Fully implicit integration scheme for the GTN model

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The GTN model

The advanced Gurson-Tvergaard-Needleman model

Yield surface

$$F_p(\boldsymbol{\sigma}, \boldsymbol{X}, H_\alpha) = \frac{\widetilde{q}^2}{{\sigma_Y}^2} - 1 + 2q_1 f^* \cosh\left(-\frac{3q_2\widetilde{p}}{\kappa\sigma_Y}\right) - q_3 f^{*2} = 0$$

Porosity evolution

$$\dot{f} = \dot{f}_g + \dot{f}_n$$
$$\dot{f}_g = (1-f)\mathrm{tr}\left(\dot{\boldsymbol{\epsilon}}^P\right)$$
$$\dot{f}_n = \mathcal{A}(\epsilon_M^P)\dot{\epsilon}_M^P$$

Plasticity

- Isotropic and kinematic hardening.
- Hill anisotropy.

$$\widetilde{\boldsymbol{\sigma}} = -\widetilde{p}\boldsymbol{I} + 2\widetilde{q}(\mathbb{H}^{-1} : \boldsymbol{n}) \Longrightarrow \boldsymbol{n} = \frac{\mathbb{H} : \boldsymbol{\sigma}_{dev}}{2\widetilde{q}}$$

Coalescence

$$f^* = \begin{cases} f & \text{if } f < f_{cr} \\ f_{cr} + K_f(f - f_{cr}) & \text{if } f > f_{cr} \end{cases}$$

Shear extensions

Xue 2008

$$D = q_1 f^*; \quad \dot{D} = K_f\left(q_1\dot{f} + \dot{D}_{shear}\right)$$
$$\dot{D}_{shear} = k_g f^{1/3} g_\theta(J_2, J_3)\epsilon_{eq}\dot{\epsilon}_{eq}$$

Nahshon and Hutchinson 2008

$$\dot{f} = \dot{f}_n + \dot{f}_g + \dot{f}_s$$
$$\dot{f}_s = k_\omega f \omega(J_2, J_3)\frac{\widetilde{\boldsymbol{\sigma}}_{dev} : \dot{\boldsymbol{\epsilon}}^P}{\widetilde{q}}$$

Pure phenomenological approaches

Integration scheme

Equations set

$$F_p(\boldsymbol{\sigma}, \boldsymbol{X}, \boldsymbol{H_\alpha}) = 0$$
$$d\boldsymbol{\epsilon}^P = \lambda\frac{\partial F_p}{\partial\boldsymbol{\sigma}}$$
$$dH_\alpha = h_\alpha(d\boldsymbol{\epsilon}^P, \boldsymbol{\sigma}, H_\beta)$$

Backward Euler Aravas 1987

$$\phi^{t+\Delta t} = \phi^t + \Delta t\dot{\phi}^{t+\Delta t}$$

Ben Bettaieb et al. 2011

$$\Gamma(\boldsymbol{Y}) = 0$$
$$Y_i = \{\Delta\epsilon_p, \Delta\epsilon_q, n_1, n_2, n_3, n_4, n_5, H_\alpha\}$$
$$H_1 := \Delta\epsilon_M^P \quad H_2 := f \quad H_3 := D$$

N-R iteration

$$\Gamma_i + \sum_{j=1}^{10}\frac{\partial\Gamma_i}{\partial Y_j}dY_j = 0$$

Consistent tangent matrix

$$\boldsymbol{D} = \frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\epsilon}} \Longleftrightarrow \boldsymbol{K} : \partial\Delta\boldsymbol{\epsilon}^P = \boldsymbol{L} : \partial\boldsymbol{\sigma}$$

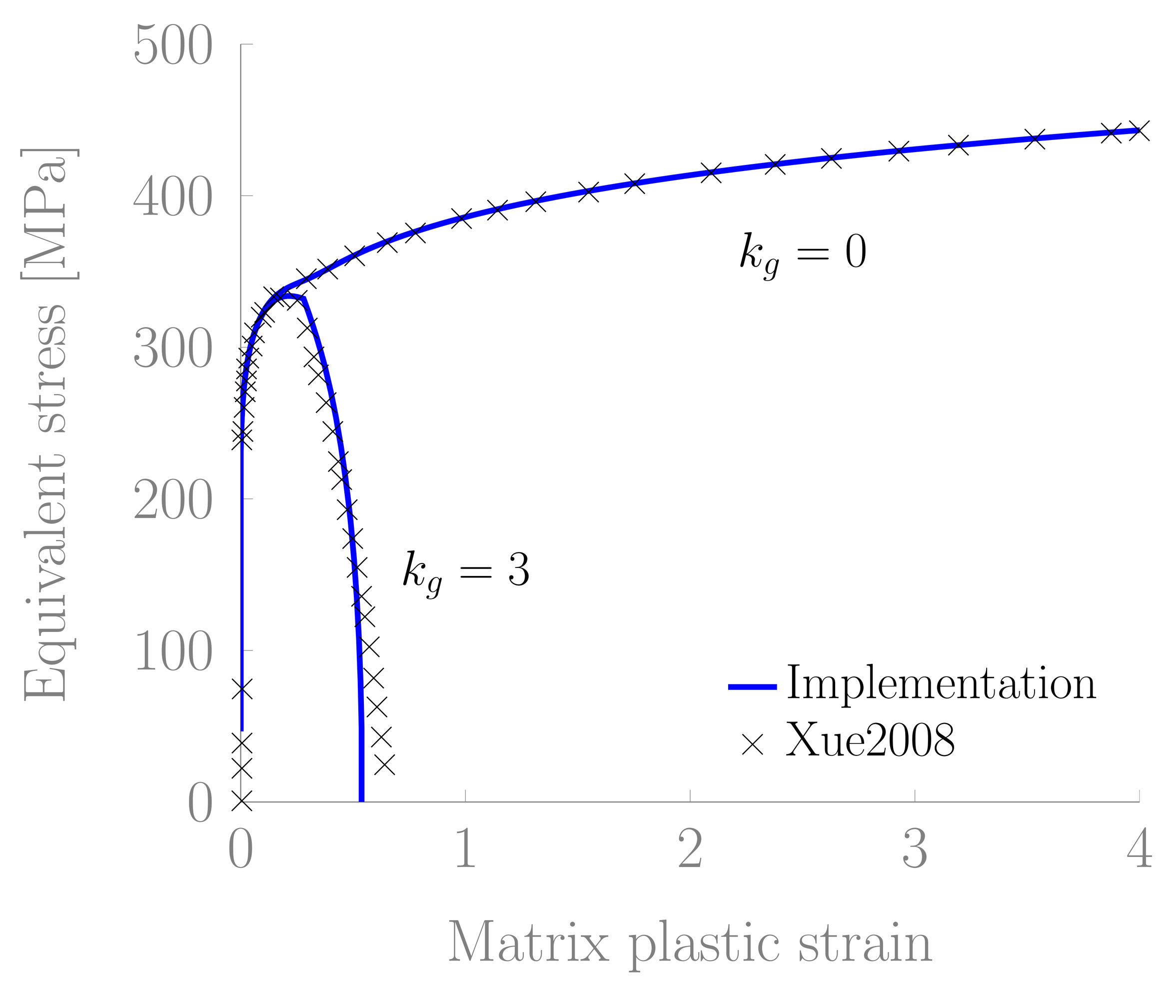
Kim and Gao 2005 approach

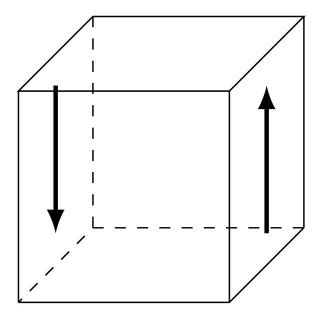
$$\boldsymbol{D} = \boldsymbol{C}^e - \boldsymbol{C}^e(\boldsymbol{K} + \boldsymbol{L}\boldsymbol{C}^e)^{-1}\boldsymbol{L}\boldsymbol{C}^e$$

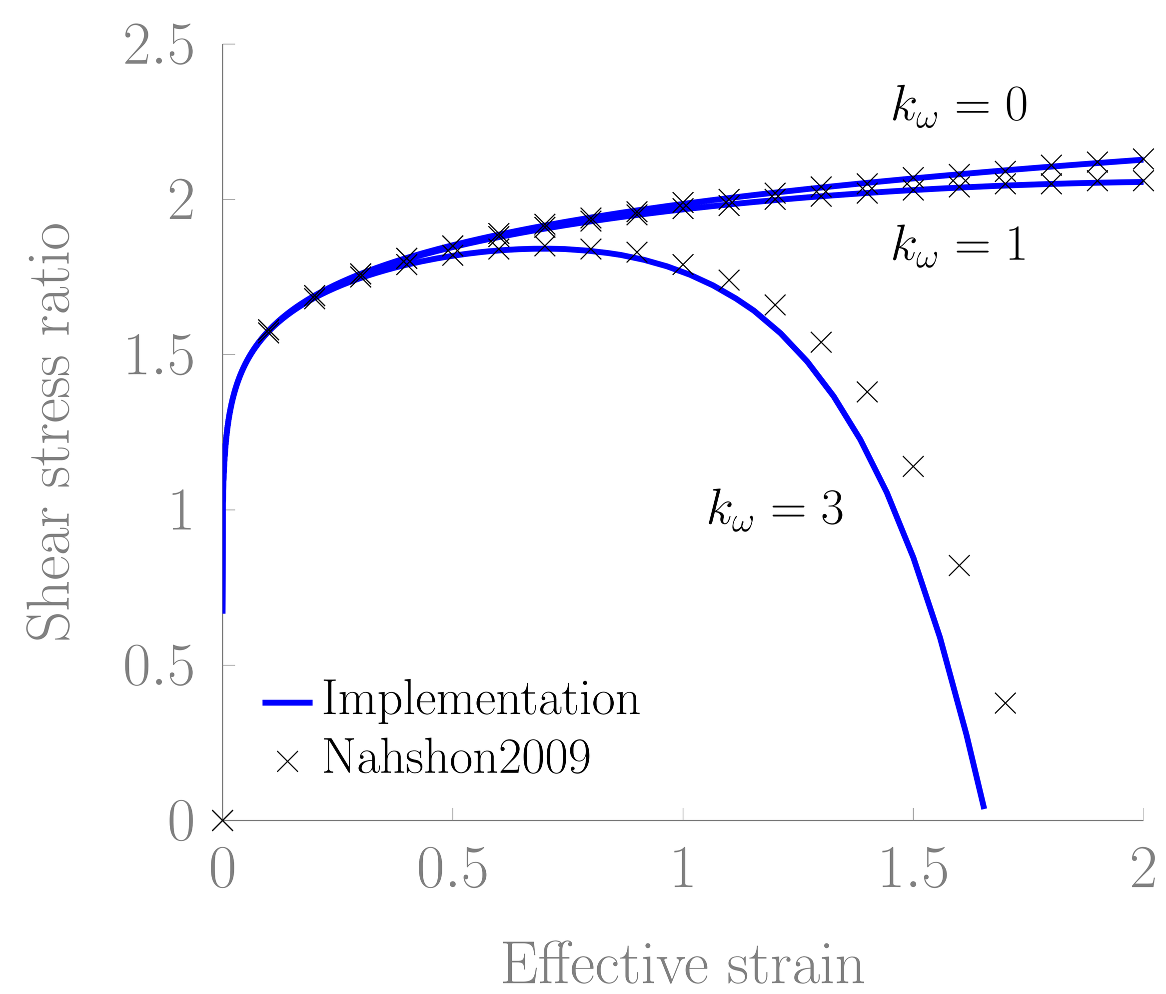
$$\frac{\partial F_p}{\partial\boldsymbol{\sigma}}, \frac{\partial F_p}{\partial\Delta\boldsymbol{\epsilon}^P}, \frac{\partial F_p}{\partial H_\beta}, \frac{\partial^2 F_p}{\partial\boldsymbol{\sigma}^2}, \frac{\partial^2 F_p}{\partial\boldsymbol{\sigma}\partial\Delta\boldsymbol{\epsilon}^P}, \ldots, \frac{\partial^2 F_p}{\partial\boldsymbol{\sigma}\partial H_\beta}, \frac{\partial h_\alpha}{\partial\boldsymbol{\sigma}}, \frac{\partial h_\alpha}{\partial\Delta\boldsymbol{\epsilon}^P}, \frac{\partial h_\alpha}{\partial H_\beta}$$

Extension to Kinematic hardening Ben Bettaieb et al. 2011

Validation: Monotonic simple shear







Comments

- Effective implementation of the GTN model.
- Shear extensions are still to be improved for high values of k_g and k_ω.
- Future study on damage anisotropy and/or shear/void coalescence.

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