Orbel 27

"Multiperiod vehicle loading with stochastic release dates"

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1. Main Message

From manual optimization over decomposed deterministic sub-problems to a multi-period stochastic policy.

"Local optima over current data
vs
global policy including uncertainty".

New blend of wellknown OR problems and techniques
Closest problem Petrol Stations Replenishment (Laporte)
- for the Problem bin-packing and set-covering Model
- for the Optimization technique CPLEX default setting
- Stochastic optimization (Birge and Louveaux)
- Consensus and Restricted Expectations algorithms (R. Bent and P. Van Hentenryck)

2. Industrial Motivation

Coils to be loaded on truck : **BIN-PACKING**
Objective function *min cost* :
Truck (fixed + tons) + Penalty for double un/loadings
Constraints : Weight constraint
Usually 1-2, sometimes 3, exceptionally 4 coils per truck

Data :
1 production site Liège (B) with several warehouses
800 customers in Europe (Mostly Germany and France)
350 trucks per day

MANUALLY INTRACTABLE TO OPTIMALITY

3. Rules and Manual Optimization

Consequence : Problem decomposed over

1. Time = period per period with the current stock
2. Space = ZIP code, lander or department
3. Customer = customer per customer

RULES : DIVIDE TIME AND SPACE TO GET SMALLER SUB-PROBLEMS

Results : Tractable instances manually optimized including up to 10 coils representing 7-8 trucks

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4.1 One-period Model(LO)

MIP approach to handle larger instances

Indices : i for M coils, j for N patterns

Parameters :

- A_{ij} pattern j contains coil i Truckload
- C_j cost of shipping pattern j

Variables : $x_j \in \{0, 1\} \forall j = 1, ..., N$

Objective Function : $min\ Z = min \sum_{j=1}^{j=N} C_j x_j$

Constraints :

$\sum_{j=1}^{j=N} A_{ij} x_j \geq 1 \forall i = 1, ...M$ every coil is sent

Advantages : Pattern includes weight constraint, pattern costs penalties and complex truck cost function

Options : different kinds of trucks and fleet size limits

4.2 Results

Merge ZIP codes or departments (up to 10) to create large sizes instances up to 100 coils and act over

SPACE

Optimization technique : EXACT

Patterns Generation and Set Covering Problem

Generation of all feasible loaded trucks, their costs and selection of the cheapest composition (CPLEX)

4.2 Results

In 'Bavaria' compare to individual optimization on ZIP Codes 80 to 89, over industrial instances,

1. the **number of trucks** is reduced by 16,9%
2. and the **cost** by 12,7 % (double unloadings)

These are well-known techniques.

5.1 Multi-period Model (O*)

New dimension **TIME** => Multi-period setting

NB : Multi-period not

1. Periodic (Bus, Train...)
2. "On-line" (task : taxi)
3. "Dynamic" (parameters : time-dependent)
4. Multiples periods (split long term planning)

Creation of a new model taking into account :
production forecasts over a rolling horizon H

Penalties related to Time Windows : INV, EAR, LATE
Typically 4 possible periods for the release dates

[EAR, INV, INV, LATE]

5.2 : Biases Policy and "End of horizon"

Model formulation : set covering problem of patterns

A pattern is a truckload of coils
At any time *t*, a given pattern is available or not

Pattern cost indexed by *t* includes the truck cost based on the weight + Un/loads + INV, EAR, LATE
For any pattern there is a cheapest shipping period !
Consequence : model size is reduced (RAM)
Implementation lead to :

2 biases

1. **Policy** over a rolling horizon H not a solution
2. **End of horizon**

5.2 Biases : Policy and "End of horizon"

A Policy is an iterative process that generates a sequence of decisions and not a full-horizon planning, it depends on H size :

1. Evaluate the best decision over P1 to P(1+H)
2. Implement decisions for P1 Always feasible ! !
3. Update extra period P(2+H) and remaining coils
4. Reevaluate the best decision from P2 to P(2+H)
5. Implement new P2 repeat...until... P(i+H)=End

End of horizon

The set of coils sent might be different

=> Average variable expedition costs $/T

5.3 Theoretical cases

Extreme case : Improvement ratio = T

TW length T

Truck capacity C

Coil weight $\leq$ C/T

1 coil per period

One-period : T * 1 truck with 1 coil per period = T trucks

Multi-period : 1 truck with T coil of weight C/T = 1 truck

	Periods					
Coils Weight	**P1**	**P2**	**P...**	**Pi**	**P...**	**PT**
A *C/T*	1		...			
B *C/T*		1	...			
... *C/T*			...			
i *C/T*				1		
... *C/T*					...	
T *C/T*						1

5.3 Theoretical cases

4 coils case with penalties and limits

	Periods				
Coils Weight	**P1**	**P2**	**P3**	**P4**	**P5**
A *0.6*	1	*TW*			
B *0.8*		1	*EAR*	*TW*	
C *0.2*	1	*TW*	*TW*	*TW*	
D *0.4*	1	*EAR*	*TW*		

- P-INV < P-EAR < P-LAT < Truck cost
- late or early delivery TW +/- 1 period P-EAR or P-LAT
- not allowed before EAR and after LAT (semi-soft TW)
- one period delivery time

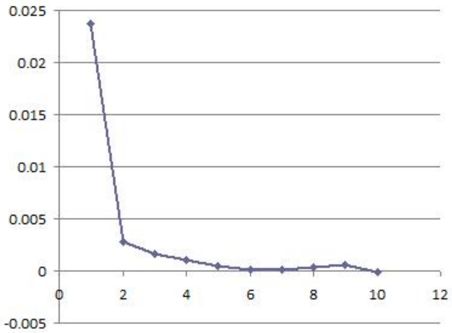
Decisions : WAIT or SEND available coils in P1

e.g. : AC(P1) + D(P2) + B(P3) vs AD(P1) + BC(P3)

TW (OK) + 1 P-INV + 1 P-INV vs P-EAR + 2 P-INV

5.4 Results

Transportation costs variation according to rolling horizon length



Rem : 400 loads, 22 periods, unit €/10kg
H=1 cost 0.065, H=10 cost 0.034 €/10kg, reduction 50% !
After H = 5P, minor improvements, yet $O*_{\infty} \leq\leq O*_{H=5P}$
But, in practice, forecasts are uncertain

6.1 Theoretical case : 2 coils

Forecasts contain uncertainty on production release dates r(i)
Example : 2 coils cases

Weights	**P1**	**P2**	**P3**	**P4**
A *0.5*	1	**TW**	Late	X
B *0.5*		0.49	0.51	**TW**

Stochastic :
SEND A en (P1) : cost 2 trucks
WAIT A en (P1) : cost P-INV(A) +
If B is available in P2 : 1 truck (AB) + P-LATE(A) + P-EAR(B)
If B is available in P3 : 1 truck (A) + P-LATE(A) + 1 truck (B)
Average : cost 1.5 truck + penalties

Weights	**P1**	**P2**	**P3**	**P4**
A *0.5*	1	**TW**	Late	X
B *0.5*			1	**TW**

"Deterministic approach" : "Modal Period" 0,51 => 1
SEND A in (P1) : cost 2 trucks

6.2 Optimal representation : Scenario Tree

	Periods				
Weights	**P1**	**P2**	**P3**	**P4**	**P5**
A *0.6*	1	*TW*			
B *0.8*		0.9	0.1	*TW*	
C *0.3*			0.2	0.8	
D *0.2*	1	*TW*	*TW*	*TW*	
E *0.4*	1		*TW*		

New objective function : *"Minimize expected cost"* **E***
Scenarios tree : Deterministic equivalent with scenarios and non-anticipativity constraints (IP Problem)

- e.g. : 4 scenarios
  1. B(P2) C(P3) Pr(0.18)
  2. B(P2) C(P4) Pr(0.72) Modal Periods EMod
  3. B(P3) C(P3) Pr(0.02)
  4. B(P3) C(P4) Pr(0.08)

Scenarios tree expected cost should be better than EMod
Aim : find a model or algorithm that approximates E*

6.2 Optimal representation : Scenario Tree

Drawback : huge number of scenarios
Limit for optimization 12 coils, H=3 periods
Distribution law over 2 periods, 2^{12} scenarios
=> Intractable (CPLEX)
"Heuristic" because of the model and/or the method
Basic ideas : Simplified solution in P1 is valid for the whole problem in P1

1. A single representative scenario
2. A subset of independent scenarios solutions aggregated in a consensus one
3. A subset of independent scenarios solutions cross-tested to select one

SCENARIOS GENERATION

1. Monte-Carlo random generation of scenarios

SOLUTION VALIDATION

- Variance due to scenario sampling (N=30)
- Compare policies from a collection of results

6.3 O*, LO, EMod, EMean, EOpt, C_i, RE

Heuristics comparison H = 5P, TW = 4P, N = 30

1. **O*** full information revealed = **LB**
2. **EMod** Deterministic equivalent modal period
3. **EMean** Deterministic equivalent expected period
4. **EOpt** Deterministic equivalent earliest period
5. **C1** Consensus : Send in P1 if Yes $\geq$ 6 /10 scenarios
   => Send only those coils in P1 (LO) Always feasible !
6. **C2** Consensus : Send in P1 if Yes $\geq$ 6/10 scenarios
   => Send **at least** those coils in P1 (LO)
   => and coils others available in P1, if it is for free !
7. **RE** Restricted Expectation : Solve 10 scenarios
   => Apply each decision for P1 in all other scenarios
   => Evaluate the cost for sending all remaining coils
   => Select the decision with the lowest cumulated cost
   => cross-evaluation 100 computations vs (10+1) or 1
8. **LO** One-period only P1 (revealed info) $\cong$ **UB**

6.3 O*, LO, EMod, EMean, EOpt, C_i, RE

Validation scenarios $\neq$ Calibrating scenarios 30 Validation scenarios

- called test scenarios or realizations
- similar for all algorithms
- cover the whole horizon
- independent of performed decisions
- purpose : statistical validation of a policy

For C_i and RE : 10 Calibrating scenarios

- algorithmic parameter (sensitivity analysis)
- different sets as generated by each algorithm
- valid at a specific decision period
- dependent from the past decisions
- purpose : generate a local decision

6.3 O*, LO, EMod, EMean, EOpt, C_i, RE

Theoretical notions

The gap between **O* (LB)** and **LO (UB)** gives the **"Value of the multi-period and perfect information model VMPM"**

The gap between **O*** and **E*** gives the **"Value of the Perfect Information VPI"**

The gap between **EMean** and **E*** gives the **"Expected Value of the Stochastic Solution EVSS"**

The average gap between **O*** and **approximate models** gives an **"Upper Bound on the VPI, the Expected Value of the Perfect Information EVPI"**

The average gap between **O*H5** and **the best approximate models** gives an **"Upper Bound on the Expected Value of the Available Information EVAI"**

6.4 Distribution laws

No industrial data for a distribution law

Test : Distribution laws over TW 4P, H=5, r(i) in %

1. Uniform [25,25,25,25]
2. "Early" [40,30,20,10] Positive Skew
3. "Late" [10,20,30,40] Negative Skew
4. Binomial [12.5,37.5,37.5,12.5]

6.5 Results

Binomial	4/P	6/P	8/P	10/P
O*	100	100	100	100
O*H5	107.184	105.799	104.379	106.105
EMean	123.440	117.196	114.687	116.709
EMod	123.426	**109.904**	112.417	115.022
C2	114.659	113.988	113.525	115.325
RE	**112.961**	111.584	**112.085**	**111.940**
LO	184.799	179.027	160.901	157.282
VMPM	84.799	79.027	60.901	57.282
EVPI	12.961	9.904	12.085	11.940
EVAI	5.777	4.105	7.706	5.835
EVSS	10.479	7.292	1.837	4.769

=> C1 underperforms compared to C2
=> High value of multi-period and perfect information model
=> Quality of solutions is better with more coils
=> Average cost reduces with number of coils
=> Average cost increases with a negative skew law
=> EMod, EMean, C2 underperform for at least 1 distribution law

6.5 Results

Early	4/P	6/P	8/P	10/P
O*	100	100	100	100
O*H5	107.235	105.307	103.751	105.586
EMean	116.451	113.892	111.090	110.430
EMod	112.236	**108.478**	**106.982**	**107.597**
C2	122.368	119.503	113.135	117.379
RE	**111.050**	111.671	109.198	111.762
LO	193.517	172.529	168.748	159.332
VMPM	93.517	72.529	68.748	59.332
EVPI	11.050	8.478	6.982	7.597
EVAI	3.815	3.171	3.231	2.011
EVSS	5.401	5.414	4.108	2.833

RE outperforms, but is outclassed for the early law

EMod Early => r(i)=[1,2,3,4] = EOpt

The single optimistic scenario performs well

6.5 Results

EOpt and RE(calibrated Early) over 4 distribution laws

Late	**4/P**	**6/P**	**8/P**	**10/P**
O*	100	100	100	100
O*H5	102.710	102.951	102.797	103.831
RE_{Late}	**109.541**	110.999	109.004	109.747
RE_{Early}	111.703	**108.643**	**106.873**	108.785
$E\ Opt$	110.142	109.246	107.331	**108.279**

Binomial	**4/P**	**6/P**	**8/P**	**10/P**
O*	100	100	100	100
O*H5	107.184	105.799	104.379	106.105
RE_{Bino}	**112.961**	111.584	112.085	111.940
RE_{Early}	117.337	113.462	112.123	111.731
$E\ Opt$	116.010	**109.044**	**109.526**	**110.291**

6.5 Results

EOpt does not always provide the best solution, yet statistically the model based on the most optimistic scenario outclasses all other models without being outclassed and is robust to any tested distribution laws !

	RE(Calibration laws) and Instances			
EOpt	**RE(Early)**	**16**	**RE(Bino)**	**4**
Outclass 95%	**YES**	**NO**	**YES**	**NO**
μ(EOpt)$\neq$ μ(X)	5	11	2	2
μ(EOpt)<μ(X)	5		2	
μ(X)$\neq$ μ(EOpt)				
EOpt	**RE(Uni)**	**4**	**RE(Late)**	**4**
Outclass 95%	**YES**	**NO**	**YES**	**NO**
μ(EOpt)$\neq$ μ(X)	2	2	2	2
μ(EOpt)<μ(X)	2		2	
μ(X)$\neq$ μ(EOpt)				

=> EOpt is a fast, efficient and robust heuristic ! Note : the influence of providing information at the earliest generates more "Wait" decisions !

7. Conclusions and Perspectives

Conclusions

- New model Transportation/Production
- Pattern generation seems an appropriate formulation
- Multi-period model is better than mono-period model
- Distribution laws and number of coils seem to influence the quality of the solution obtained for the EVPI
- Managerial advices can be provided
- A fast, efficient and robust heuristic

Perspectives

- Non-independent identically distributed laws, so less scenarios EVPI Increase
- Study other problems such as FTL with stochastic arrivals of orders

Thank you for

Global opinion (:-)) (:-()

Questions ?

Advices !

Remarks ! ! !

Comments...