Optimal procurement decisions in the presence of total quantity discounts and alternative product recipes

Y. Crama[1], R. Pascual J.[2] and A. Torres[3]

October 2001

[1]University of Liège, Boulevard du Rectorat 7 (B31), 4000 Liège, Belgium, y.crama@ulg.ac.be

[2]Department of Mechanical Engineering, Universidad de Chile, Beaucheff 850, Santiago, Chile, rpascual@cec.uchile.cl

[3]Ferrocarriles del Pacifico, Alfredo Barros Errazuriz 1960, Providencia, Santiago, Chile, andrea.torres@entelchile.net

Abstract

We describe the cost-minimization problem faced by the purchasing department of a multi-plant company when its suppliers offer discounts based simultaneously on plant and on corporate purchases, when discount schedules depend on the total quantity (rather than cost) of ingredients purchased, and when alternative production recipes exist for each final product. We formulate the problem as a nonlinear mixed 0-1 programming problem and we propose various ways to linearize this formulation. The quality of these models is evaluated on real-world data.

Keywords: purchasing, supply chain management, mixed integer programming, quantity discounts, flexible recipes.

1 Introduction

Today's fierce competitive environment and strong emphasis on efficient supply chain management entice companies to select suppliers that score high on a broad array of performance criteria, with *price*, *quality* and *delivery reliability* ranking among the most important ones. Several publications have addressed this issue in recent years. Weber, Current and Benton [26], for instance, discuss the complexity of the procurement process from an OR perspective and review the literature on this subject (see also Weber and Current [25]). Recent papers by Degraeve, Labro and Roodhooft [7] or Ghodsypour and O'Brien [9] position the supplier selection issue in the context of the "total cost of ownership" or "total cost of logistics" criterion and propose a brief overview of the literature. Roethlein and Mangiameli [14] stress the point of view of the supplier. More references will be cited below.

Assuming that a company has made a preselection of suppliers who are able to satisfy its requirements for quality and reliability, a main selection criterion remains the purchasing cost of ingredients. Even in this restricted framework, however, the process leading to optimal purchasing decisions may be complicated by various factors. In this paper, we consider the medium-term purchasing decisions faced by a major chemical company and we develop an integer programming model which provides useful support to the planners.

The medium-term production plan of the company specifies the quantity of each *product* to be manufactured over the next year, based on forecasts provided by the marketing department. Each product is manufactured by blending various *ingredients* purchased from several *suppliers*. The suppliers are assumed to be reliable, and Just-In-Time strategies significantly reduce the inventory costs, which can therefore be disregarded from the analysis.

Two main characteristics concur to render the purchasing decision especially complex:
• each supplier offers *quantity discounts based on the total quantity of ingredients* purchased by the company;
• each product made by the company can be processed according to several *alternative recipes*, where each recipe specifies which proportion of each ingredient should be included in the blend.

Observe that, were it not for the second feature, and under the assumption that each ingredient can be purchased from a single supplier, the company would have no real decision to make: it would simply buy the ingredients prescribed by each product recipe from the appropriate suppliers and collect the discounts at the end of the year.

Also, were it not for the first feature, the company's problem would be easily solved: it would suffice, for each product, to adopt the recipe with the lowest cost.

The combination of both features, however, makes the problem more intricate and more interesting. To the best of our knowledge, it has not been previously investigated in this form.

Classical inventory models traditionally involve two types of discounts: either *quantity discounts*, i.e. discounts based on the quantity of each ingredient ordered from a supplier, or *business volume discounts*, i.e. discounts based on the total dollar value of all ingredients ordered from a supplier.

Quantity discount models involve distinct price breaks for each ingredient and supplier. This type of model is well-known and is discussed for instance in [3], [5], [11], [22], [23], [24]. Chaudry, Forst and Zydiak [5], in particular, consider a supplier selection problem involving multiple side-contraints: capacity, delivery performance, ingredient quality, etc. They propose a mixed integer programming formulation to minimize the purchasing costs for each ingredient separately.

Business volume discounts yield advantages both to the buyer and to the suppliers. This framework is described in detail by Sadrian and Yoon [19] and Katz, Sadrian and Tendick [12]. Buyers benefit because they diminish the number of active suppliers, which leads to reductions in the administrative costs and better relations with the suppliers. Suppliers simplify their discount schedules and promote more balanced sales over multiple ingredients. The application of this discount strategy may sometimes enable suppliers to sell at higher prices than those of the competitors. Also, larger orders reduce order processing costs (paperwork, setups, packaging, shipping) both for suppliers and buyers [20].

Sadrian and Yoon [19], [20] proposed a mixed-integer programming model to optimize the total cost of purchases in the presence of business volume discounts. Their model, just like ours, considers only one period, and thus does not take inventory costs and other time-dependent parameters into account. It is solved using a standard commercial mathematical progamming package. Related models dealing with *bundling* are discussed by Rosenthal, Zydiak, and Chaudhry [16] and Sarkis and Semple [21].

A third class of discounting strategies, to be further considered in this paper, is based on the concept of *total quantity discounts*. As mentioned above, we encountered this situation in the chemical industry. Here, the discount schedule of each supplier is expressed as a function of the total *quantity* of ingredients purchased over the year (rather than the total *dollar value* of these purchases, as in the previous case).

The problem gets even more complex when consumption forecasts for each *ingredient* are not known, but only demand forecasts for the *final products* are available. As mentioned above, final products are mixtures of ingredients and, as is very common in process industries, several alternative *recipes* are available to manufacture each final product (see e.g. Crama, Pochet and Wera [6], Rutten [17], Rutten and Bertrand [18]). Thus, in this case, consumption forecasts are expressed in terms of the final products, discount schedules are expressed in terms of the ingredients and there is no unique relationship between the demand for ingredients and the demand for products. These features result in a very complex problem, where decisions relating to production planning and to purchasing should be tightly integrated.

Finally, in the industrial situation that we encountered, the company operates several plants which purchase their ingredients from the same suppliers. Each supplier offers discounts based either on the *local* purchases of each plant or on the *consolidated* purchases of the company (we make this more precise below). It turns out that this last feature is, in a sense, the most difficult one to model, as it forces the company to consider several alternative discount schedules as well as alternative suppliers and alternative recipes.

The article is organized as follows. A more precise description of the problem is given in Section 2. Section 3 proposes a mathematical programming formulation for the single plant case, and Section 4 extends it to multiple plants. Some computational results are presented in Section 5. Finally, conclusions and future research directions are outlined in Section 6.

2 Problem statement and notations

We consider a company manufacturing a set of products $j = 1, 2, \ldots, J$. Each product can be obtained by blending a set of ingredients $i = 1, 2, \ldots, I$ according to certain recipes. Several recipes $r = 1, 2, \ldots, R$ are actually available for any given product and the company is free to choose among them (the production costs are not affected by this choice, beyond the cost of the ingredients themselves). Ingredients are purchased from a set of suppliers $s = 1, 2, \ldots, S$ (see Figure 1). Let us first consider the simplest case, where the company operates a single plant.

Each supplier s offers a discount schedule which only depends on the *total quantity* (expressed in tons, for example) of all the ingredients purchased by the plant over a year. The schedule is described by $D(s) + 1$ *cutoff points* $u_{s,0} = 0 < u_{s,1} < \ldots < u_{s,D(s)} = +\infty$ and by $D(s)$ corresponding discount rates $r_{s,1} < \ldots < r_{s,D(s)}$. If the plant buys Q_s tons from supplier s and Q_s belongs to the interval $[u_{s,d-1}, u_{s,d})$ of the discount schedule, then the supplier awards the discount rate $r_{s,d}$ on the total dollar value of the purchases. Thus, the schedule of supplier s can also be viewed as a piecewise constant curve consisting of D segments.

Discount programs based on the total quantity of purchases generate discontinuities in the cost of the purchases from a given supplier. Since there are several suppliers, the superposition of the discount schedules generates an intrincate discontinuous cost surface.

Demand forecasts for each product are available. The problem is to determine which recipe(s) should be used for each product and, simultaneously, which quantity of each ingredient should be purchased from each supplier, in order to satisfy demand. Note that the plant may decide to produce a fraction of the demand for product j according to recipe r_1 and another fraction according to a different recipe r_2. As discussed in the Introduction, we assume that the company only aims at minimizing its total purchasing costs.

$$\textit{plant} \left\{ \begin{array}{l} \textit{product } 1 \left\{ \begin{array}{l} \textit{recipe } 1 \left\{ \begin{array}{l} \textit{ingredient } 1 \\ \vdots \\ \textit{ingredient } I \end{array} \right. \\ \vdots \\ \textit{recipe } R \quad \dots \end{array} \right. \\ \vdots \\ \textit{product } J \quad \dots \end{array} \right. \quad \xLeftarrow{\textit{quantity discounts}} \quad \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \textit{supplier } 1 \\ \vdots \\ \textit{supplier } S \end{array}$$

Figure 1: The Simple Plant Total Quantity Discount model

The *business volume discount* problem described in [19] handles a simpler situation where a unique recipe is available for each product. In this case, if the demand for the final references is known, the demand for the input ingredients is also known (see Figure 2). Discount curves in [19] are similar to those for the *total quantity discount* problem, except that the total dollar value of purchases is used in order to define the curve segments, instead of the total quantity purchased.

$$\textit{plant} \left\{ \begin{array}{l} \textit{product } 1 \left\{ \begin{array}{l} \textit{ingredient } 1 \\ \vdots \\ \textit{ingredient } I \end{array} \right. \\ \vdots \\ \textit{product } J \quad \dots \end{array} \right. \quad \xLeftarrow{\textit{volume discounts}} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \textit{supplier } 1 \\ \vdots \\ \textit{supplier } S \end{array}$$

Figure 2: The Business Volume Discount model in [19]

Consider now the situation where the company operates several *plants* $p = 1, 2, \dots, P$ which manufacture different products. We assume that the company purchasing decisions are centralized and optimized at the company level, rather than by each individual plant. The suppliers offer *two* discount schedules to each plant. The first one, to be called *plant* or *local* schedule, is specific to each plant and is based exclusively on this plant's purchases as in the simple plant situation. The other one, to be called *company* or *group* schedule, is identical for all plants; it carries a rebate on the purchases of each individual plant, but its discount levels are a function of the consolidated purchases of the whole company. More precisely, consider a supplier s and a plant p. If the purchases from supplier s at plant p yield the discount rate rl on the plant schedule, while the consolidated company purchases yield the discount rate rg on the company schedule, then s awards the discount rate $\max\{rl, rg\}$ on the total dollar value of all purchases of plant p. In this case, the decision criterion consists in minimizing the total consolidated purchasing costs of the company, rather than the individual plant costs.

$$\left.\left\{\begin{array}{l} \textit{plant } 1 \left\{\begin{array}{l} \textit{product } 1 \left\{\begin{array}{l} \textit{recipe } 1 \left\{\begin{array}{l} \textit{ingredient } 1\\ \vdots \\ \textit{ingredient } I \end{array}\right. \\ \vdots \\ \textit{recipe } R \quad \dots \end{array}\right. \\ \vdots \\ \textit{product } J \quad \dots \end{array}\right. \\ \vdots \\ \textit{plant } P \quad \dots \end{array}\right. \qquad \begin{array}{c} \overset{\textit{plant discounts}}{\Longleftarrow} \\ \overset{\textit{company discounts}}{\Longleftarrow} \end{array} \right\} \begin{array}{l} \textit{supplier } 1 \\ \vdots \\ \textit{supplier } S \end{array}$$

Figure 3: The Multi-plant Total Quantity Discount model

Several *mixed integer linear programming* (*MILP*) formulations of the total quantity discount problem are presented in the next sections. For the sake of clarity, we start with a formulation of the simple-plant problem before proceeding with the more complex multi-plant formulations.

Since a standard commercial package will be used to solve these problems, we do not concentrate on the solution process, but only on the choice of the most appropriate model. In order to stress the structure of the models and to facilitate their understanding, we use lower case letters for parameters and upper case letters for decision variables. The following indices are used in all models: $p = 1, 2, \ldots, P$ for *plants*, $i = 1, 2, \ldots, I$ for *ingredients*, $j = 1, 2, \ldots, J$ for *products*, $r = 1, 2, \ldots, R$ for *recipes*, $s = 1, 2, \ldots, S$ for *suppliers*, $d = 1, 2, \ldots$ for *discount segments*. Unless explicitly stated otherwise, these indices always run over their full range of possible values. (Note that the range of d may depend on s, p, and on whether we consider local or global discounts. For simplicity of notations, we usually do not indicate this dependence explicitly, but it should be clear from the context).

Finally, all variables are implicitly constrained to be non-negative.

3 Simple plant formulation

3.1 Parameters

Let us introduce the following parameters.

$o_{i,s}$	$\begin{cases} 1 & \text{if ingredient } i \text{ is offered by supplier } s \\ 0 & \text{otherwise} \end{cases}$
$w_{i,r}$	quantity of ingredient i used to obtain one unit of product according to recipe r (where the recipe implicitly defines the product)
dem_j	demand forecast for product j
p_i	unit price of ingredient i
$r_{s,d}$	discount rate associated with segment d of supplier s' schedule
$u_{s,d}$	upper cutoff point for segment d of supplier s' schedule
$pmax_s$	upper bound on the price of ingredients offered by supplier s

The formulation is simplified if we assume that each ingredient is offered by one supplier only ($\sum_s o_{i,s} = 1$ for all i). This was actually the case in the industrial environment that we encountered, where ingredients sold by different suppliers were sometimes close substitutes, but never exactly identical. In case a same ingredient would be offered by several suppliers, then the recipes could be artificially duplicated in order to abide by our assumption.

We further assume that total mass is conserved in each recipe ($\sum_i w_{i,r} = 1$ for all r), and that the schedule cutoff points are listed in increasing order ($u_{s,d-1} < u_{s,d}$ for all s, d).

3.2 Variables

Four classes of decision variables are defined. In a first attempt, one may want to introduce variables Q_s and V_s, indicating respectively how many tons of ingredients are purchased from supplier s and the total dollar business volume awarded to supplier s, for $s = 1, 2, \ldots, S$. These variables, however, do not allow for an appropriate expression of the discounts. Therefore, it is necessary to split each of these variables into D copies $Q_{s,d}$ and $V_{s,d}$, respectively, corresponding to D segments in the discount schedule. In every feasible solution, at most one of $Q_{1,s}, \ldots, Q_{s,d}$ and at most one of $V_{1,s}, \ldots, V_{s,d}$ will be nonzero (this is similar to [12, 20]).

$F_{j,r}$ quantity of product j produced using recipe r

$I_{s,d}$ $\begin{cases} 1 & \text{if the total quantity purchased from supplier } s \text{ gives right to the discount rate } r_{s,d} \\ 0 & \text{otherwise} \end{cases}$

$Q_{s,d}$ total quantity of ingredients purchased from supplier s and carrying the discount rate $r_{s,d}$

$V_{s,d}$ business volume awarded to supplier s and carrying the discount rate $r_{s,d}$.

3.3 Model

The aim is to minimize the total purchasing costs of the company over the given horizon:

$$\min \ \text{Cost} \ = \sum_s \sum_d (1 - r_{s,d}) \cdot V_{s,d}. \tag{1}$$

A first set of constraints express that demand must be satisfied for each product:

$$\sum_r F_{j,r} = dem_j \qquad \text{for all } j. \tag{2}$$

The next constraints define the total quantity of ingredients purchased from each supplier and the corresponding business volume:

$$\sum_d Q_{s,d} \ = \ \sum_i o_{i,s} \sum_j \sum_r \ w_{i,r} \cdot F_{j,r} \qquad \text{for all } s, \tag{3}$$

$$\sum_d V_{s,d} \ = \ \sum_i o_{i,s}\, p_i \sum_j \sum_r \ w_{i,r} \cdot F_{j,r} \qquad \text{for all } s. \tag{4}$$

For each supplier s, at most one of the variables $Q_{s,d}$ can be non-zero. Some modelling languages allow to express this constraint by simply stating that "the variables $Q_{s,d}$ form a *special ordered set of type* 1" (SOS1) for each s (see e.g. [2, 27]). We adopt here a more generic approach, as in [20]. Namely, we use the auxiliary variables $I_{s,d}$ which link the total quantities $Q_{s,d}$ to the appropriate segments of the discount schedules:

$$Q_{s,d} \ \leq \ u_{s,d} \cdot I_{s,d} \qquad \text{for all } \ s, d, \tag{5}$$

$$Q_{s,d} \ \geq \ u_{s,d-1} \cdot I_{s,d} \qquad \text{for all } s, d, \tag{6}$$

$$\textstyle\sum_d I_{s,d} \ = \ 1 \qquad \text{for all } s \tag{7}$$

(recall that each $I_{s,d}$ is a zero-one variable). Observe that, if the total quantity supplied by s is exactly equal to some cutoff point $u_{s,d}$, then feasible solutions are defined by setting either $I_{s,d} = 1$ and $Q_{s,d} = u_{s,d}$, or $I_{s,d+1} = 1$ and $Q_{s,d+1} = u_{s,d}$. But since $r_{s,d} < r_{s,d+1}$, the higher discount rate will be selected in the optimal solution, in agreement with our definition of the discount intervals.

Finally, we relate the variables $V_{s,d}$ to the variables $Q_{s,d}$ by imposing the constraints:

$$V_{s,d} \leq pmax_s \cdot Q_{s,d} \qquad \text{for all } s, d. \tag{8}$$

These constraints ensure that $V_{s,d}$ is zero whenever $Q_{s,d}$ is zero. Together with constraints (4), this is sufficient to enforce the correct value of $V_{s,d}$, and model (1)-(8) provides a complete formulation of the simple plant problem.

4 Multi-plant formulation

We now turn to the more general case involving multiple plants. We start with a definition of the problem parameters.

4.1 Parameters

$o_{i,s}$	$\begin{cases} 1 & \text{if ingredient } i \text{ is offered by supplier } s \\ 0 & \text{otherwise} \end{cases}$
$c_{p,j}$	$\begin{cases} 1 & \text{if product } j \text{ is manufactured at plant } p \\ 0 & \text{otherwise} \end{cases}$
$w_{i,r}$	quantity of ingredient i used to obtain one unit of product according to recipe r
dem_j	demand forecast for product j
p_i	unit price of ingredient i
$rl_{p,s,d}$	discount rate associated with segment d of supplier s' local schedule for plant p
$ul_{p,s,d}$	upper cutoff point for segment d of supplier s' local schedule for plant p
$rg_{s,d}$	discount rate associated with segment d of supplier s' group schedule for the company
$ug_{s,d}$	upper cutoff point for segment d of supplier s' group schedule for the company
$pmax_{p,s}$	upper bound on the price of ingredients offered by supplier s at plant p
$vmax_{p,s}$	upper bound on the total purchases from supplier s by plant p.

As in the single-plant case, we assume that ingredients are supplier-exclusive ($\sum_s o_{i,s} = 1$ for all i), that total mass is conserved in each recipe ($\sum_i w_{i,r} = 1$ for all r), and that the schedule cutoff points are listed in increasing order ($ul_{p,s,d-1} < ul_{p,s,d}$ and $ug_{s,d-1} < ug_{s,d}$ for all p, s, d). We also assume that each product is manufactured in a unique plant ($\sum_p c_{p,j} = 1$ for all j).

Except for $vmax_{p,s}$, all parameters are readily available as part of the problem description. A value for $vmax_{p,s}$ is easily obtained by assuming for instance that, for each product, plant p always selects the recipe which entails the largest amount of purchases from supplier s.

4.2 Variables

Unfortunately, we need to introduce quite a lot of variables. Their meaning will hopefully become clear in subsequent sections. For now, let us just stress that IL, QL and VL refer to local discounts to which each plant is entitled *independently of* the company's consolidated purchases; IG and QG refer to group discounts to which each plant is entitled *as a consequence of* the company's cumulative purchases; I, L and G refer to the discounts which are *actually applied* on each plant's purchases after comparing local and group discounts.

$F_{j,r}$ quantity of product j produced using recipe r

$$IL_{p,s,d} \begin{cases} 1 & \text{if the total quantity purchased by plant } p \text{ from supplier } s \\ & \text{entitles } p \text{ to the discount rate } rl_{p,s,d} \text{ on the \textit{local} schedule;} \\ 0 & \text{otherwise;} \end{cases}$$

$$IG_{s,d} \begin{cases} 1 & \text{if the total quantity purchased by the company from supplier } s \\ & \text{entitles each plant to the discount rate } rg_{s,d} \text{ on the \textit{group} schedule;} \\ 0 & \text{otherwise;} \end{cases}$$

$$I_{p,s} \begin{cases} 1 & \text{if the discount rate associated to the local \textit{schedule} is eventually} \\ & \text{applied to the total quantity purchased by plant } p \text{ from supplier } s\text{;} \\ 0 & \text{if the discount rate associated to the \textit{group} schedule is eventually} \\ & \text{applied to the total quantity purchased by plant } p \text{ from supplier } s\text{.} \end{cases}$$

$V_{p,s}$	business volume awarded to supplier s by plant p;
$QL_{p,s,d}$	total quantity of ingredients purchased from supplier s by plant p if it entitles p to the discount rate $rl_{p,s,d}$ on the *local* schedule; 0 otherwise;
$VL_{p,s,d}$	business volume awarded to supplier s by plant p if it entitles p to the discount rate $rl_{p,s,d}$ on the *local* schedule; 0 otherwise;
$QG_{s,d}$	total quantity of ingredients purchased from supplier s by the *company* if it entitles each plant to the discount rate $rg_{s,d}$ on the *group* schedule; 0 otherwise;
$L_{p,s,d}$	business volume awarded to supplier s by plant p if the *local* rate $rl_{p,s,d}$ is eventually applied to this volume; 0 otherwise;
$G_{p,s,d}$	business volume awarded to supplier s by plant p if the *group* rate $rg_{s,d}$ is eventually applied to this volume; 0 otherwise.

4.3 Model

The cost function (9) decomposes into two terms: the first one models the costs to which local discounts apply; the second one models the costs to which group discounts apply.

$$\min \ \text{Cost} = \underbrace{\sum_p \sum_s \sum_d (1 - rl_{p,s,d}) \cdot L_{p,s,d}}_{\textit{with local discounts}} + \underbrace{\sum_p \sum_s \sum_d (1 - rg_{s,d}) \cdot G_{p,s,d}}_{\textit{with group discounts}}. \quad (9)$$

- **Demand constraints**

Demand constraints have the usual form:

$$\sum_r F_{j,r} = dem_j \qquad \text{for all } j. \quad (10)$$

- **Quantities ordered and business volumes - plant level**

The total quantity purchased by each plant from each supplier is expressed as:

$$\sum_d QL_{p,s,d} = \sum_i o_{i,s} \sum_j c_{p,j} \sum_r w_{i,r} \cdot F_{j,r} \qquad \text{for all } p, s. \quad (11)$$

Next, we find it convenient to express in two different ways the business volume of plant p with each supplier s, as the auxiliary variables $V_{p,s}$ will be used separately below (see constraints (29)-(30)).

$$\sum_d VL_{p,s,d} = V_{p,s} \qquad \text{for all } p, s, \tag{12}$$

$$V_{p,s} = \sum_i o_{i,s}\, p_i \sum_j c_{p,j} \sum_r w_{i,r} \cdot F_{j,r} \qquad \text{for all } p, s. \tag{13}$$

- **Positioning in the discount schedule - plant level**

The next constraints link the local variables $IL_{p,s,d}$, $QL_{p,s,d}$, $VL_{p,s,d}$ in the same way as in the simple plant model.

$$VL_{p,s,d} \leq pmax_{p,s} \cdot QL_{p,s,d} \qquad \text{for all } p, s, d, \tag{14}$$

$$QL_{p,s,d} \leq ul_{p,s,d} \cdot IL_{p,s,d} \qquad \text{for all } p, s, d, \tag{15}$$

$$QL_{p,s,d} \geq ul_{p,s,d-1} \cdot IL_{p,s,d} \qquad \text{for all } p, s, d, \tag{16}$$

$$\textstyle\sum_d IL_{p,s,d} = 1 \qquad \text{for all } p, s. \tag{17}$$

- **Quantities ordered - company level**

The cumulated variables $QG_{s,d}$ are defined from their plant counterparts.

$$\sum_d QG_{s,d} = \sum_p \sum_d QL_{p,s,d} \quad \text{for all } s. \tag{18}$$

- **Positioning in the discount schedule - company level**

Similarly to constraints (15)-(17), we impose:

$$QG_{s,d} \leq ug_{s,d} \cdot IG_{s,d} \qquad \text{for all } s, d, \tag{19}$$

$$QG_{s,d} \geq ug_{s,d-1} \cdot IG_{s,d} \qquad \text{for all } s, d, \tag{20}$$

$$\textstyle\sum_d IG_{s,d} = 1 \qquad \text{for all } s. \tag{21}$$

- **Linking the business volume variables**

Note that the variables $L_{p,s,d}$ and $G_{p,s,d}$ appearing in the objective function (9) have not been linked, yet, to other variables representing business volumes. In view of the interpretation of the variables, there holds

$$L_{p,s,d} = I_{p,s} \cdot VL_{p,s,d} \qquad \text{for all } p, s, d, \tag{22}$$

and

$$G_{p,s,d} = (1 - I_{p,s}) \cdot IG_{s,d} \cdot V_{p,s} \qquad \text{for all } p, s, d. \tag{23}$$

Unfortunately, these equations are nonlinear. Since most commercial optimization systems are unable to handle large scale mixed 0-1 nonlinear programming problems, additional modelling work is required here. In the next sections, we examine alternative ways to remove these nonlinearities.

4.4 Removing nonlinearities - I

Constraints (22) and (23) both involve products of binary and continuous variables. Several authors have proposed generic ways to *linearize* such constraints, i.e. to replace them by equivalent linear constraints (see e.g. [1, 2, 4, 10, 13, etc.]). A standard trick to linearize a constraint of the form $Y = I_1\, I_2\, X$, where I_1, I_2 are binary variables and X, Y are real variables subject to the range constraint $0 \leq X \leq u$, consists in imposing the following constraints:

$$Y \leq u\, I_1$$

$$Y \leq u\, I_2$$

$$Y \leq X$$

$$Y \geq X - u\,(1 - I_1) - u\,(1 - I_2)$$

$$Y \geq 0.$$

It is easy to see that these constraints force Y to take the value of the product $I_1\, I_2\, X$ for every assignment of 0-1 values to I_1 and I_2.

Let us apply this procedure in order to linearize (22) (here, one of the binary variables is viewed as identically 1). Remember that $vmax_{p,s}$ is, by definition, an upper-bound on $V_{p,s}$, and hence on $VL_{p,s,d}$, for all p, s, d. Besides the nonnegativity contraints (which we always treat implicitly), we obtain

$$L_{p,s,d} \leq vmax_{p,s} \cdot I_{p,s} \qquad \text{for all } p, s, d, \tag{24}$$

$$L_{p,s,d} \leq VL_{p,s,d} \qquad \text{for all } p, s, d, \tag{25}$$

$$L_{p,s,d} \geq VL_{p,s,d} - vmax_{p,s} \cdot (1 - I_{p,s}) \qquad \text{for all } p, s, d. \tag{26}$$

We next turn to (23), and we obtain

$$G_{p,s,d} \leq vmax_{p,s} \cdot (1 - I_{p,s}) \qquad \text{for all } p, s, d, \tag{27}$$

$$G_{p,s,d} \leq vmax_{p,s} \cdot IG_{s,d} \qquad \text{for all } p, s, d, \tag{28}$$

$$G_{p,s,d} \leq V_{p,s} \qquad \text{for all } p, s, d, \tag{29}$$

$$G_{p,s,d} \geq V_{p,s} - vmax_{p,s} \cdot I_{p,s} - vmax_{p,s} \cdot (1 - IG_{s,d}) \qquad \text{for all } p, s, d. \tag{30}$$

We claim that the cost function (9) and the contraints (10)-(21), (24)-(30) give a complete formulation of the multi-plant problem. Since the model is non trivial, it may actually be useful to establish the validity of this claim. Before we proceed with a proof, we first observe that we can safely discard the constraints (24)-(25) and (27)-(29) without affecting the optimal solutions of the model.

Lemma. Every optimal solution of the model (9)-(21), (26), (30) satisfies the constraints (24)-(25) and (27)-(29).

Proof. Note that the variables $L_{p,s,d}$ and $G_{p,s,d}$ only appear in the cost function, in the constraints (24)-(26), (27)-(30), and in the implicit nonnegativity constraints. Moreover, $L_{p,s,d}$ and $G_{p,s,d}$ have positive coefficients in the cost function. Therefore, in every optimal solution of (9)-(21), (26), (30), there holds

$$L_{p,s,d} = \max\{0, VL_{p,s,d} - vmax_{p,s} \cdot (1 - I_{p,s})\}$$

and

$$G_{p,s,d} = \max\{0, V_{p,s} - vmax_{p,s} \cdot I_{p,s} - vmax_{p,s} \cdot (1 - IG_{s,d})\}.$$

Now, it is easy to check that these values satisfy (24)-(25) and (27)-(29). □

In the sequel, we refer to the model (9)-(21), (26), (30) as model *Multi-plant 1*, or *MP1* for short. (In this model, the variables $V_{p,s}$ could actually be eliminated by substituting the left-hand side of (12) for $V_{p,s}$ in all other constraints. But, except for reducing the number of variables and constraints, this substitution does not result in any improvement of the formulation, i.e., it does not tighten the mixed-integer programming model.)

We now sketch a proof that the model is indeed correct.

Proposition 1. Model *MP1* provides a correct description of the multi-plant total quantity discount problem.

Proof. Consider any optimal mixed 0-1 solution of the model. Total demand is satisfied, since constraint (10) holds. Note that the remaining constraints can be partitioned into S independent subsystems, corresponding to the different suppliers. Therefore, we focus in the sequel on a fixed supplier s.

(a) For every plant p, by (15)-(17), there exists a unique discount rate $d_1(p)$ such that $QL_{p,s,d} = IL_{p,s,d} = 0$ for all $d \neq d_1(p)$. Moreover, by (14), $VL_{p,s,d} = 0$ for all $d \neq d_1(p)$.

(b) Similarly, because of (19)-(21), there exists a unique discount rate d_2 such that $QG_{s,d} = IG_{s,d} = 0$ for all $d \neq d_2$.

(c) In view of (a), (b) and (18), there holds $QG_{s,d_2} = \sum_p QL_{p,s,d_1(p)}$.

(d) For every plant p, we deduce from (a) and (11) that the total quantity of ingredients purchased by plant p from supplier s is equal to $QL_{p,s,d_1(p)}$. Similarly, from (a) and (12)-(13), the business volume awarded by plant p to supplier s is equal to $VL_{p,s,d_1(p)} = V_{p,s}$.

(e) From (c) and (d), we deduce that QG_{s,d_2} is the total quantity of ingredients purchased from supplier s.

(f) By Lemma 1, (24)-(25), (27)-(29) hold, and hence (22) and (23) also hold.

(g) We would like now to conclude that all quantities $L_{p,s,d}$ and $G_{p,s,d}$ have their intended meaning. Fix a plant p. There are two cases.
• If $I_{p,s} = 1$, then $G_{p,s,d} = 0$ for all d, by (23). Moreover, (a), (d) and (22) together imply that $L_{p,s,d} = 0$ for all $d \neq d_1(p)$, and $L_{p,s,d_1(p)} = VL_{p,s,d_1(p)} = V_{p,s}$ as required.
• Assume now that $I_{p,s} = 0$. Then, by (22), $L_{p,s,d} = 0$ for all d. By (b), (e) and (23), $G_{p,s,d} = 0$ for all $d \neq d_2$, and $G_{p,s,d_2} = V_{p,s}$ as required.

In conclusion, for each supplier s and plant p, at most one of the quantities $L_{p,s,d}$ and $G_{p,s,d}$ does not vanish. This nonzero quantity (either $L_{p,s,d_1(p)}$ or G_{p,s,d_2}) exactly represents the business volume of plant p with supplier s, and the corresponding discount rate (either $rl_{p,s,d_1(p)}$ or rg_{s,d_2}) is indeed applicable to this business volume. Hence, the objective function (9) correctly computes the costs incurred. $\square$

4.5 Removing nonlinearities - II

We next propose another approach to linearize the constraints (22) and (23). Although this alternative approach is more *ad hoc* than the first one, it will prove, in our experiments, to provide a tighter formulation of the problem.

The main idea consists in substituting the constraints (26) and (30) (which essentially force one of $L_{p,s,d_1(p)}$ or G_{p,s,d_2} to take the value $V_{p,s}$ - see point (g) in the proof of Proposition 1) by a new aggregated constraint

$$\sum_d L_{p,s,d} + \sum_d G_{p,s,d} = \sum_d VL_{p,s,d} \qquad \text{for all } p, s. \tag{31}$$

For the resulting formulation to be complete, we need to add a constraint which guarantees that $G_{p,s,d}$ is zero when $IG_{s,d}$ is zero. Contraint (28) would do the job, but we prefer to use the following contraint, which is usually stronger:

$$G_{p,s,d} \leq pmax_{p,s} \cdot QG_{s,d} \qquad \text{for all } p, s, d. \tag{32}$$

Note that the relative strength of (28) and (32) really depends on the numerical value chosen for the bound $vmax_{p,s}$ in (28). For instance, if one replaces $vmax_{p,s}$ by $pmax_{p,s} \cdot ug_{s,d}$ (a rather reasonable choice), then (19) implies that (32) is a tighter constraint then (28).

Another valid constraint which can also be substituted for (32) (or better yet, added to it) is the aggregated constraint

$$\sum_p G_{p,s,d} \leq \max_p (pmax_{p,s}) \cdot QG_{s,d} \qquad \text{for all } s, d. \tag{33}$$

This constraint is valid, as it expresses that the d-th discount rate will not be applied by supplier s when $QG_{s,d} = 0$.

The model (9)-(21), (25), (31)-(33) will be referred to as model *Multi-plant 2*, or *MP2* for short. Note that the variables $I_{p,s}$ do not play any role in this model, and can be suppressed altogether. The variables $V_{p,s}$ could also be eliminated, exactly as in model *MP1*.

Proposition 2. Model *MP2* provides a correct description of the multi-plant total quantity discount problem.

Proof. The proof of Proposition 1 remains valid, up to and including step (e).

(f') Fix p and s. By step (a), $VL_{p,s,d} = 0$ for all $d \neq d_1(p)$. Hence, constraint (25) implies that $L_{p,s,d} = 0$ for all $d \neq d_1(p)$. Similarly, step (b) and (32) (or (33)) imply that $G_{p,s,d} = 0$ for all $d \neq d_2$. So, there follows from constraint (31) that

$$L_{p,s,d_1(p)} + G_{p,s,d_2} = VL_{p,s,d_1(p)} \tag{34}$$

and, in view of step (d), this quantity represents exactly the business volume awarded by plant p to supplier s. As a matter of fact, (34) is the only equation restricting the values of $L_{p,s,d_1(p)}$ and G_{p,s,d_2} in the model. Therefore, because of the form of the cost function (9), there is an optimal solution of the model in which at most one of the variables $L_{p,s,d_1(p)}$ and G_{p,s,d_2} is nonzero (namely, that variable corresponding to the maximum of the discounts $rl_{p,s,d_1(p)}$ and rg_{s,d_2}). $\square$

4.6 A relaxed formulation

In models $MP1$ and $MP2$, the purchased quantities of each ingredient allow to satisfy *exactly* the demand, by virtue of the demand constraints (10). In practice, however, the company may want to exploit the discontinuities of the discount curves by buying *more* ingredients than strictly necessary. Indeed, as a bizarre implication of the discount policies, total purchasing costs can sometimes be reduced by raising the purchased volume into the next segment of a discount curve (a similar effect has already been observed by Sarkis and Semple [21] in the context of bundled purchases). We now propose a model which accounts for this possibility.

Remark. The practical side-implications of overbuying are not entirely clear, and may vary with each specific industrial setting. For instance, inventory costs may rise, or some of the superfluous ingredients may have to be eliminated. In other contexts, they may also be put to some alternative use, or sold on a secondary market. A reasonable assumption may actually be that the additional quantities would not be purchased at all, even when it would prove advantageous to do so, but that the mere existence of this possibility could be used in negotiations with the suppliers in order to obtain further rebates. Therefore, in our formulation, we limit ourselves to the consideration of purchasing costs.

In order to relax the link between demand and purchase levels, we introduce a new class of variables $QE_{p,s}$, representing the number of additional units of ingredients which plant p purchases from supplier s. Since the only use of these units is to increase the total quantity purchased, we may as well assume that they correspond to the cheapest ingredient offered by s to p. These extra units are added to constraints (11) and (13), which respectively become

$$\sum_d QL_{p,s,d} = \sum_i o_{i,s} \sum_j c_{p,j} \sum_r w_{i,r} \cdot F_{j,r} + QE_{p,s} \qquad \text{for all } p, s, \tag{35}$$

$$V_{p,s} = \sum_i o_{i,s}\, p_i \sum_j c_{p,j} \sum_r w_{i,r} \cdot F_{j,r} + pmin_{p,s} \cdot QE_{p,s} \qquad \text{for all } p, s, \tag{36}$$

where $pmin_{p,s}$ is the lowest price of an ingredient offered by supplier s to plant p.

The model obtained when substituting (35) and (36) for (11) and (13) in *MP2* will be referred to as model *MP3*. Conversely, *MP2* can be viewed as the restriction of *MP3* where extra purchases are frozen to zero.

4.7 Extensions

Various extensions of the above models can easily be formulated, for instance in order to limit the volume of business with certain suppliers (e.g., when their reliability is questionable), or to express their capacity restrictions on certain ingredients.

In the chemical industry application that we tackled, we had to face another extension of the basic model. Besides the nominal price p_i of ingredient i, each supplier can quote a second price p_i^σ, called the *spot price* of i. The spot price is lower than the nominal price. When both prices are offered for a particular ingredient, each plant can decide whether the ingredient will be acquired at its nominal price or at its spot price. However, if the spot price is selected, then the quantities purchased at that price *are not* taken into consideration when computing the discount to be awarded.

To account for this added complexity, we duplicate every ingredient for which a spot price has been quoted and we introduce a new parameter σ_i for each ingredient, where $\sigma_i = 1$ if i is a spot ingredient, and $\sigma_i = 0$ otherwise. The cost function (9) is now replaced by the following expression:

$$\begin{aligned} \min\ \text{Cost} \quad = \quad & \sum_p \sum_s \sum_d (1 - rl_{p,s,d}) \cdot L_{p,s,d} + \sum_p \sum_s \sum_d (1 - rg_{s,d}) \cdot G_{p,s,d} \\ + \quad & \sum_p \sum_s \sum_i o_{i,s}\, \sigma_i\, p_i^\sigma \sum_j c_{p,j} \sum_r w_{i,r} \cdot F_{j,r}. \end{aligned} \tag{37}$$

The last term in the expression (37) corresponds to the purchasing cost of the spot ingredients. The right-hand sides of (11) and (13) have to be adapted accordingly. Constraint (11) becomes

$$\sum_d QL_{p,s,d} = \sum_i o_{i,s}\,(1 - \sigma_i) \sum_j c_{p,j} \sum_r w_{i,r} \cdot F_{j,r} \qquad \text{for all } p, s \tag{38}$$

and constraint (13) becomes

$$V_{p,s} = \sum_i o_{i,s}\,(1 - \sigma_i)\, p_i \sum_j c_{p,j} \sum_r w_{i,r} \cdot F_{j,r} \qquad \text{for all } p, s. \tag{39}$$

These changes yield a valid formulation of model *MP2* in the presence of spot prices. Similar modifications of (35) and (36) are necessary when using the relaxed model *MP3*.

5 Experimental evaluation

5.1 Main test case

Data for the test case were obtained from a large chemical company that operates two plants. Plant I manufactures a mix of 30 distinct products and plant II manufactures 7 additional products. Each product can be processed according to several alternative recipes: from 1 to 15 recipes per product, with a grand total of 52 recipes. Each recipe uses from 1 to 3 basic ingredients. Altogether, 25 different ingredients can be purchased from eight suppliers.

All three mixed integer models *MP1, MP2* and *MP3* have been formulated in the AIMMS modeling language version 3.2 [2] and solved by branch-and-bound using the XA solver on a 750 MHz Pentium III with 256Mb RAM. The main goal of the experiments was to evaluate and to compare the difficulty of solving these alternative formulations. When interpreting the results, it is necessary to remember that *MP3* is slightly different from the other formulations, since it allows for extra purchases.

Table 1 displays the size of the models.

	$MP1$	$MP2$	$MP3$
Nr of variables	901	888	904
Nr of constraints	886	966	966

Table 1: Problem size

An important user parameter of the XA solver is the Relative Optimality Criterion - *ROC* - which stops the branch-and-bound process if the solver can guarantee that the cost of the best current solution is less than *ROC*% away from the global optimum. Three tests were performed. The first one uses a *ROC* value of 0%. The second one uses a value of 1%. The third one uses a value of 5%. Moreover, the maximum number of branch-and-bound iterations was set to 10000. The main results of the tests are displayed in Tables 2 -4. The first line of each table indicates the number of iterations performed by the solver. The second line gives the total computing time in seconds. The third line contains the cost of the best available solution (i.e., purchasing plan). The fourth line gives the optimal value of the linear relaxation of each model (i.e., of the LP model obtained by relaxation of the integrality constraints). Finally, the last line indicates whether the *ROC* criterion was satisfied upon termination of the branch-and-bound process.

	$MP1$	$MP2$	$MP3$
Iterations	10000	10000	1634
Computing time (s)	9	9	1
Cost	179.67	178.86	178.45
LP relaxation	0.00	178.45	178.45
ROC criterion satisfied	*no*	*no*	*yes*

Table 2: ROC 0%

Only model *MP3* can be solved to optimality (i.e., with $ROC = 0\%$) within 10000 iterations, but *MP2* is almost as easy, since it can be solved within 1% of optimality in

	$MP1$	$MP2$	$MP3$
Iterations	10000	725	363
Computing time (s)	9	1	1
Cost	181.18	179.59	178.64
LP relaxation	0.00	178.45	178.45
ROC criterion satisfied	*no*	*yes*	*yes*

Table 3: ROC 1%

	$MP1$	$MP2$	$MP3$
Iterations	10000	725	363
Computing time (s)	9	1	1
Cost	183.03	179.03	178.64
LP relaxation	0.00	178.45	178.45
ROC criterion satisfied	*no*	*yes*	*yes*

Table 4: ROC 5%

1 second of CPU time. Model *MP1*, however, appears to be more difficult to handle: for this model, good solutions are obtained by the solver in a reasonable amount of CPU time, but the quality of these solutions can only be assessed by reference to the lower bound ("LP relaxation") computed for model *MP2*. As a matter of fact, both *MP2* and *MP3* yield linear programming relaxations whose optimal value is very close to the optimal value of the *MIP* models (within 1%), thus giving rise to small branch-and-bound trees. The LP relaxation of model *MP1*, by contrast, is extremely weak, and this prevents the branch-and-bound process from converging within an acceptable time limit (we checked that for this model, the best available lower bound is still equal to zero after 10^6 branch-and-bound iterations).

In conclusion, both models *MP2* and *MP3* appear to be computationally tractable, while model *MP1* is more complex to solve. In a real-world setting, the choice between *MP2* and *MP3* should probably be based on their practical relevance, rather than on pure computational considerations.

5.2 Auxiliary tests

Besides the main test case described above, we also carried out some additional experiments with modified data sets. These experiments confirmed our previous conclusions regarding the computational behavior of all three models. For instance, Table 5 displays the results obtained in a case involving two plants, with 20 distinct products in plant I and 17 products in plant II. Here again, we see that models *MP2* and *MP3* are much easier to solve to optimality than model *MP1*.

	$MP1$	$MP2$	$MP3$
Iterations	10000	888	482
Computing time (s)	9.71	2.19	1.61
Cost	176.11	172.36	171.62
LP relaxation	0.00	171.49	171.44
ROC criterion satisfied	no	yes	yes

Table 5: ROC 1%

In order to illustrate how the models could be used in a negotiation setting, we also propose the following simulation. In the initial test case, one of the suppliers (supplier 7) did not offer any quantity discount to the company and, as a result of the optimization process, none of the ingredients sold by this supplier appeared in the optimal purchasing plan. Therefore, we examined what would happen if, prompted by these disappointing conclusions, supplier 7 agreed to adopt the same discount curves as the most generous supplier. A run of the models with these new parameters leads to a new purchasing plan whereby supplier 7 is awarded 4.2% of the business volume of plant 1.

6 Final remarks

We have described a tactical purchasing planning problem involving total quantity discounts and alternative product recipes, and we have presented several mixed integer programming models for this problem. Depending on the industrial and planning context, these models could or should be enriched by a number of additional features. Such extensions have already been discussed in Section 4.7. We now sketch a few more.

- Typically, demand and production forecasts may be revised several times over the course of a year. If the initial purchasing decisions are non-committal, then the MIP models could be run whenever the forecasts are updated, so as to reoptimize the procurement plan. Only small modifications would be required to take into account the quantities already ordered from different suppliers.

- Our models do not take the production capacity of the company into account: implicitly, the procurement decisions are supposed to be made on the basis of a medium-term (e.g., annual) production plan. It may be interesting, however, to integrate the purchasing issues directly and explicitly into the production planning models. In this framework, inventory costs may have to be considered in the models, as well as decisions related to the allocation of products to the plants.

- Many companies are currently striving to reduce the number or their suppliers in order to promote better relations with the few chosen ones (e.g., exchanges of commercial, technical or planning information). Therefore, in a strategic, rather than tactical use of our models, it may be interesting to introduce constraints which limit the number of active suppliers.

7 Acknowledgments

The authors wish to express their gratitude to the company which provided the information and the confidential data used in this case-study. The authors are also grateful to Yves Pochet for suggesting the investigation of the relaxed model described in Section 4.6. Yves Crama acknowledges the partial financial support of ONR (grant N00014-92-J-1375) and research grants from the Natural Sciences and Engineering Research Council of Canada (NSERC).

References

[1] W.P. Adams and H.D. Sherali, Linearization techniques for a class of zero-one mixed integer programming problems, *Operations Research* 38 (1990) 217-226.

[2] J. Bisschop and R. Entriken, *AIMMS - The Modeling System*, Paragon Decision Technology, The Netherlands, 1993.

[3] R.L. Bregman and E.A. Silver, A modification of the Silver-Meal heuristic to handle MRP purchase discount situations, *Journal of the Operational Research Society* 44 (1993) 717-723.

[4] C.-T. Chang and C.C. Chang, A linearization method for mixed 0-1 polynomial programs, *Computers & Operations Research* 27 (2000) 1005-1016.

[5] S.S. Chaudhry, F.G. Forst and J.L. Zydiak, Vendor selection with price breaks, *European Journal of Operational Research* 70 (1993) 52-60.

[6] Y. Crama, Y. Pochet and Y. Wera, A discussion of production planning approaches in the process industry, Working Paper GEMME 0102, University of Liège, 2001.

[7] Z. Degraeve, E. Labro and F. Roodhooft, An evaluation of vendor selection models from a total cost of ownership perspective, *European Journal of Operational Research* 125 (2000) 34-58.

[8] Z. Degraeve and F. Roodhooft, Improving the efficiency of the purchasing process using total cost of ownership information: The case of heating electrodes at Cockerill Sambre S.A., *European Journal of Operational Research* 112 (1999) 42-53.

[9] S.H. Ghodsypoura and C. O'Brien, The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint, *International Journal of Production Economics* 73 (2001) 15-27.

[10] F. Glover and R.E. Woolsey, Converting the 0-1 polynomial programming problem to a 0-1 linear programming problem, *Operations Research* 22 (1974) 180-182.

[11] J.V. Juker and M.J. Rosenblatt, Single period inventory models with demand uncertainty and quantity discounts: behavioural implications and a new solution procedure, *Naval Research Logistics Quarterly* 32 (1985) 537-550.

[12] P. Katz, A.A. Sadrian and P. Tendick, Telephone companies analyze price quotations with Bellcore's PDSS software, *Interfaces* 24 (1994) 52-63.

[13] M. Oral and O. Kettani, A linearization procedure for quadratic and cubic mixed-integer problems, *Operations Research* 40 (1992) S109-S116.

[14] C.J. Roethlein and P.M. Mangiameli, The realities of becoming a long-term supplier to a large total quality management customer, *Interfaces* 29 (1999) 71-81.

[15] F. Roodhooft and J. Konings, Vendor selection and evaluation: An activity based costing approach, *European Journal of Operational Research* 96 (1997) 97-102.

[16] E.C. Rosenthal, J.L. Zydiak, and SS. Chaudhry, Vendor selection with bundling, *Decision Sciences* 26 (1995) 35-48.

[17] W.G.M. Rutten, *The Use of Recipe Flexibility in Production Planning and Inventory Control*, Ph.D. dissertation, Technische Universiteit Eindhoven, The Netherlands, 1995.

[18] W.G.M. Rutten and J. W. M. Bertrand, Balancing stocks, flexible recipe costs and high service level requirements in a batch process industry: a study of a small scale model, *European Journal of Operational Research*, 110 (1998) 626-642.

[19] A.A. Sadrian and Y.S. Yoon, Business volume discount: A new perspective on discount pricing strategy, *Journal of Purchasing and Materials Management* 28 (1992) 43-46.

[20] A.A. Sadrian and Y.S. Yoon, A procurement decision support system in business volume discount environments, *Operations Research* 42 (1994) 14-23.

[21] J. Sarkis and J.H. Semple, Vendor selection with bundling: A comment, *Decision Sciences* 30 (1999) 265-271.

[22] E. Silver and R. Peterson, *Decision Systems for Inventory Management and Production Planning*, John Wiley & Sons, New York, 1985.

[23] J.W. Toomey, *MRPII, Planning for Manufacturing Excellence*, 1st ed., Chapman & Hall, New York, 1996.

[24] T.E. Vollman, W.L. Berry and D.C. Whybark, *Manufacturing Planning and Control Systems*, 3rd ed., Business One Irwin, Homewood, Illinois, 1992.

[25] C.A. Weber and J.R. Current, A multiobjective approach to vendor selection, *European Journal of Operational Research* 68 (1993) 173-184.

[26] C.A. Weber, J.R. Current and W.C. Benton, Vendor selection criteria and methods, *European Journal of Operational Research* 50 (1991) 2-18.

[27] H.P. Williams, *Model Building in Mathematical Programming*, 2nd edition, John Wiley & Sons, 1985.