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Finite difference and finite element modelling of an aquifer in Cretaceous chalk

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ABSTRACT : The studied aquifer is located in Cretaceous chalk near Liège (Belgium). It is recharged by infiltration through the overlying loess and conglomerate. Wells and pumping adits produce a daily flow of 60000 m^3. Mathematical models have been worked out to foresee the evolutions of the water table level and to get some additional informations about the drainance axis. The finite difference model NEWSAM has been used for the steady state study of the aquifer. Some values of transmissivity obtained by the calibration of this model have been further introduced into the finite element models. Two- and three- dimensional finite element models have been developped using an adapted LAGATHER thermic conductivity program in order to study unsteady conditions of the aquifer. These two models are useful and helpful to enable good location of the permeability variations due to geological features, and good estimations of hydrogeological balances. Conclusions are drawn from the comparison and the complementarity of the two models for this kind of study.

1 INTRODUCTION

Two kinds of models are considered. The finite difference model NEWSAM is applied in quasi 3D conditions and steady state. The finite element model, using some developments of the LAGAMINE code has been adapted for modelling a water table aquifer in transient conditions with a fixed mesh structure. This new computer code is tested on uniaxial and two-dimensional sections, and then on a 2D cross-section of the Hesbaye aquifer. The 3D modelling of the whole aquifer is completed with 2670 brick finite elements.

Hydrogeological basin of Hesbaye covers an area of 350 km^2 in the NW of Liège in Belgium (figure 1). This water table aquifer is mainly composed of Cretaceous chalks which are intensively fractured in some places.

The alimentation of Liège and its suburbs is provided by the 60000 m^3/day collected in the aquifer by 45 km of adits. The hydrographic network is sparse and we can distinguish several dry valleys due to the immediate infiltration of the water in the fractured chalks. In the other places and especially on the hills, infiltration occurs through the overlying loess and conglomerates.

More than 500 wells, boreholes and piezometers have given a lot of informations about the piezometry and the geological data.

The portion of the lithostratigraphic sequence of interest consists of :

1. Primary rocks composed of Visean limestones in contact with Silurian shales, siltstones and sandstones by a fault shift.

2. Secondary layers with a 20 to 100 m thickness essentially composed by Cretaceous layers
- the "smectite de Herve" (hardened calcareous clay) which is considered as the bottom of the aquifer
- the compact solid white chalk, also called lower chalk, in which the main water circulations are present in the zones where the fracturation is more important
- the Hard-Ground which is a thin layer (less than 1 m thick) of hardened chalk
- the grey chalk or upper chalk which are exposed to deconsolidation, alteration and karst phenomena
- the residual conglomerate which is the result of very large and superficial alteration phenomena.

3. Tertiary and Quaternary formations.
- the loess with a thickness of 2 to 20 m

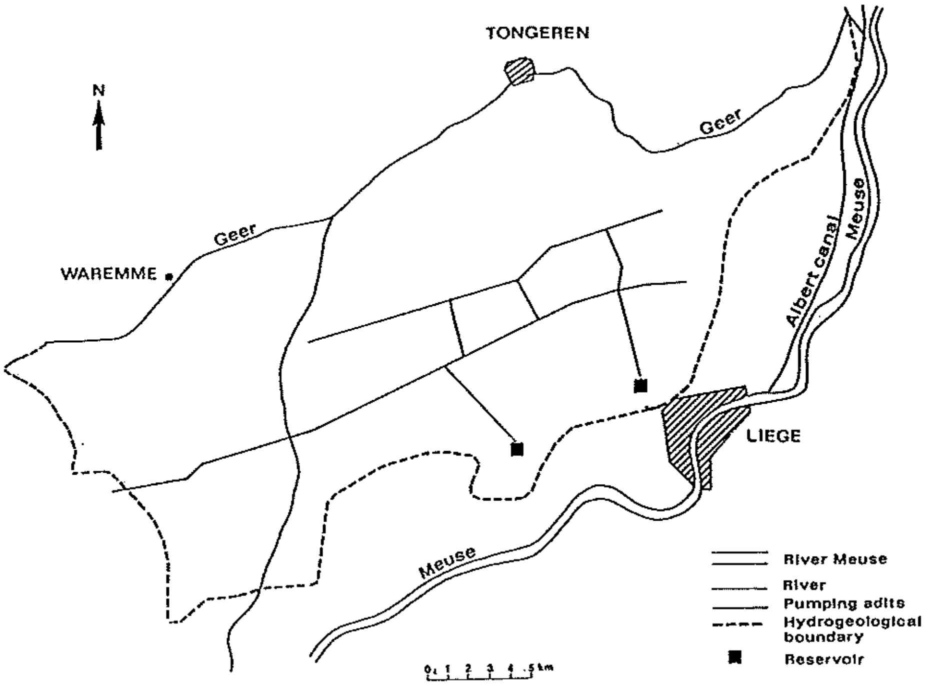

Figure 1 : Location map of Hesbaye.

- the recent alluvial and colluvial deposits especially in the valley of the River Geer

Hydrogeological parameters of the different geological units have been provided by 150 pumping tests. Between 1951 and 1984, piezometric maps have been drawn every year and piezometerhead evolution in function of the time has been recorded at different "check points".

2 FINITE DIFFERENCE MODEL

The finite difference computations are realized with an adapted NEWSAM program, solving the second-order partial differential equation of the flow in porous media. A quasi-three-dimensional idealization is obtained by the superimposition of 2 layers separated by an aquitard implicitely represented by vertical drainance coefficients.

Dirichlet conditions along the aquifer boundaries are settled to take into account the flow of the River Geer (in the North) and the lateral exchanges with the phreatic aquifer of the River Meuse (in the South-East). Initially, the pumping adits were also simulated by Dirichlet or Fourier conditions neglecting possible overpressure conditions in the adits. Later on, an explicit representation using an additional layer is prefered. The range of permeability coefficients of this special layer is from 1 to 20 m/sec, in order to compound with gradient problems (numerical stability, convergence) and with the reduction of the losses of energy.

The calibration procedure of the model is realized with a total of 4647 cells, for both years 1951 (low water level) and 1984 (high water level), assuming steady state and water table conditions.

The results appear satisfactory, seeing that :

- the main features of the water table are correctly reproduced (figure 2)
- the flows collected by the different sections of the pumping adits are correctly withdrawn, by way of reduced exchanges on the western, southern and eastern boundaries.
- the simulated global water-balances are in agreement with those calculated.

The main outputs of the model consist in :

- global groundwater balances, with quantification of the flows exchanged through the aquifer boundaries
- local groundwater balances and quantification of the laterally and

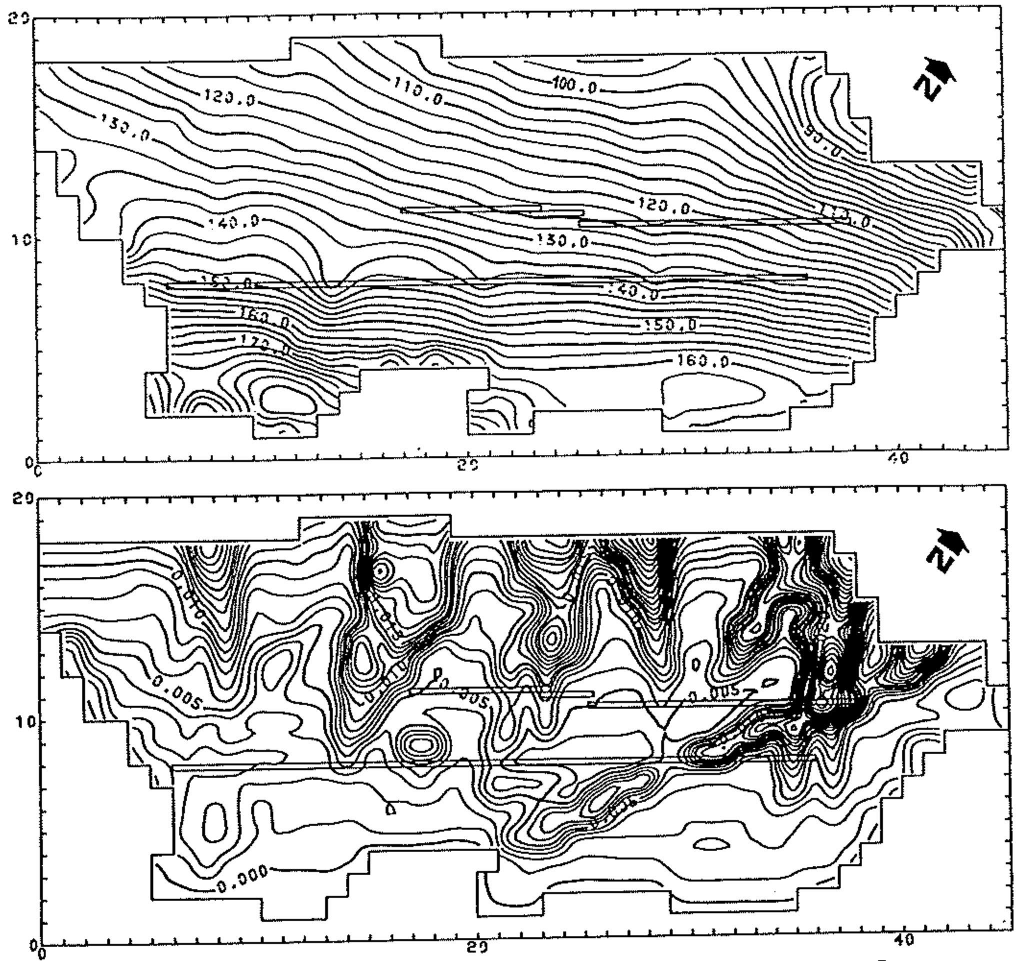

Figure 2 : Piezometerhead map and transmissivity field (equidistance 1.10^{-3} m^2/sec).

vertically discharges drained by the wells and the pumping adits
- complete fields of permeability and vertical drainance coefficients; these fields, in agreement with the results of pumping tests, show obviously the drainage produced by a dry valley system and the "screen-effect" locally induced by the presence of the Hard-Ground.

For example, the figure 2 shows the transmissivity field of the upper chalk, with its particular dry valley system.

Transmissivity values are from 8.10^{-4} to $3.7\ 10^{-2}$ m^2/sec for the upper chalk, from $1.1\ 10^{-4}$ to $6.6\ 10^{-3}$ m^2/sec for the lower chalk and the vertical drainance coefficient is from 9.10^{-6} to $1.6\ 10^{-4}$ sec^{-1} for the Hard-Ground.

This NEWSAM model is also used in unsteady state to interpret multilayer pumping tests and the results are compared with those of the BRUGGEMAN method.

3 FINITE ELEMENT MODELS

3.1 Modelling 2D cross-section

Two different discretizations have been tested for modelling the cross-section (figure 3). Calibration of this section was really uneasy because of the lack of precision in the estimation of the withdrawal data to be considered in the adit for a 1 meter thick section. Lateral exchanges are neglected. The main constatation made from the comparison between

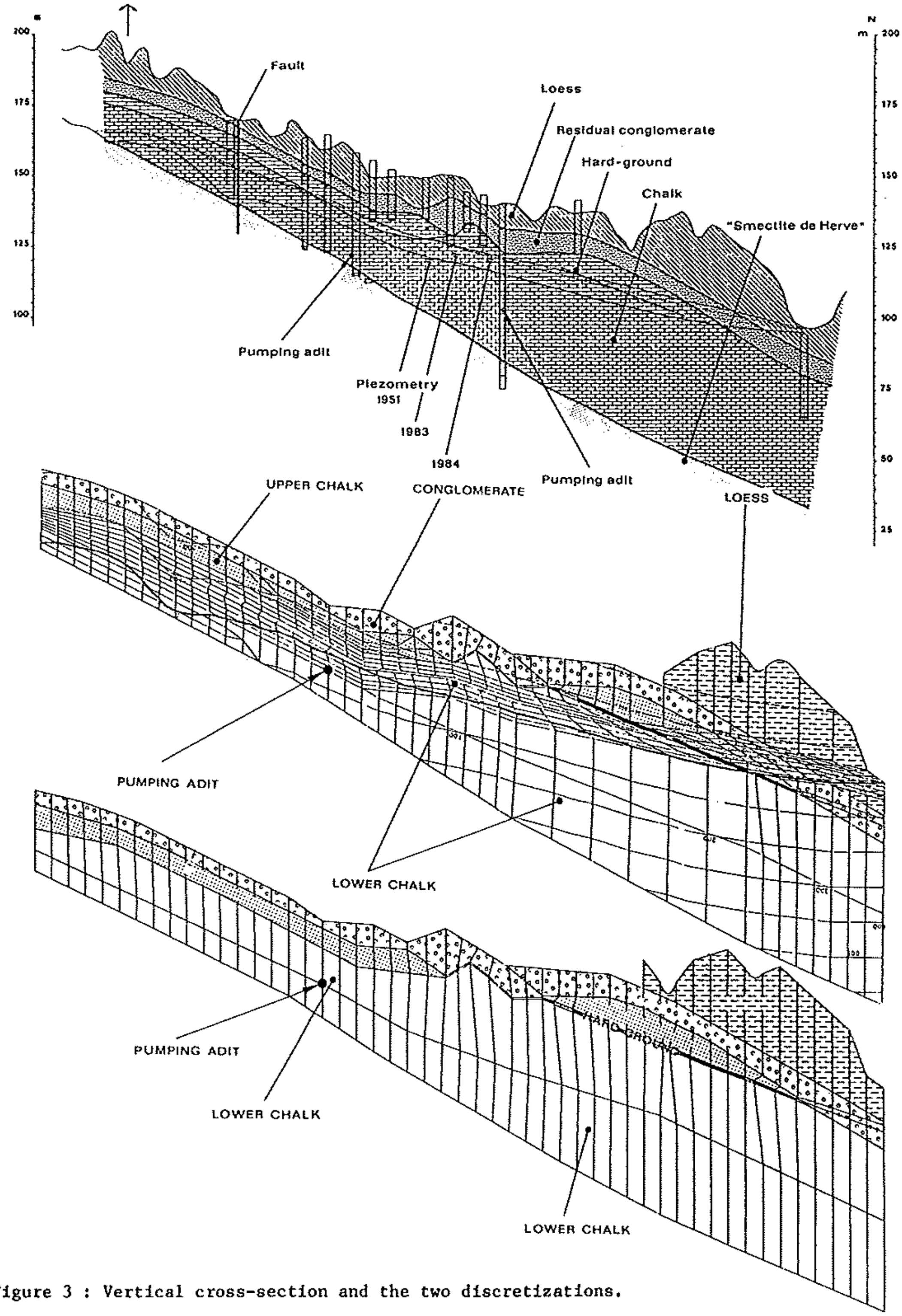

Figure 3 : Vertical cross-section and the two discretizations.

results of these two models (of the same cross-section) is that the small benefit in accuracy gained with fine mesh doesn't justify the use of so fine mesh in the structure for the 3D modelling.

3.2 3D Modelling

Finite elements used in this model are 8-nodes isoparametric bricks. Their edges are line segments and interpolating functions are linear and written in function of the three main coordinates. The change of variable is made using interpolation functions themselves which are satisfying the displacement compatibility and the geometrical compatibility of all the elements.

3D structure have to take into account a lot of geometrical data to obtain the final discretisation : pumping wells and adits, main geological faults, limits of geological units, hydrogeological basin limits ...

Five main strata of 534 elements have been distinguished : 6 levels of nodes. From one level to the next one, only Z coordinate changes. By this way, the running numbers of each node and of each element of the discretized structure can be computed from one station to the next one by simple addition of an increment. Complexity of the whole discretized structure is very high (figure 4).

Modelling of the pumping adits is realized with uniaxial "pipe elements" with infinite permeability in regard to the neighbouring elements. In the proximity of these pipe elements, the dimension of the mesh has been decreased in order to get accurate informations in this area from where a lot of water is withdrawn.

Prescribed heads (Dirichlet condition) are imposed on the northern boundary at the River Geer. The other boundaries are assumed to be impermeable (Neuman condition). As this aquifer is clearly a water table aquifer, the two hydrogeological parameters are the permeability and the effective porosity. Different "materials" have been defined corresponding essentialy to the different geological units.

Material 1 :
lower chalk $2.10^{-5} \leqslant K \leqslant 5.10^{-4}$ m/sec
$0.075 \leqslant S \leqslant 0.12$

Material 2 :
dry valley $2.10^{-4} \leqslant K \leqslant 2.10^{-3}$ m/sec
$0.075 \leqslant S \leqslant 0.20$

Material 3 :
Hard-Ground $5.10^{-6} \leqslant K \leqslant 2.10^{-4}$ m/sec
$0.075 \leqslant S \leqslant 0.10$

Material 4 :
upper chalk $2.10^{-4} \leqslant K \leqslant 1.10^{-4}$
$S \leqslant 0.15$

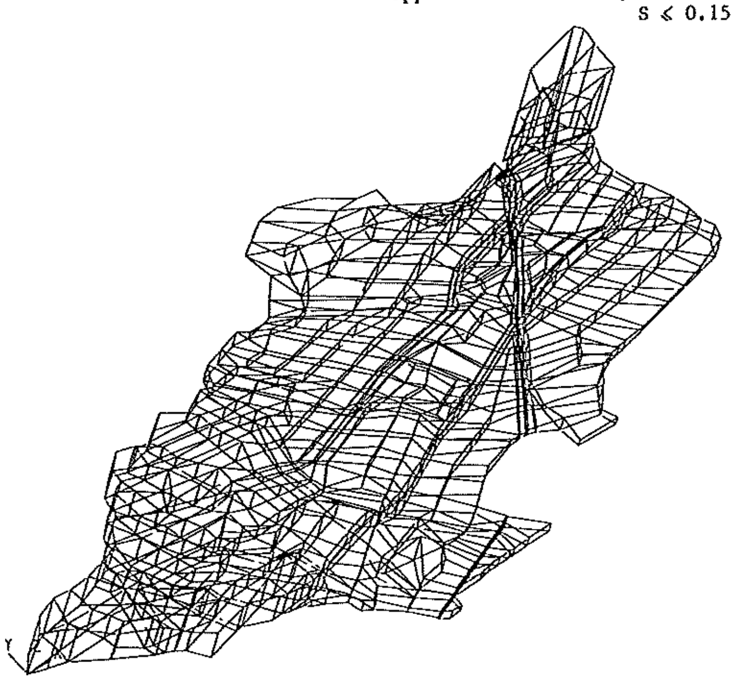

Figure 4 : The 4th layer of the discretized structure showing the complexity of the geometrical problem.

Material 5 :
alluvial deposits and/or residual conglomerate

$$3.10^{-5} \leqslant K \leqslant 1.10^{-3}$$
$$0.075 \leqslant S \leqslant 0.20$$

Material 6 :
conglomerate and loess

$$1.10^{-6} \leqslant K \leqslant 1.10^{-5}$$
$$0.075 \leqslant S \leqslant 0.10$$

Material 7 :
fault material

$$1.10^{-4} \leqslant K \leqslant 1.10^{-2}$$
$$0.075 \leqslant S \leqslant 0.15$$

The time step for the calibration procedure has been chosen to one year. The initial conditions are the piezometer-heads of the year 1951. The model has performed until 1984. Additional data are

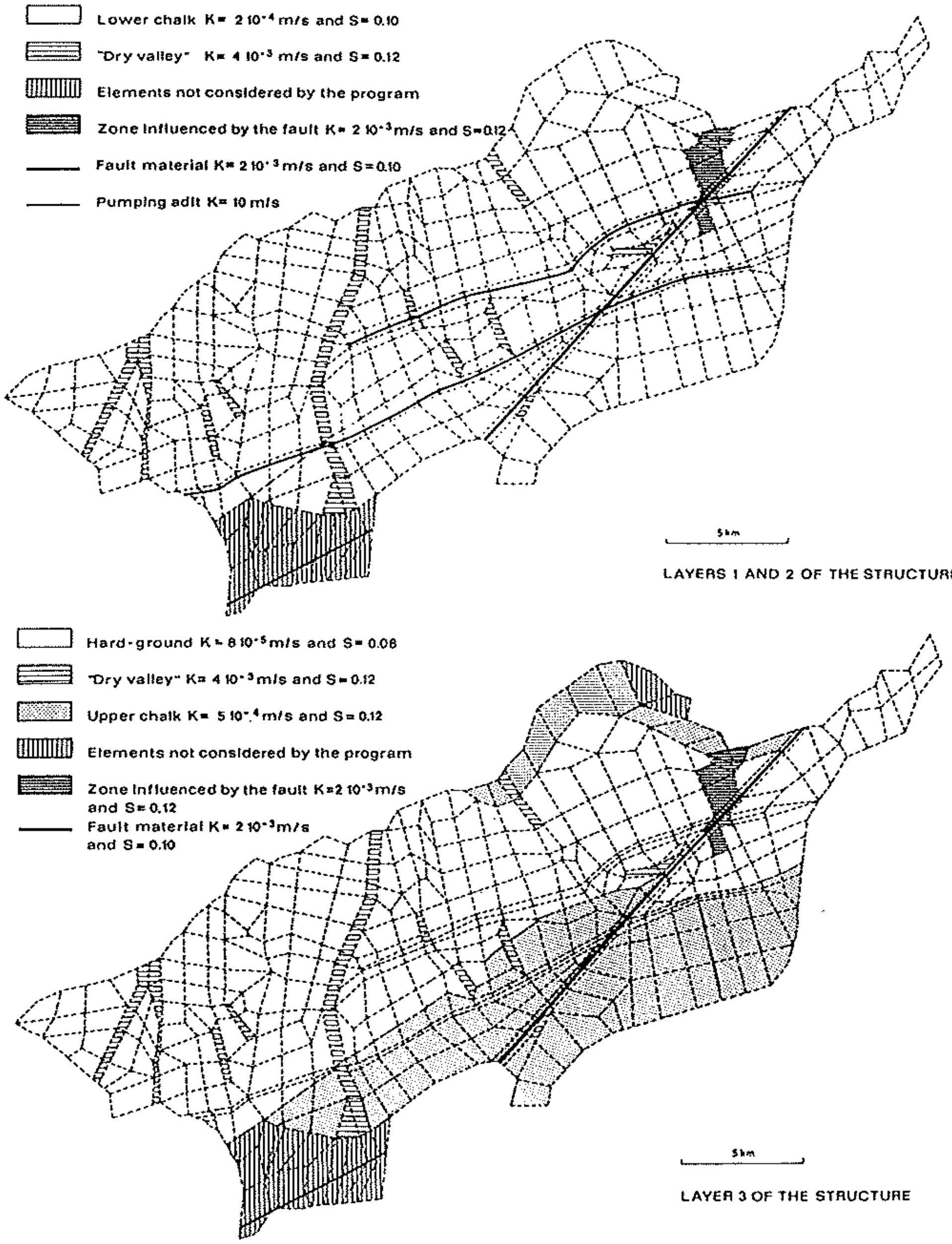

Figure 5 : Final repartition of the different materials in the structure for layers 1, 2 and 3.

the annual pumping rates in the adits and the values of effective infiltration during all these years.

The calibration procedure has been realized mainly in comparing measured piezometerheads and computed ones. This comparison is made with the piezometry maps, some cross-sections chosen in the structure and the "check points". During this procedure, values of the permeabilities and effective porosities have been setted up and some zones have been changed of "material". All these changes are made in the brackets of logical values of the parameters. At the end of the calibration, the computed flow of the River Geer is consistent with the reality and the computed piezometerheads are very close to the measurements. At this step, the final values of the materials parameters are deduced (figures 5 and 6). Figure 7 give an example of computed piezometry and flow maps of the aquifer in 1966.

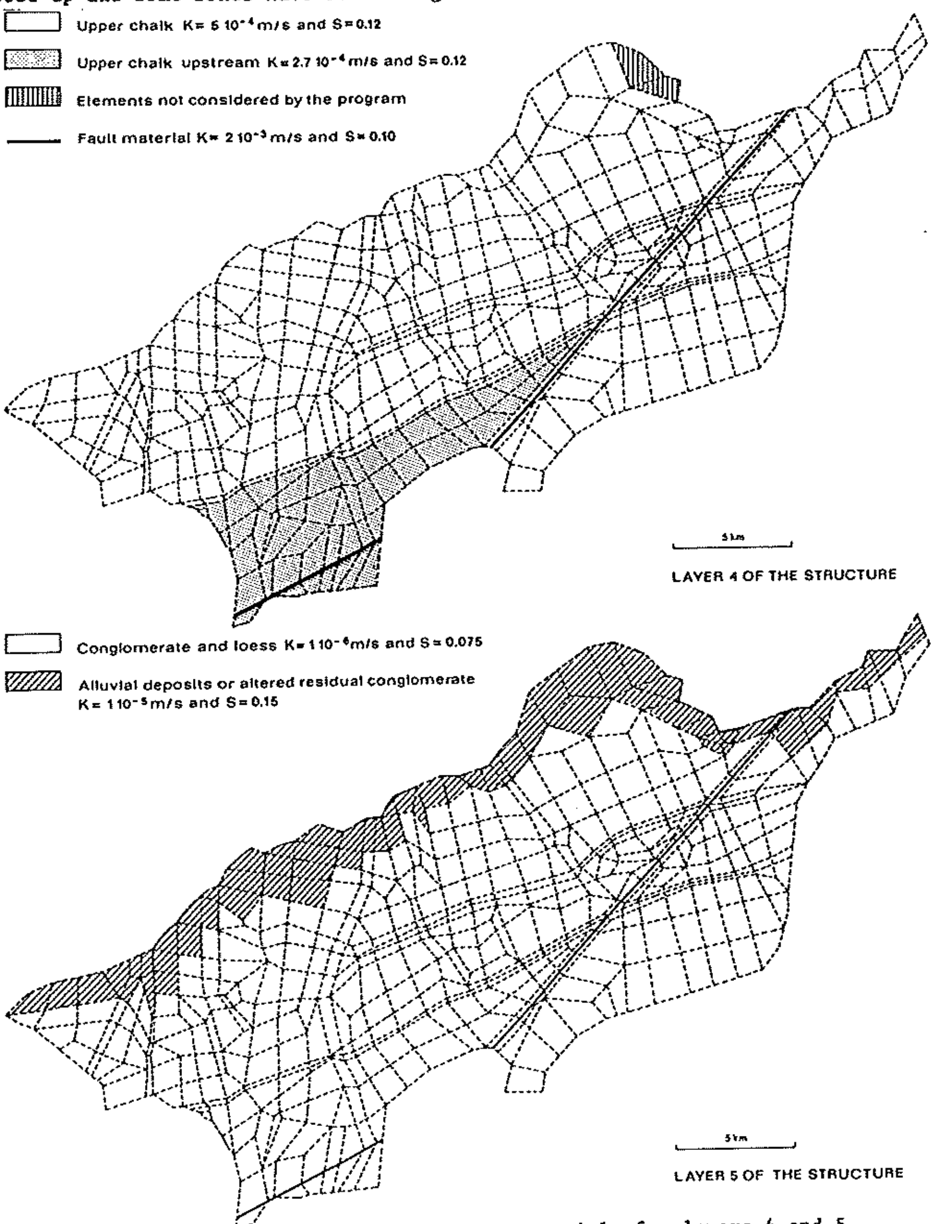

Figure 6 : Final repartition of the different materials for layers 4 and 5.

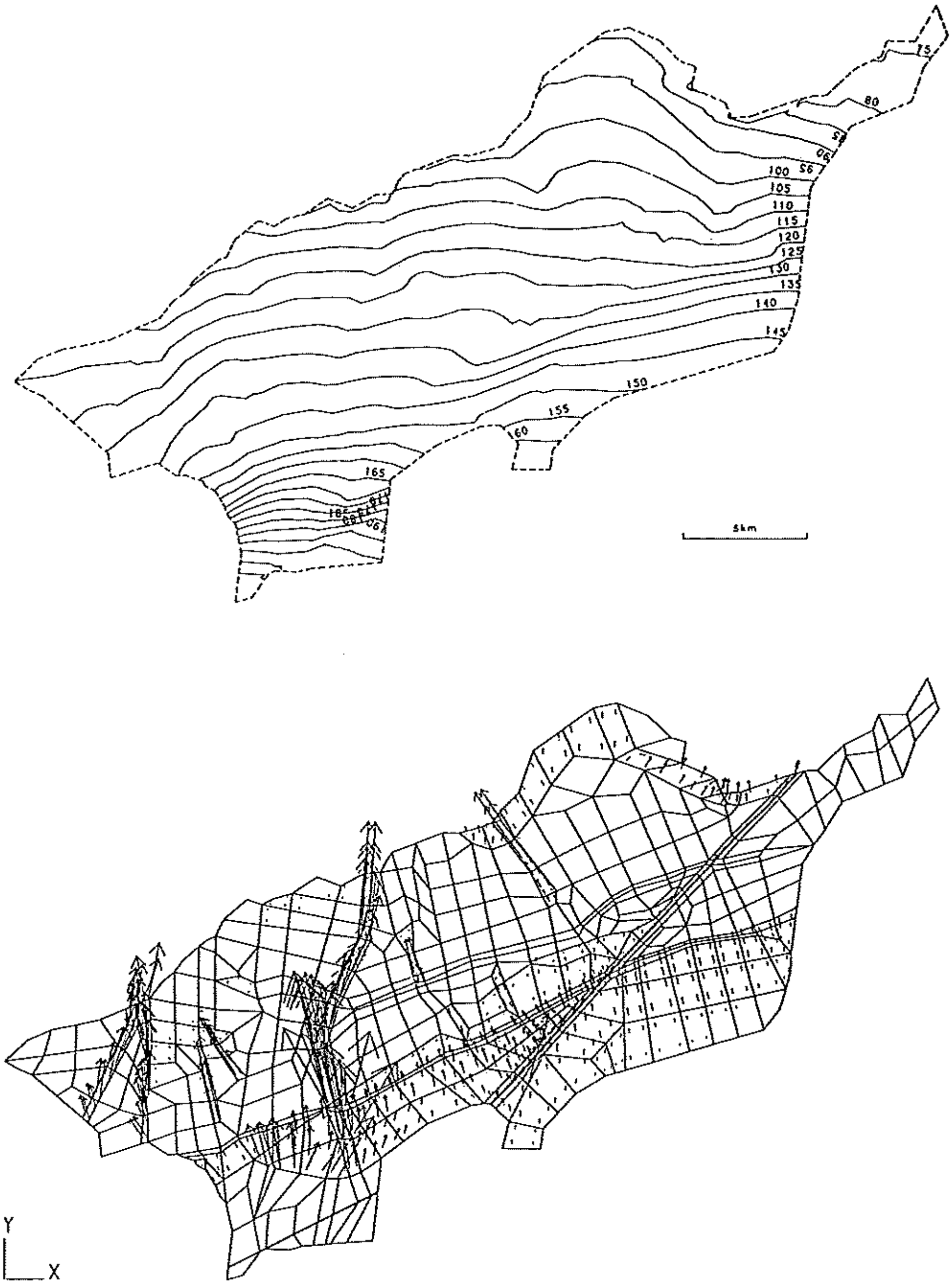

Figure 7 : Computed piezometry of the year 1966 and flow map in the 3rd layer at this time.

4 CONCLUSIONS AND FUTURE PROSPECTS

These two models allow us to start a "dynamic management" of this important aquifer. It's not our purpose to compare the approximations made in each model. In our case, we can emphasize on their good complementarity. The finite difference model in spite of big approximations about the geometry gives a good answer especially in the analysis of the transmissivity field in steady-state conditions. The finite element 3D model, more accurate in its conception gives very good results in transient conditions. The outputs of this LAGAMINE code are numerous and useful, giving piezometry maps, flow maps, cross-sections in the structure, evolution of the piezometerheads in function of the time at different "check points" ... The main problem of this model is the computation time; for a simulation from 1951 to 1984, it takes about 46 CPU hours on a Micro VAX II.

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