Experimental modal analysis of a beam travelled by a moving mass using Hilbert Vibration Decomposition

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The research is focused on the identification of time-varying systems

$$\boldsymbol{M}(t)\,\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}(t)\,\dot{\boldsymbol{x}}(t) + \boldsymbol{K}(t)\,\boldsymbol{x}(t) = \boldsymbol{f}(t)$$

Dynamics of such systems is characterized by :

- Non-stationary time series
- Instantaneous modal properties
  - Frequencies : $\omega_r(t)$
  - Damping ratio's : $\xi_r(t)$
  - Modal deformations : $\boldsymbol{q}_r(t)$

The *Hilbert Transform*

The Hilbert transform $\mathcal{H}$ of a signal $x(t)$ is the convolution product of this signal with the impulse response $h(t) = \frac{1}{\pi\, t}$

$$
\begin{aligned}
\mathcal{H}(x(t)) &= (h(t) * x(t)) \\
&= \text{p.v.} \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau \\
&= \frac{1}{\pi}\, \text{p.v.} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau}\, d\tau
\end{aligned}
$$

It is a particular transform that remains in the time domain

It corresponds to a phase shift of $-\frac{\pi}{2}$ of the signal

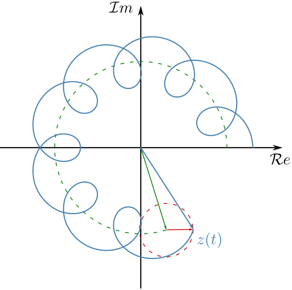
The Hilbert transform and the *analytic signal*

The analytic signal z is built as

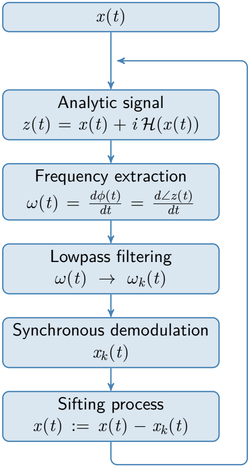
$$\begin{aligned} z(t) &= x(t) + i\,\mathcal{H}(x(t)) \\ &= A(t)\,e^{i\phi(t)} \end{aligned}$$

The instantaneous properties of the signal can then be obtained

$$\begin{cases} A(t) = |z(t)| \\ \phi(t) = \angle z(t) \\ \omega(t) = \dfrac{d\phi}{dt} \end{cases}$$



The *Hilbert Vibration Decomposition* (HVD) method



It is an iterative process

The sifting of the signal extracts monocomponents from higher to lower instantaneous amplitude

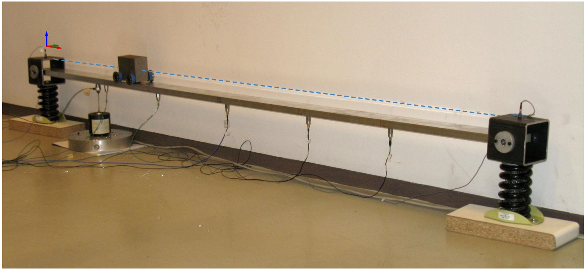
It is applicable to single channel measurement

Crossing monocomponents may be a problem

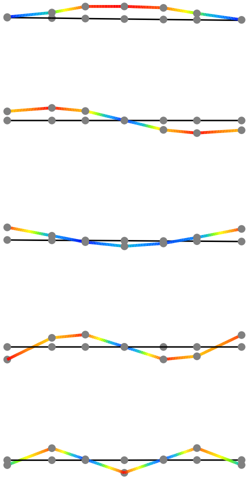
The experimental set-up

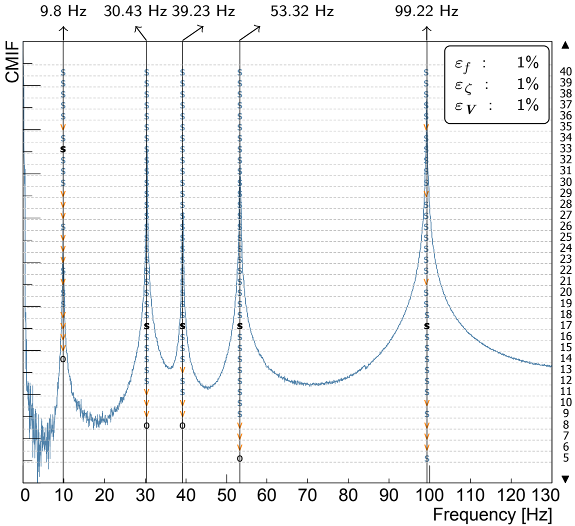
2.1 meter aluminum beam
Steel block ($\approx$ 3.5 kg, 38.6%)

1 shaker (random force)
7 accelerometers
LMS SCADAS & LMS Test.Lab system

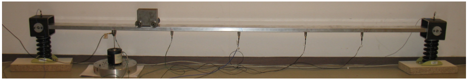


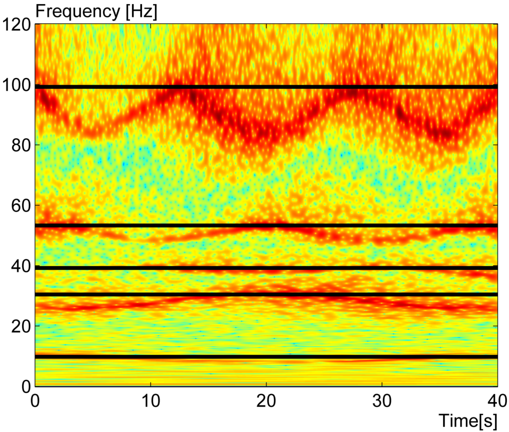
Time invariant modal identification of the beam subsystem



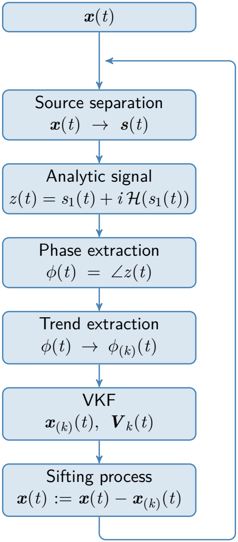


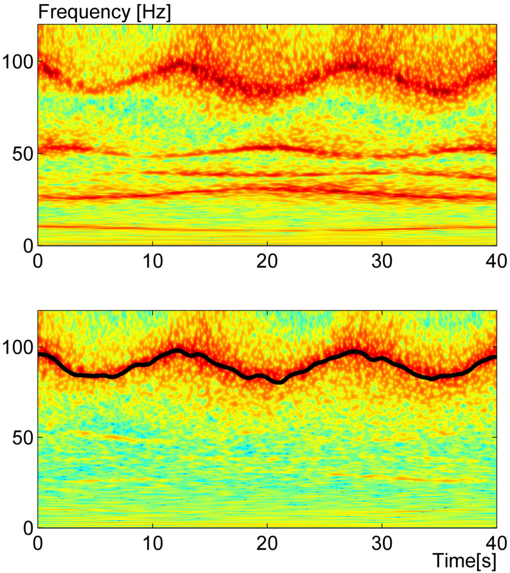
Time-varying dynamics of the system



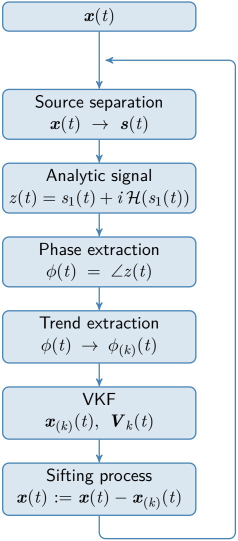


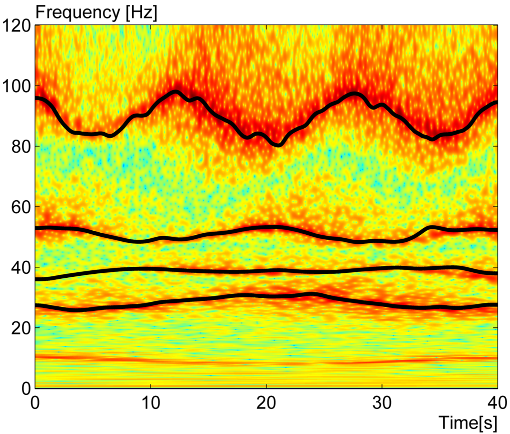
The sifting process and the benefit of the source separation





Other modes are extracted after few iterations





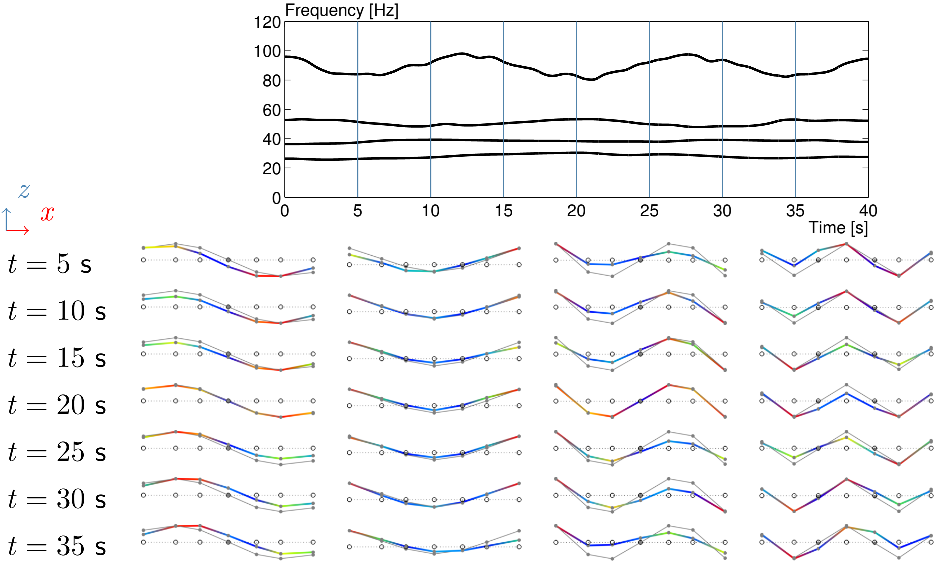
Monocomponents and complex amplitudes are extracted with a Vold-Kalman filter

The Vold-Kalman model and the modal expansion are very similar.

The extracted complex amplitudes are then considered as unscaled mode shapes

$$\begin{array}{llll} \text{Vold-Kalman filter:} & \boldsymbol{x}(t) = \sum_k & \boldsymbol{a_k}(t) & e^{i\,\phi_k(t)} \\ & & \updownarrow & \updownarrow \\ \text{Modal expansion:} & \boldsymbol{x}(t) = \sum_k & \boldsymbol{V_k}(t) & \eta_k(t) \end{array}$$

The moving mass affects both frequencies and mode shapes



Thank you for your attention