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Double Sweep Preconditioners for propagation problems solved by Optimized Schwarz Methods

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Abstract

We present a parallel version of the otherwise sequential double sweep preconditioner, used to accelerate the convergence of an optimized Schwarz domain decomposition method. The method is based on the same sweeping strategy, yet applied on a shorter scale and in parallel, on distinct groups of subdomains. The modified algorithm, unlike the original one designed for layered decompositions, has the advantage of being directly applicable to cyclic decompositions as well. The whole method is described in terms of combinations of transport operators and is therefore suitable to both Helmholtz and frequency-domain Maxwell problems.

Key words: Domain Decomposition, Preconditioner, Propagation, High Frequency.

1 Introduction

The idea of sweeping for the solution of wave propagation problems in the frequency domain, of the form $(\Delta + k^2)\mathbf{u} = 0$, is quite natural, since it somehow mimics the physical phenomenon of a wave propagating inside a medium. It is therefore not surprising that techniques inspired by this observation have proved successful [1, 2]. In the slightly different context of Domain Decomposition solvers for these problems, we have recently proposed the double sweep preconditioner for the non-overlapping optimized Schwarz algorithm [3]. A limitation of the method is the sequential nature of the sweeping process that makes the iterative part of the solution less scalable on parallel architectures—the factorization of the subproblems remaining fully parallel. This paper addresses that issue, by proposing a modification of the algorithm to partially restore its parallelism.

2 Algorithms

The double sweep preconditioner was originally designed as the inverse of the iteration operator in the particular case of a layered decomposition, supposing that perfectly non-reflecting operators are used as transmission condition. In that case, the matrix that represents the iteration operator is easy to invert. Since it is, like its inverse, made of transport operators that involve the solution of subproblems, we give it an interpretation in terms of a combination of such subproblems. It is a double sequence of solves, that we call the forward and backward sweeps. That inverse is then used to precondition more complex problems [3].

Unlike the standard algorithm, the application of the preconditioner is sequential and exploits no more than 2 CPUs simultaneously, which is a very suboptimal use of the resources if more CPUs are available. By performing the sweeps independently and concurrently over smaller groups of domains, we still benefit of the long range sharing of information provided by the sweeps, while reducing the idleness of the CPUs (Figure 1). Introducing a cut in a cyclic decomposition makes it topologically equivalent to a layered one, making the preconditioner readily applicable.

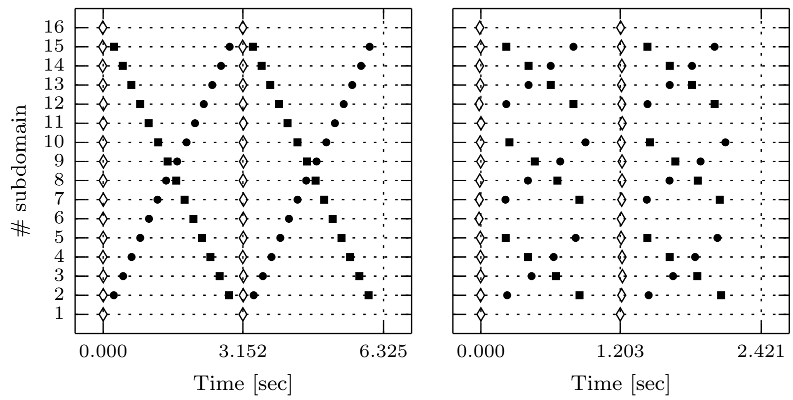

Figure 1: Timelines of the double sweep preconditioner application without cuts (left) and with 2 cuts (right). The white diamonds indicate solves performed in the iteration operator; the black circles and squares indicate solves in the forward and backward sweeps, respectively.

Table 1 shows the number of iterations and an estimation of the normalized time required to attain convergence ($||r||/||r_0|| < 10^{-4}$) for the solution of a Maxwell problem in the challenging COBRA cavity benchmark. The standard algorithm (np) failed to converge, while the (wall-clock) time to solution with the preconditioned algorithm (ds) decreases when cuts are added, though too many cuts are detrimental (the reported times do not include subproblem factorization).

#CPU	2	4	6	8	14	22
N_c	0	1	2	3	6	10
$N_{\text{it}}^{(\text{ds})}$	44	74	105	135	230	354
$T_{\text{sol}}^{(\text{ds})}$	2024	1702	1680	1485	1610	1416
$N_{\text{it}}^{(\text{np})}$	> 1000					
$T_{\text{sol}}^{(\text{np})}$	> 16016	> 8008	> 5339	> 4004	> 2288	> 1456

Table 1: COBRA test case for Maxwell with 32 subdomains (N_c cuts) at $k = 314.16$.

References

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