments (EFD)



Simulations (CFD)



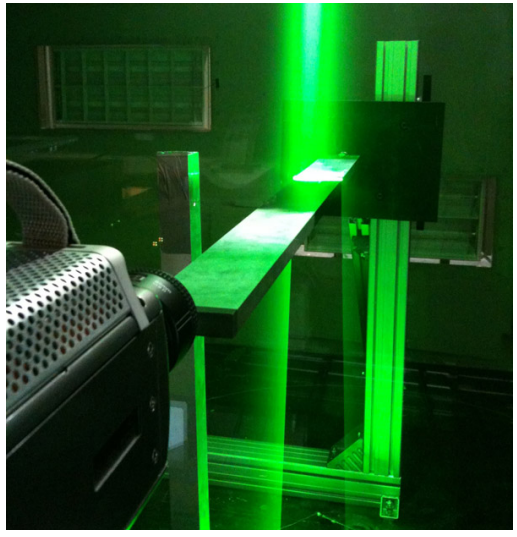
Comparison of results

✓ Global quantities

✗ **Local quantities**

Motivation

Experiments (EFD)



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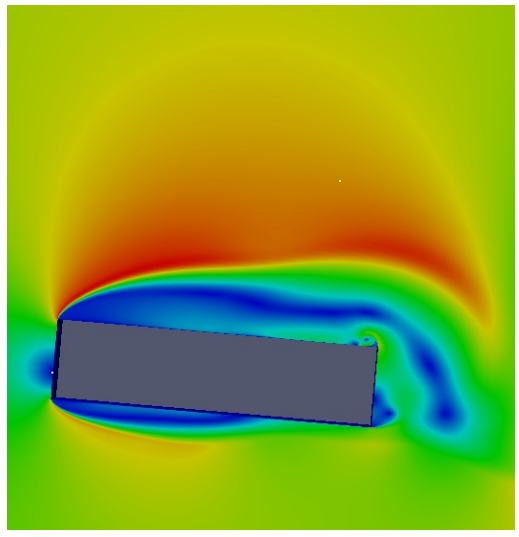

Comparison of results

✓ Global quantities

✗ **Local quantities**

Higher level of comparison is needed

POD and DMD used for comparison

POD finds the most persistent spatial structures ϕ_i^{POD}

$$\mathbf{u}(x,y,t) = \sum_{i=1}^{N} \underbrace{a_i^{\text{POD}}}_{\substack{\text{amplitude} \\ = \text{ energy}}} \underbrace{f_i^{\text{POD}}(t)}_{\substack{\text{time} \\ \text{evolution}}} \underbrace{\phi_i^{\text{POD}}(x,y)}_{\substack{\text{spatial} \\ \text{mode}}}$$

DMD finds single frequency modes ϕ_i^{DMD}

$$\mathbf{u}(x,y,t) = \sum_{i=1}^{N} \underbrace{a_i^{\text{DMD}}}_{\text{amplitude}} \underbrace{\exp\left(\lambda_i^{\text{DMD}} t\right)}_{\substack{\text{time} \\ \text{evolution}}} \underbrace{\phi_i^{\text{DMD}}(x,y)}_{\substack{\text{spatial} \\ \text{mode}}}$$

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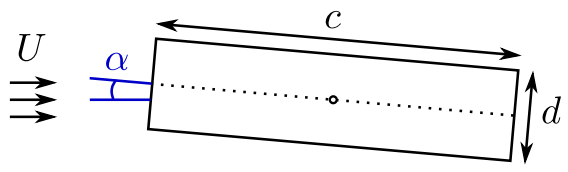
$\Rightarrow$ Different parts of decompositions are compared

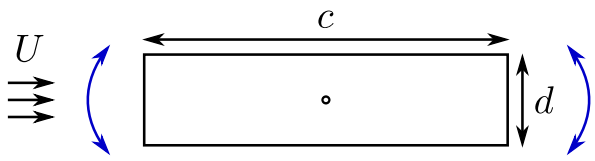
Methodology

Static rectangle



- $\mathrm{Re} = Uc/\nu = 10^5$
- $c = 4d$
- $\alpha = 5^\circ$

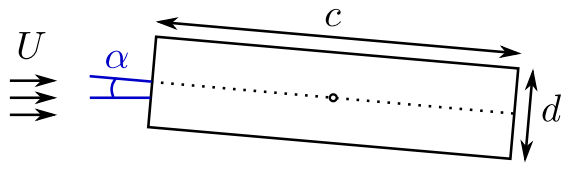
Pitching rectangle



- $\mathrm{Re} = 4 \cdot 10^4$
- $k = \pi fd/U = 6.85 \cdot 10^{-2}$
- $\alpha_{MAX} = 10^\circ$

Methodology

Static rectangle



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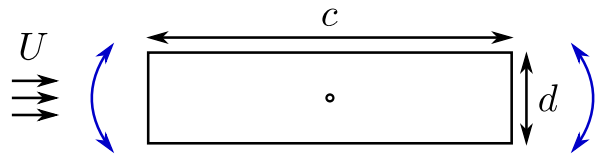

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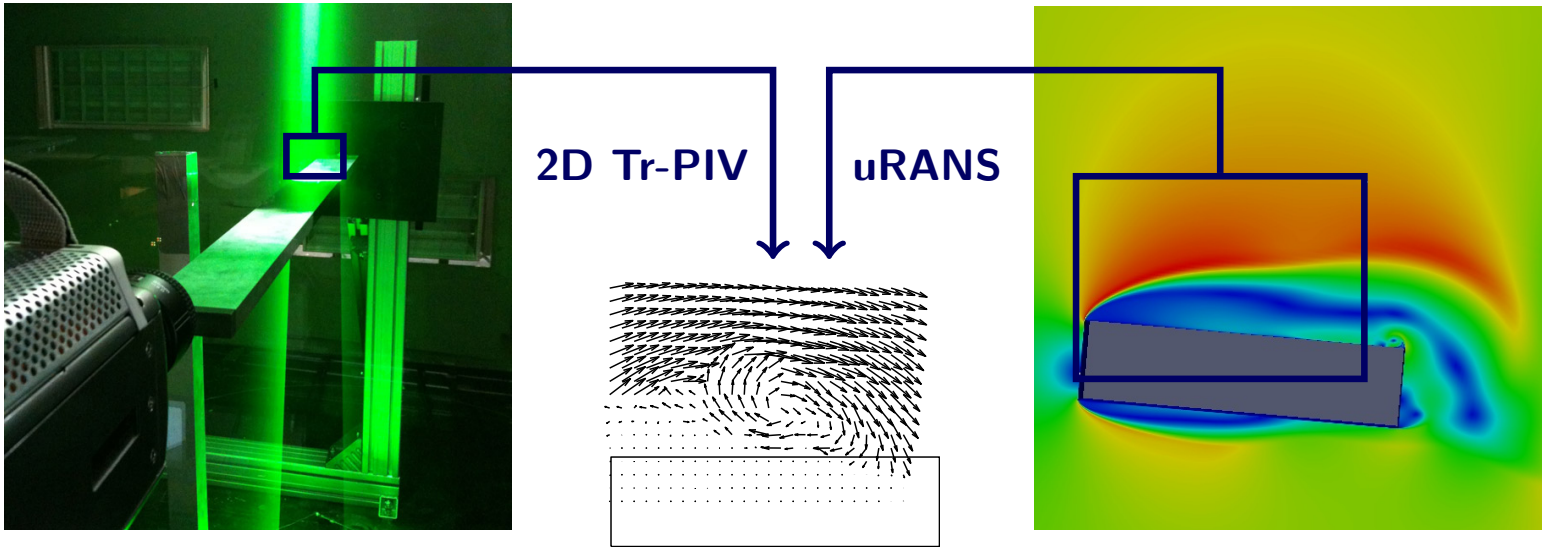
Data Collection ⇒ **Decomposition** ⇒ **Comparison**

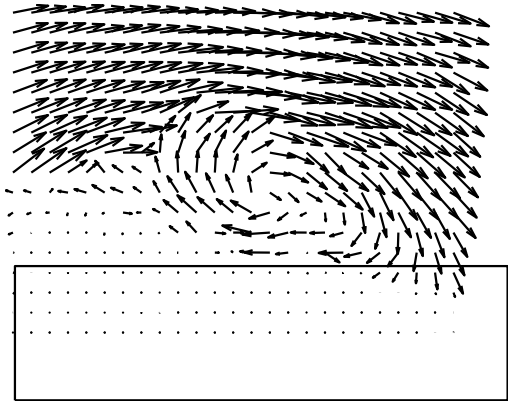
Methodology

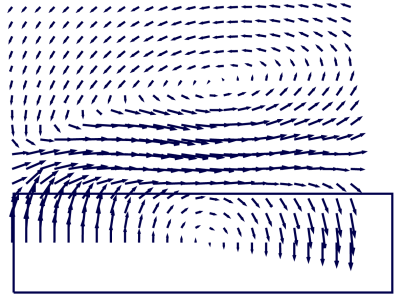


Data Collection

1. EFD → Tr-PIV
2. CFD → uRANS

Methodology





$$a_i^{\mathrm{DMD}} \quad \exp\left(\lambda_i^{DMD} t\right) \quad \phi_i^{\mathrm{DMD}}(x,y)$$

amplitude **time** **spatial**

Data Collection ⇒ **Decomposition**

1. EFD → Tr-PIV
2. CFD → uRANS

1. POD: $\phi_i(x,y)$ and a_i
2. DMD: $\phi_i(x,y)$, a_i and λ_i

Methodology

$$\mathbf{u}(x,y,t) = \sum_{i=1}^{N} \underbrace{a_i^{\mathrm{DMD}}}_{\text{amplitude}} \underbrace{\exp\left(\lambda_i^{\mathrm{DMD}} t\right)}_{\substack{\text{time}\\ \text{evolution}}} \underbrace{\phi_i^{\mathrm{DMD}}(x,y)}_{\substack{\text{spatial}\\ \text{mode}}}$$

Modes shapes comparison

$$\mathrm{MAC}\left(\phi_i^{EFD}, \phi_j^{CFD}\right) = \left(\frac{\phi_i^{EFD} \cdot \phi_j^{CFD}}{\|\phi_i^{EFD}\| \|\phi_j^{CFD}\|}\right)^2$$

Amplitudes and frequencies comparison

$$\mathrm{ERR}\left(a_i^{EFD}, a_i^{CFD}\right) = \frac{\|a_i^{EFD}\| - \|a_i^{CFD}\|}{\|a_i^{EFD}\|}$$

Data Collection ⇒ **Decomposition** ⇒ **Comparison**

1. EFD → Tr-PIV
2. CFD → uRANS

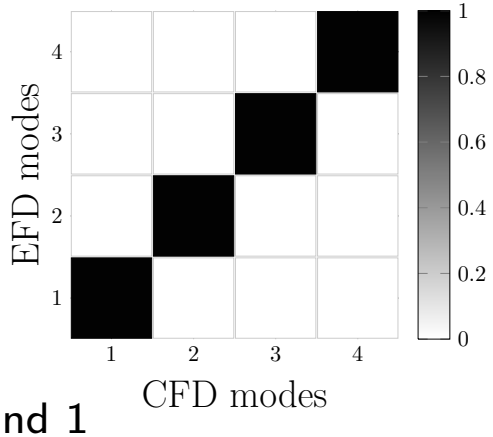
1. POD: $\phi_i(x,y)$ and a_i
2. DMD: $\phi_i(x,y)$, a_i and λ_i

1. $\phi_i(x,y)$
2. a_i and λ_i

Methodology

Modal Assurance Criterion

$$\mathrm{MAC}\left(\phi_i^{EFD}, \phi_j^{CFD}\right) = \left(\frac{\phi_i^{EFD} \cdot \phi_j^{CFD}}{\|\phi_i^{EFD}\| \|\phi_j^{CFD}\|}\right)^2$$



MAC takes values between 0 and 1

0 if modes are not correlated and $\rightarrow$ 1 as correlation increases

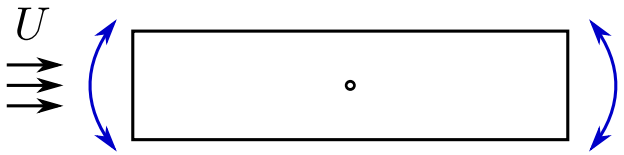
Data Collection $\Rightarrow$ **Decomposition** $\Rightarrow$ **Comparison**

1. EFD $\rightarrow$ Tr-PIV
2. CFD $\rightarrow$ uRANS

1. POD: $\phi_i(x, y)$ and a_i
2. DMD: $\phi_i(x, y)$, a_i and λ_i

1. $\phi_i(x, y)$
2. a_i and λ_i

Oscillating rectangle



- EFD: rectangle undergoing **LCO**
- CFD: pitching frequency and amplitude **imposed** (same as EFD)

Problem

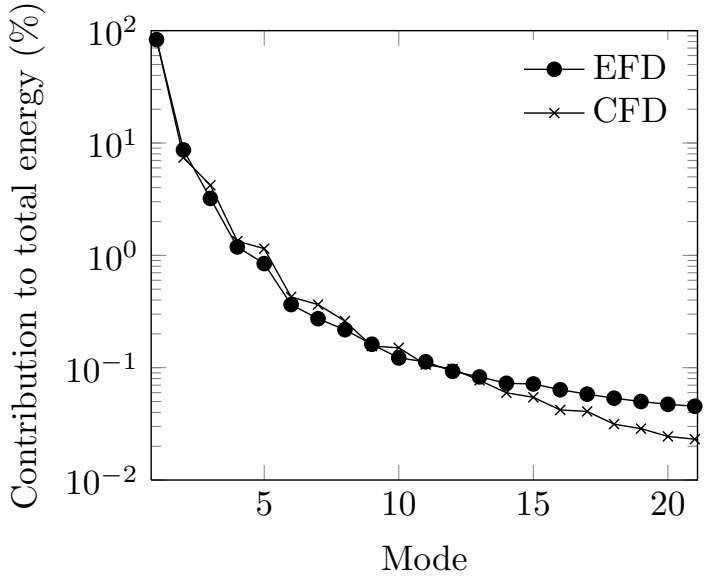
- No other EFD values than velocities
- CFD data need to be validated

$\Rightarrow$ **Use decomposition methods!**

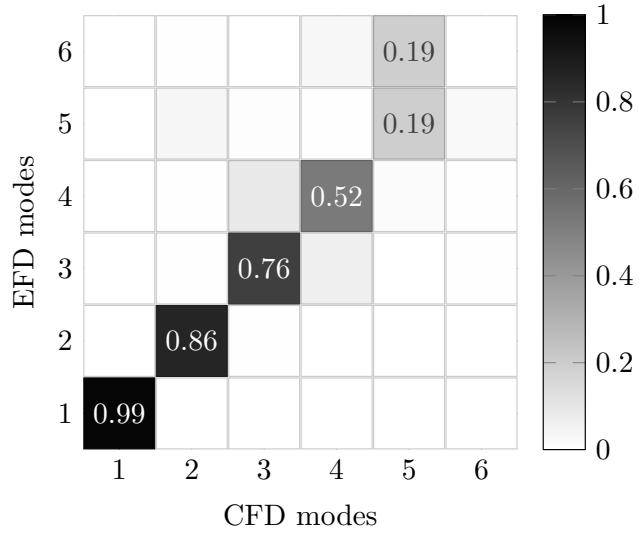
Oscillating rectangle

POD results

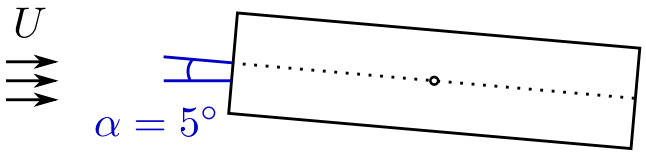
Energy content



MAC matrix



Static rectangle



Global values			
	$\overline{c_l}$	$\overline{c_d}$	St
CFD	0.83	0.46	0.136
EFD	0.53	0.45	0.152

Problem

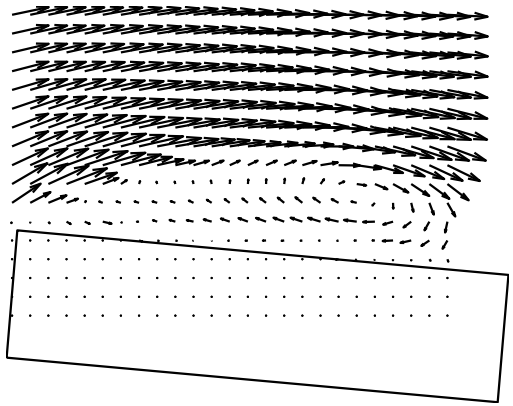
- $\overline{c_l}$ from CFD far from EFD value
- Why?

$\Rightarrow$ **Use decomposition methods!**

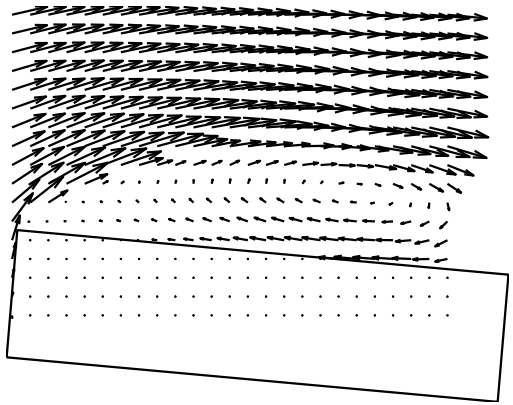
Static rectangle

DMD results

EFD: 2 modes



CFD: 2 modes



Static rectangle

DMD results

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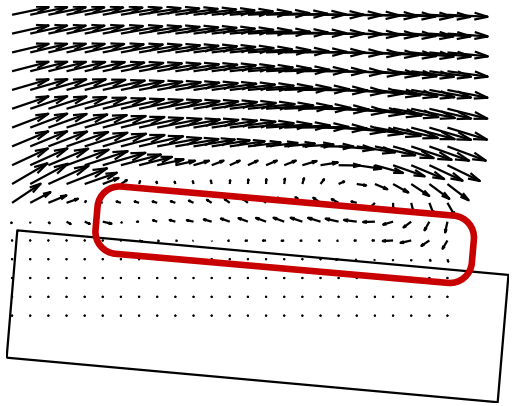

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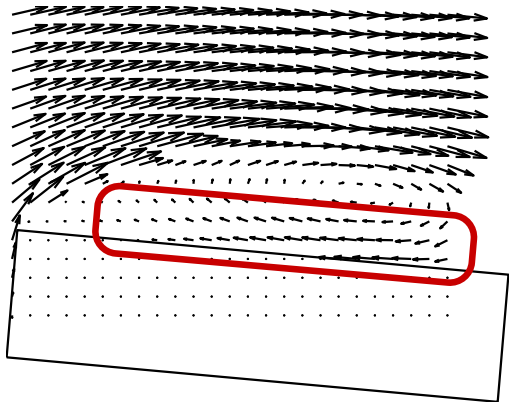


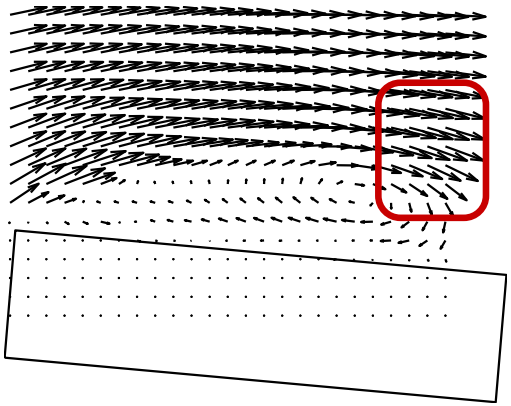
In CFD reconstruction

- More important reverse flow

Static rectangle

DMD results

EFD: 2 modes



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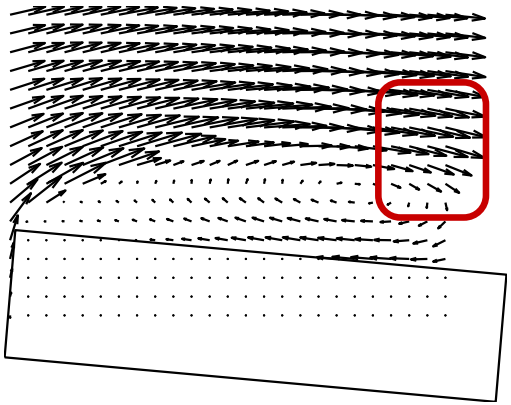


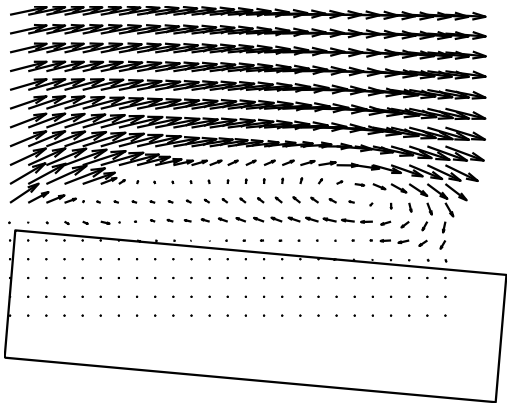
In CFD reconstruction

- More important reverse flow
- Discrepancies on the rear part

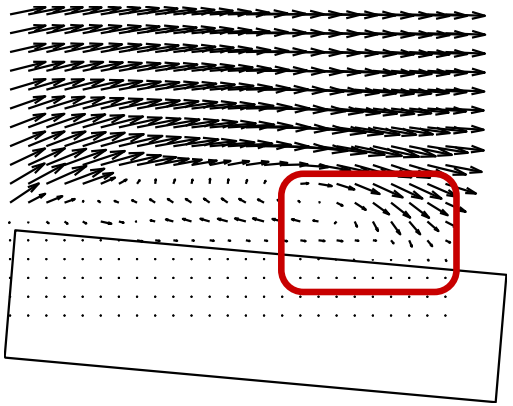
Static rectangle

DMD results

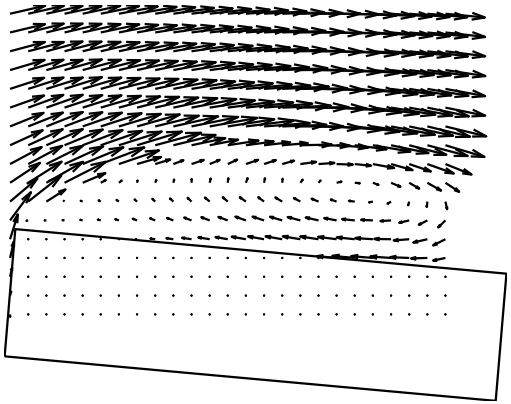
EFD: 2 modes



EFD: 3 modes



CFD: 2 modes



In CFD reconstruction

- More important reverse flow
- Discrepancies on the rear part
- Different dynamic of reattachment

Conclusion and future work

DMD and POD are useful

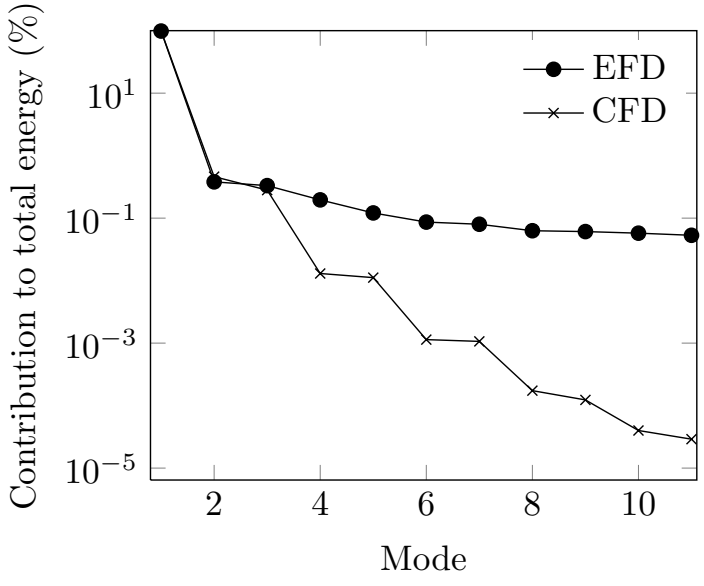
- Compare and validate CFD results
- Highlight and understand potential discrepancies

- Enlarge PIV window to get the rear part
- Apply DMD on lift evolution

Static rectangle

POD results

Energy content



MAC matrix