Model of electrostatic actuated deformable mirror using strongly coupled electro-mechanical finite element

Véronique Rochus · Jean-Claude Golinval · Christophe Louis · Carlos Mendez · Igor Klapka

Received: 29 June 2006/Revised: 7 September 2006/Accepted: 10 May 2007 © Springer Science+Business Media, LLC 2007

Abstract The aim of this paper is to deal with multiphysics simulation of micro-electro-mechanical systems (MEMS) based on an advanced numerical methodology. MEMS are very small devices in which electric as well as mechanical and fluid phenomena appear and interact. Because of their microscopic scale, strong coupling effects arise between the different physical fields, and some forces, which were negligible at macroscopic scale, have to be taken into account. In order to accurately design such micro-electro-mechanical systems, it is of primary importance to be able to handle the strong coupling between the electric and the mechanical fields. In this paper, the finite element method (FEM) is used to model the strong coupled electro-mechanical interactions and to perform static and transient analyses taking into account large mesh displacements. These analyses will be used to study the behaviour of electrostatically actuated micro-mirrors.

Keywords Electro-mechanical coupling · Micro-mirrors · Finite element modelling

1 Introduction

Classical methods used to simulate the coupling between electric and mechanical fields are usually based on staggered procedures, which consist in computing quasi-static

V. Rochus (🖂) · J.-C. Golinval

C. Louis · C. Mendez · I. Klapka

Open Engineering sa, Rue des Chasseurs Ardennais, Angleur 4031, Belgium

configurations using two separate models: a structural model loaded by electrostatic forces predicted at the current iteration step and an electrostatic model defined on the current deformed structure. Staggered iterations then lead to the static equilibrium position. In this paper, a fully coupled electro-mechanical FE formulation is proposed, which allows static equilibrium positions to be computed in a non-staggered way, and which provides fully consistent tangent stiffness matrices that can be used for transient analyses.

In the classical approach, staggered coupling procedures are typically used to simulate electro-mechanically coupled systems. The electrostatic and the mechanical domains are discretised and solved independently in different analysis steps. Iterations are then performed: the electrostatic field is first computed using, for instance, the boundary element method or the finite element method and it provides the electrostatic forces acting on the mechanical structure. Then a mechanical FEM code computes the structure deformation under the effect of electrostatic forces. The deformed structure defines new boundaries for the electrostatic problem and the electric field has to be computed again. This method is commonly presented in the literature (see e.g. Lee et al. [1]) and is illustrated in Fig. 1. The staggered method can also be understood as block-Gauss Seidel procedure.

The method proposed here consists in considering the monolithically coupled problem. The electric and mechanical fields are computed *simultaneously* in a unified formulation. Since the problem is non-linear, it must be solved by an iterative algorithm such as the Newton-Raphson method or Riks-Crisfield method. The solution strategy consists in the following steps. Given an electric potential applied on the boundaries of the structure, a first solution is obtained by considering the coupled problem

LTAS, Vibrations et Identification des Structures, University of Liège, Chemin des Chevreuils, 1, B52, Liege 4000, Belgium e-mail: v.rochus@ulg.ac.be



Fig. 1 Staggered method

linearised around an initial configuration. The resulting structural deformation then defines a modified electric domain and a new linearised problem is defined around the modified configuration. This process continues until the solution has converged, namely until the electric and mechanical equilibrium are satisfied up to a prescribed tolerance (Fig. 2).



Fig. 2 Monolithical method

The monolithic formulation has many advantages compared to the staggered method. Indeed the tangent stiffness matrix can be explicitly constructed for the entire system. Therefore, the stability and the natural frequency of the electrostatically coupled structure can easily be evaluated around a given equilibrium configuration [2]. Also the staggered coupled method becomes numerically unstable when the pull-in voltage is reached [3]. Finally, the coupled method permits to pass over the pull-in voltage. This method allows us to determine more precisely the pull-in voltage which is an important characteristic of MEMS devices.

Strong coupled methods are also proposed by ANSYS and FEMLAB to obtain the tangent stiffness matrix.

1.1 ANSYS coupled formulation

The software ANSYS proposes different solutions to achieve the equilibrium between electrostatic forces and mechanical structure [3–5]. The first one is a sequential coupling electro-mechanical solver called *ESSOLV* which uses the staggered methodology as presented previously. The second possibility is to use *TRANS126* element which is a strong coupled transducer formed by 2-nodes line [5]. The vacuum between the plates is discretised by one-dimensional condensers

To simulate very strong coupling in two dimensions TRANS109 element is usually used [4]. This transducer is based on virtual principle. It is represented by a finite element 3-nodes triangle with electric and mechanical degrees of freedom. A standard Ritz-Galerkin approach is applied to compute the energy in the triangle. The forces in each node are obtained by deriving this energy by the nodes displacement. This methodology is used when the capacitance-stroke relation is difficult to obtain. The advantage of this element is its very accurate precision to compute the electrostatic forces and the electro-mechanical coupling, but the convergence is sensitive to the mesh discretisation and it is less robust for devices that experience large deformation [3]. An extension in three dimensions of this element has been performed by Avdeev [6]. It is called "3D strongly coupled tetrahedral transducer". Displacement and voltage are the variables assigned to the nodes. The energy in one element is computed by the relation:

$$W = \frac{1}{2} \int_{\Omega} \varepsilon_0 (\nabla V)^2 d\Omega = \frac{\varepsilon_0 \Omega}{2} \sum_{i=1}^4 \sum_{j=1}^4 \frac{\mathbf{n}_i \cdot \mathbf{n}_j}{h_i h_j} V_i V_j \tag{1}$$

where V is the electric potential, ε_0 the void permittivity, Ω the volume, h_i is the tetrahedral altitudes and \mathbf{n}_i the interface normal vector of the plane *i* as shown in Fig. 3.



Fig. 3 Avdeev coupled element

To compute electrostatic forces and tangent stiffness matrix, the altitudes and the normal vector are derived by the displacement of the nodes. This element as well as the *TRANS109* element, has an automatic mesh morphing based on equilibrium considerations. The initial mesh is automatically transformed to follow the displacement of the structure according to the force equilibrium criteria. With this method the electro-mechanical coupling is modelled strongly, but the formulation has been implemented only for linear tetrahedral elements.

1.2 FEMLAB coupled formulation

FEMLAB Software proposes a hybrid method to compute the electro-mechanical coupling. The mechanical structure is modelled with a classical mechanical model. For the electric domain new elements are created taking mesh deformation into account. The electrostatic field is solved and the electric forces are computed using the relation [7]:

$$\mathbf{F}_{elec} = \frac{\varepsilon_0 \mathbf{E}^2}{2} = \frac{\varepsilon_0 (\nabla V)^2}{2}$$
(2)

where E is the electric field.

The deformed electrostatic mesh is based on the coordinate transformation of the electric mesh. A new system to solve is added to the usual purely electric and mechanical problem creating a relation between the degrees of freedom of the mechanical structure and the position of the deformed electric mesh. The final system is the following:

$$\begin{cases} \mathbf{K}_{qq} \mathbf{q} = \mathbf{F}_{elec}(V) \\ \mathbf{K}_{uu}(\mathbf{u})\mathbf{u} = \mathbf{F}_{boundary}(V) \\ \mathbf{K}_{VV}(\mathbf{u}, V)\mathbf{V} = \mathbf{V}_{boundary} \end{cases}$$
(3)

The first equation is the pure mechanical system taking the electrostatic forces \mathbf{F}_{elec} into account. \mathbf{q} is the displacements vector of the mechanical structure and \mathbf{K}_{qq} is the mechanical stiffness matrix. The second system is the equilibrium equation of the mesh. \mathbf{u} is the displacement of the mesh nodes and \mathbf{K}_{uu} is the stiffness matrix related to the moving mesh. Finally, the third equation is the electrostatic system with \mathbf{K}_{VV} the electrical stiffness matrix depending on the displacement of the mesh **u**. The electro-mechanical coupling appears through this deformed mesh. This methodology provides a strong electro-mechanical coupling. Problems appear in the electrostatic forces computation. First and second order elements do not provide the same converged electrostatic forces.

2 Strong finite element formulation

The finite element formulation proposed in this paper is based on a variational principle applied to the entire energy of the problem. The energy includes together the electric and mechanical energies. The expression of the energy density of the system results from thermodynamical considerations [8]. Depending on the chosen variables, different thermodynamics functions have to be used. If the mechanical strain and the electric field are chosen, electric Gibbs' free energy density [9] or electric enthalpy [10] have to be considered. The difference between these two expressions appears in the thermal variables. Usually when analyzing piezoelectric behaviour of systems, the electric Gibbs' energy G is considered:

$$\mathbf{G} = \frac{1}{2}\mathbf{S}^{T}\mathbf{T} - \frac{1}{2}\mathbf{D}^{T}\mathbf{E}$$
(4)

where T is the stress tensor, S the strain tensor, D the electric displacement tensor and E the electric field.

The coupled electromechanical equations are obtained using a variational approach [11]. It leads to a consistent way to derive a finite element discretisation for the coupled electro-mechanical problem. Starting from the energy of the coupled system, nodal forces are obtained for an element by derivation of the energy. The tangent stiffness matrix of the coupled problem is then obtained by linearisation of the equilibrium equations in the vicinity of an equilibrium position.

The total energy on a volume Ω representing the electromechanical domain is:

$$W_{\text{int}} = \int_{\Omega} \left(\frac{1}{2}\mathbf{S}^{T}\mathbf{T} - \frac{1}{2}\mathbf{D}^{T}\mathbf{E}\right)d\Omega = W_{m} - W_{e}$$
(5)

assuming linear elasticity and constant permittivity. W_m is the mechanical energy and W_e the electric one. The integration volume for the electric energy W_e depends on the mechanical displacement **u**. For the mechanical energy W_m however the integration domain is the reference (i.e. undeformed) domain since **u** represents the displacement as a function of the initial positions (Lagrangian description). The constitutive equations and the compatibility relations for both fields are used. The variation of the internal energy due to the virtual displacement (compatible with kinematical constraints) yields the mechanical internal forces and its variation due to virtual perturbations of the potential yields the electric charges:

$$\begin{cases} \mathbf{f}_m \cdot \delta \mathbf{u} = \delta_u W_{\text{int}} = \delta_u W_m - \delta_u W_e \\ q_e \cdot \delta \phi = \delta_\phi W_{\text{int}} = \delta_\phi W_m - \delta_\phi W_e \end{cases}$$
(6)

Three of these variations can be evaluated in a straightforward way. In fact, $\delta_u W_m$ and $\delta_\phi W_e$ can be treated as in the standard variational calculus for uncoupled electrostatics and mechanics. The mechanical energy is independent on the voltage: $\delta_\phi W_m = 0$.

On the other hand, the variation of the electric energy due to the displacement **u** is not common. To compare the energy before and after variation, they have to be expressed on the same reference domain. In order to show the dependence of the volume of the electric part with respect to the structural displacement, an initial volume Ω is considered and a perturbation of the displacement $\delta \mathbf{u}$ is applied to obtain the perturbed volume Ω .

After developments the variation of the electric energy corresponding to virtual displacements can also be written as

$$\mathbf{f}_{elec}.\delta\mathbf{u} = \frac{1}{2} \int_{\Omega} \mathbf{D}^T \mathbf{F}(\nabla \delta \mathbf{u}) d\Omega$$
(7)

To obtain the tangent stiffness matrix the linearisation of the internal forces around an equilibrium position (\mathbf{u}_e, ϕ_e) is performed and gives us:

$$\begin{cases} \mathbf{f}_m \approx \mathbf{f}_m^e + (\frac{\partial^2 W_m}{\partial u^2} - \frac{\partial^2 W_e}{\partial u^2}) d\mathbf{u} - \frac{\partial^2 W_e}{\partial \phi \partial u} d\phi \\ q_e \approx q_e^e - \frac{\partial^2 W_e}{\partial u \partial \phi} d\mathbf{u} - \frac{\partial^2 W_e}{\partial \phi^2} d\phi \end{cases}$$
(8)

which can be written in the matrix form:

$$\begin{pmatrix} \frac{\partial^2 W_m}{\partial u^2} - \frac{\partial^2 W_e}{\partial u^2} & \frac{\partial^2 W_e}{\partial u \partial \phi} \\ - \frac{\partial^2 W_e}{\partial u \partial \phi} & \frac{\partial^2 W_e}{\partial \phi^2} \end{pmatrix} \begin{pmatrix} du \\ d\phi \end{pmatrix} = \begin{pmatrix} \Delta f_m \\ \Delta q_e \end{pmatrix}$$
(9)

$$\begin{pmatrix} K_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi \phi} \end{pmatrix} \begin{pmatrix} du \\ d\phi \end{pmatrix} = \begin{pmatrix} \Delta f_m \\ \Delta q_e \end{pmatrix}$$
(10)

The tangent stiffness matrix takes into account the electromechanical coupling in the system. It is observed that the coupling matrix is symmetric. The matrices $\frac{\partial^2 W_e}{\partial \psi^2}$ and $\frac{\partial^2 W_m}{\partial u^2}$ are the same as the stiffness matrices of the purely electric problem and the purely mechanical problem. Then only two new terms $\frac{\partial^2 W_e}{\partial u^2}$ and $\frac{\partial^2 W_e}{\partial u \partial \phi}$ have to be computed. After long developments an analytical expression of the

tangent stiffness matrix is obtain using the following expressions:

$$\begin{cases} \delta_{u\phi} W_e = \int_{\Omega} (\nabla \delta \mathbf{u})^T \mathbf{F}_1 (\nabla \delta \phi) d\Omega \\ \delta_{uu} W_e = \int_{\Omega} (\nabla \delta \mathbf{u})^T \mathbf{F}_2 (\nabla \delta \mathbf{u}) d\Omega \end{cases}$$
(11)

where \mathbf{F}_1 and \mathbf{F}_2 are matrices depending on the electric field. Thanks to the tangent stiffness matrix a dynamical study may be performed on electro-mechanical problem.

3 Applications

This strong finite element formulation has been implemented on the commercial code OOFELIE(r). Both following applications have been realised using it.

3.1 Clamped-clamped beam micro-mirror

The micro-device studied here is a micro-bridge who may be used as an electrostatically actuated micro-mirror. The dimensions of the beam are the following: a length of 300 μ m, a thickness of 0.5 μ m and the gap between the electrodes is about 6 μ m. Due to the large aspect ratio, large displacement hypothesis has to be used. Three small electrodes are deposited on the substrate to actuate the beam. The length of these electrodes is about 60 μ m and a voltage V₁, V₂ and V₃ may be applied between one electrode and the beam (see Fig. 4).

Two different configurations will be considered: a voltage V_2 is applied to the centred electrode, then V_1 and V_3 will be set to a same predefined voltage. First a static study is performed to evaluate the static pull-in voltage. Then the dynamic behaviour of both configurations is studied when a voltage is suddenly applied between the electrodes.

3.1.1 Static pull-in voltage

When a voltage is applied between the membrane and the electrodes, electrostatic forces appear which force the



Fig. 4 Clamped-clamped beam micro-mirror



Fig. 5 Static pull-in voltage

membrane to bend. When the applied voltage increases, the electrostatic forces become dominant and for some limit value, the plates might stick together. The corresponding critical voltage is called the static pull-in voltage. It is one of the most important design parameters in this type of micro-systems. Indeed if the system achieved the pull-in state, the mirror may be destroyed.

Using a Riks–Crisfield algorithm the displacement of the centre of the beam may be computed when the voltage changes. The static pull-in voltage is estimated to 129.3 V for the centred electrode and to 141.6 V for the two electrodes as shown in Fig. 5. The displacement necessary to reach the static pull-in voltage is larger in the case of two electrodes than when there is only one.

3.1.2 Dynamic pull-in voltage

In this section, the transient dynamic response of the system is computed when the voltage is suddenly applied between the two electrodes i.e. when the time-history of the voltage corresponds to a step function as illustrated in Fig. 6.

To investigate the dynamic pull-in, several voltages V were considered and applied to the beam as a step in time, the system being at rest initially. Using the Newmark scheme [12] to time integrate the problem one obtains the results depicted in the phase diagram in Fig. 7 and Fig. 8.



Fig. 6 Step of voltage applied to the beam



Fig. 7 Phase diagram for the centred electrode

From the dynamic study of the electrostatically actuated beam behaviour, a new parameter is defined: the dynamic pull-in voltage. The definition of this voltage is the following ([13, 14]):

The dynamic pull-in voltage is defined as the voltage amplitude such that, when applied suddenly, it leads to the dynamical instability of the system.

The dynamic pull-in voltage in the case of a single centred electrode is about 114.8 V, 10.5% lower than the static pull-in voltage with the same hypothesis. For the second problem, the dynamic pull-in voltage is about 125.1V, i.e. 11.6% lower than the static one. So due to the dynamic effect the system may become unstable before reaching the static pull-in voltage.



Fig. 8 Phase diagram for the two electrodes



Fig. 9 OKO technology micro-mirror



Fig. 10 Dissymmetric Deformation



Fig. 11 Circular deformation

3.2 Circular micro-mirror

The second application example treated in this paper is the 37-channel Micromachined Deformable Mirror System [15]. This micro-mirror is fabricated by the company OKO technologies in cooperation with the Institute of Micro-electronic and Submicron Technology of the University of Delft. It consists in a circular deformable membrane with a diameter of 15 mm which is coated to form the mirror. 37 actuators are located under the membrane as shown in Fig. 9. The mirror shape is controlled by applying voltage between the membrane and some actuator electrodes. This micro-device is usually used for fast dynamic correction of low-order optical aberration such as defocus, astigmatism, coma, ... in laser telescopes, ophthalmology, displays and general imaging optics. So it is important to know exactly the shape of the mirror during its actuation.

A voltage of 100 V is applied to the electrodes. First one fourth of the electrodes are activated. The applied electric potential and the deformation are presented in Fig. 10. Then a ring of electrodes is activated and the results are shown in Fig. 11.

4 Conclusions

The method proposed in this paper is a strong coupled formulation to compute the electro-mechanical coupling. This method differs from the other strong coupling methods due to its thermodynamics energy origin. The computation of electrostatic force derives from this energy and a second derivative provides the analytical expression of the tangent stiffness matrix. These expressions may be implemented for different types of elements (and not only for the tetrahedral linear element as in ANSYS). The electrostatic forces may be computed with linear and quadratic elements and converge to a single solution (not as FEMLAB). This methodology has been implemented in the commercial software called *oofelie* and has been used to study the dynamic behaviour of micro-mirrors.

Acknowledgments The first author acknowledges the financial support of the Belgian National Fund for Scientific Research. Part of the work presented in this text is supported by the Communauté Française de Belgique – Direction Générale de la Recherche Scientifique in the framework Actions de Recherche Concertées (convention ARC 03/08-298). This paper has been written in memory of Yves Rubin and Hermann Metens both from Open Engineering without whom this paper could not have been achieved.

References

- Lee, W. S., Kwon, K. C., Kim, B. K., Cho, J. H., & Young, S. K. (2001). Frequency-shifting analysis of electrostatically tunable micro-mechanical actuator. *Journal of Modeling and Simulation* of Micro-systems, 2, 83–88.
- 2. ANSYS Training Manual: Introduction to ANSYS 5.7 for MEMS, USA ANSYS, 2001.
- 3. Rochus, V., Rixen, D. J., & Golinval, J. -C. (2003). Consistent vibration analysis of electrostatically coupled structures: application to microsystems. *Tenth international congress on sound and vibration*. Stockholm, Sweden.
- 4. Gyimesi, M., & Ostergaard, D. (1999). Electro-mechanical transducer for MEMS analysis in ANSYS. *International conference on modeling and simulation of microsystems MSM 1999*. Puerto Rico, USA: NSTI.
- Gyimesi, M., Ostergaard, D., & Avdeev, I. (2002). Triangle transducer for micro electro mechanical systems (MEMS) simulation in ANSYS finite element program. *International Conference on Modeling and Simulation of Microsystems - MSM* 2002 (pp. 380–383). Puerto Rico, USA: NSTI.
- 6. Avdeev, I. (2003). *New formulation for finite element modeling electrostatically driven microelectromechanical systems*. Phd. thesis, University of Pittsburg, United States of America.
- 7. FEMLAB (2004). Comsol, Femlab 3.1 Documentation.
- 8. Ikeda, T. (1996). Fundamentals of piezoelectricity, Oxford.

- 9. Piefort, V. (2001). *Finite element modeling of piezoelectric active structures*. Phd thesis, University of Brussels, Belgium.
- Landis, C. M. (2004). Energetically consistent boundary conditions for electromechanical fracture. *International Journal of Solids and Structures*, 41, 6291–6315.
- Rochus, V., Rixen, D. J., & Golinval, J. -C. (2006) Monolithical Modeling of Electro-Mechanical Coupling in Micro-Structures. *International Journal for Numerical Methods in Engineering*, 65/ 4, 461–493.
- 12. Geradin, M., & Rixen, D. J. (1997). Mechanical vibrations theory and application to structural dynamics (2nd ed.). Wiley.
- Elata, D., & Bamberger, H. (2006). On the dynamic pull-in of electrostatic actuators with multiple degrees of freedom and multiple voltage sources. *IEEE JMEMS*, 15(1), 131–140.
- Rochus, V., Rixen, D. J., & Golinval, J. -C. (2005). Electrostatic Coupling of MEMS Structures: Transient simulations and dynamic pull-in. *Nonlinear Analysis* 63, 1619–1633.
- OKO Technologies, 37 (19) Channel Micromachined Deformable Mirror System: Technical Passport.



Véronique Rochus received the degree of Physics Engineer from University of Liège, Belgium in 2001. In 2002 she achieved a master in nanostructures and microsystems for electronics and communications at University of Paris XI, France. Financed by the Belgian National Fund for Scientific Research, she passed her Ph.D. thesis on "Finite Element Modelling of Strong Electro-Mechanical Coupling in MEMS'' in 2006 at the University of Liège, Belgium. Currently she is continuing her

research on MEMS modelling (still financed by the Belgian National Fund for Scientific Research) by a postdoctoral stay at the TU Delft University, The Netherlands.



Jean-Claude Golinval is the Head of the Department of Aerospace and Mechanical Engineering at the University of Liège (Belgium). He has developed a strong expertise in the field of identification of linear and nonlinear structures using model updating methods. He graduated as electromechanical engineer from the University of Liège in 1980 and got his Ph.D. from the same university in 1989. He is the author and co-author of more than a hundred scientific publications in international journals and confer-

ence proceedings. He became Associate Professor in 1991, Professor in 1998 and Full Professor at the University in 2001. He was also the Chairman of the European COST Action F3 « *Structural Dynamics* » (European Co-operation in the Field of Scientific and Technical Research) during the period 1997–2001. This action concerned the fields of model updating, structural health monitoring and damage detection and identification of nonlinear mechanical systems. Since 2005, Professor Golinval is Associate Editor of the Journal of Vibration and Acoustics (ASME).





Christophe Louis is an Aerospace Civil Engineer from University of Liège (Belgium), Development Engineer at Open Engineering for Electrotechnics numerical simulation.

Carlos Mendez is Doctor in Physics from the Universidad Nacional de Cuyo (Argentina) on the subject of Transport phenomena in the fractional quantum Hall effect regime, Post doctoral position at the Pontificia Universidad Católica of Rio de Janeiro, Brazil (1997–1998), Post doctoral position at the CIMEC (International Center for Computer Methods in Engineering), Santa Fe, Argentina (2001–2003).

Igor Klapka is Teaching assistant of Professor Michel Geradin at Aerospace Laboratory of University of Liège, Belgium (1990– 2001), CTO of Open Engineering sa, scientific software editor in multiphysics 3D CAD-CAE.