SOME THEORETICAL AND NUMERICAL FEATURES OF STRAIN GRADIENT MEDIA REGARDED AS MICROMORPHIC MEDIA

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Strain localization is a phenomenon occuring very often in anelastic media and it is very important to take it into account in most of applications, for instance in geotechnics. The usual Cauchy's theory of continuous media is unable to properly take into account strain localization, at best it can predict its onset by using a Rice's analysis of shear bands. One of the reasons of the inability of the classical theory to properly describe localization is that the usual continuum model does not incorporate any internal length which could determine the localization band width. Moreover, it has been shown [2] that, in numerical computations, the uniqueness of the solution is lost when localization occurs. A simple but instructive 1 dimensional classical traction problem with softening, developed in [1], enlightens the loss of uniqueness of solutions and the impossibility to define a localized zone width. It is shown in [1] that the enrichment of the previous model to a strain gradient model enables to define a strain localization length and to drastically diminish the number of solutions.

In [1], a comparison is performed between analytical and finite element solutions of the previous 1D strain gradient problem. Conformal finite element discretizations of strain gradient problems need C^1 interpolation functions. In 1D, this requirement is easily fulfilled by using Hermite polynomials, in 2D or 3D, C^1 conformal elements exist but are not so easy to handle. An alternative to the use of conformal elements is to consider the strain gradient medium as a micromorphic medium for which the microdeformation tensor E is set to be equal to the displacement gradient tensor ∇u, that has been known to be possible since the work by Germain [3]. For a BV problem, the equality $E=\nabla u$ can be realized with the help of Lagrange multipliers. Thus the variational formulation of the problem of a strain gradient medium regarded as a micromorphic medium involves only first derivatives, either of the microdeformation E or of the displacement u, and not second derivatives and so, it is possible to use C^0 elements in FE computations. In such a formulation, a peculiar attention has to be given to the boundary conditions. Indeed, on the boundaries of a strain gradient medium, only the values of the displacement u and of its normal derivative can be prescribed. Setting separately the values of u and of E on the boundary for a micromorphic-like strain gradient medium could lead to inconsistencies. By duality, the same kind of snag, even more tricky, occurs when considering stress boundary conditions. For a strain gradient medium, it is not possible to set separately the usual forces and the double forces, a part of the double forces being, like reactions, unknowns. The situation is quite similar at the interface between two different media for the continuity conditions of stresses are not the same for strain gradient or micromorphic media.

All those points occurs only for 2D or 3D problems, they will be illustrated with a 2D axisymmetric example in strain gradient elasticity for which we have determined analytical solutions and perfomed comparison with different FE computations.

References

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