Optimized Schwarz Algorithm with Double Sweep Preconditioner for the Helmholtz Equation

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In previous work (Y. Boubendir, X. Antoine and C. Geuzaine, A quasi-optimal non-overlapping domain decomposition algorithm for the Helmholtz equation, JCP, 231, pp. 262-280, 2012), it was shown how a Domain Decomposition Method (DDM) formulation of the Helmholtz problem, using impedance-matching boundary conditions, can be set up and accelerated with a Krylov solver. This optimized Schwarz algorithm involves the so-called Dirichlet-to-Neumann (DtN) map for the transmission of information between subdomains, which can be approximated in different ways. While the quality of this approximation is critical to the rate of convergence of the algorithm, recent works have succesfully addressed this problem. Another difficulty with this algorithm resides in the spectral properties of the iteration operator, that still prevent fast convergence in the case of many subdomains and a layered decomposition.

By exploiting the impedance properties of the DtN map, that lets outgoing waves leave the subdomains without reflection at the artificial boundaries, we obtain an approximation of the inverse of the iteration operator. It provides an efficient preconditioner, even when the transmission operator used in the algorithm is an imperfect approximation of the DtN map. We show how this preconditioner can be rewritten as a double sequence of subproblems solves, forward and backward. While the original iteration operator only transmits information between adjacent subdomains, hence the slow convergence, the double sweep acts by transporting information over all subdomains. In the DDM community, a technique that enables such long range communication is known as a *coarse grid*.

Numerical experiments show that, for very good approximations of the DtN map, the convergence rate is independent of the wavenumber and the number of subdomains. This is a very desirable property, as domains that are large compared to the wavelength can be decomposed in many subdomains, without degrading the performance of the algorithm. Let us mention that the recently introduced technique of matrix probing is an efficient way of accurately approximating the DtN map, using geometrical optics to build a small-size basis for the DtN map (R. Bélanger-Rioux and L. Demanet, Compressed absorbing boundary conditions via matrix probing, preprint, 2012). To reduce the complexity of this non-parallelizable preconditioner, future work will investigate approximate solutions for the direct transmission of information between interfaces, without solving in the interior of the subdomains.